13.012 Quiz #1

FALL 2003

PART A:

(1) @ sea floor there is no velocity normal to the ground $\vec{V}_{a} \cdot \hat{n} = 0$; $\frac{\partial \phi}{\partial n} = 0$ on $z = -H$

(2) Streakline

(3) pathline

(4) $\frac{\partial \Gamma}{\partial t} = 0$

(5) $\vec{\nabla} \cdot \vec{V} = 0$

$\boxed{c = -4}$

(6) $\omega = 2\pi f = 2.19\ rad/s \stackrel{?}{=} \sqrt{gk}$ $\quad k \stackrel{?}{=} 0.49 \frac{1}{m}$ $\quad kH \sim 4.8 > \pi$ ok deep

- waves are <u>deep</u>
- $\omega^2 = gk$
- $\lambda = 13.1 m$
- $V_p = \omega/k = 4.5 m/s$
- $V_g = \frac{1}{2} V_p = 2.3 m/s$

(7) $\omega = 5.6\ rad/s$ $\quad k = \omega^2/g$ $\quad k = 3.14/m$ $\quad kH = 3.51 > \pi$

still deep
$\omega^2 = gk$ $\quad \lambda = 2\pi/k$
$v_p = \omega/k$ $\quad V_g = \frac{1}{2} V_p$

(8) $p = -\rho g z = 1000 \cdot 10 \cdot 100 = 10^6\ N/m^2$

(9) $P_d = \frac{1}{2} \rho V^2$

$P_d = -\rho \frac{\partial \phi}{\partial t}$

(10) irrotational $\vec{\nabla} \times \vec{v} = 0$ [DOES] ✓

continuity $\vec{\nabla} \cdot \vec{v} = 0 \Rightarrow$ [DOES NOT] ✓

PART B:

"2" waves in central Atlantic

deep water

$\omega^2 = gk$

$k = 0.324\ ^1/_m$

$\lambda = 19.4\ m$

$\omega = 1.8\ ^{rad}/_s$

$a = 1\ m$

(a) $\Rightarrow 2a/\lambda = {}^2/_{19.4} = 0.103 < {}^1/_7$ (0.143)

$\therefore$ yes, linear assumption $\underline{\underline{\text{is}}}$ valid but for $\underline{a = 2a}$ $\frac{4}{19.4} = 0.2 > 0.14$

(b) $kH > \pi$ so @ $kH \sim \pi$ then too shallow $\therefore$ not $\underline{\text{linear}}$

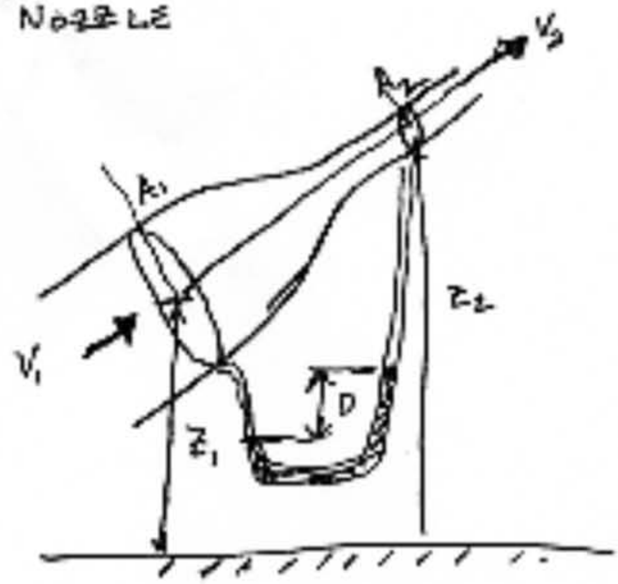
$kH = 0.324 H \sim \pi$

$$\boxed{H \sim 9.7\ m}$$

(c) $\bar{E} = \frac{1}{2} \rho g a^2 = 5000\ ^J/_{m^2}$

$\overline{KE} = \frac{1}{2} \bar{E} = {}^1/_4 \rho g a^2 = 2500\ ^J/_{m^2}$

"1" NOZZLE



<u>Flow Through Nozzle</u>

Continuity: $\rho_1 V_1 A_1 = \rho_1 V_2 A_2$

$V_1 A_1 = V_2 A_2$

$\therefore V_2 = V_1 \frac{A_1}{A_2}$

$\left(\frac{A_1}{A_2} = \frac{V_2}{V_1} \right)$

<u>Bernoulli's eqn gives ΔP</u>

$P_1 + \frac{1}{2}\rho_1 V_1^2 + \rho_1 g z_1 = P_2 + \frac{1}{2}\rho_1 V_2^2 + \rho_1 g z_2$

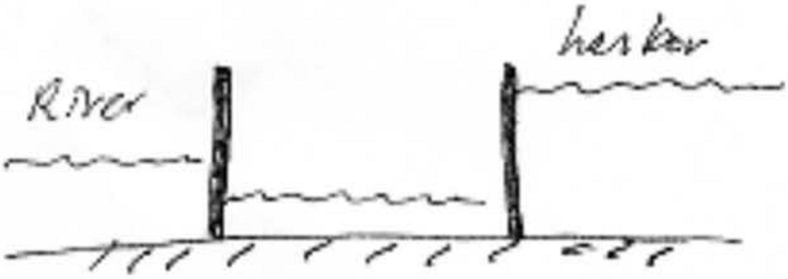
$(P_1 - P_2) = \frac{1}{2}\rho_1 (V_2^2 - V_1^2) + \rho_1 g (z_2 - z_1)$

<u>From manometer</u>

$$\boxed{(P_1 - P_2) = \rho_2 g D}$$

$$\rho_2 g D = \frac{1}{2}\rho_1 V_1^2 \left(\frac{A_1^2}{A_2^2} - 1 \right) + \rho_1 g (z_2 - z_1)$$

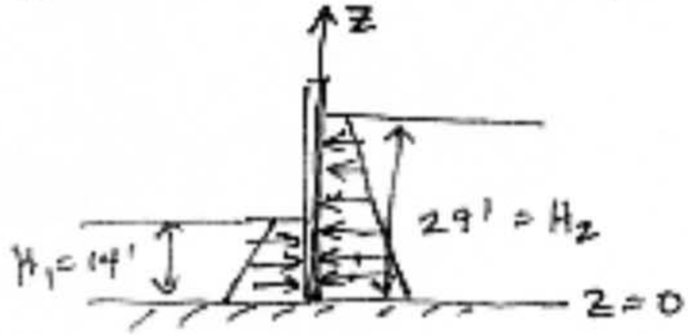
$$\boxed{\therefore \sqrt{\frac{\rho_2 g D + \rho_1 g (z_1 - z_2)}{\frac{1}{2}\rho_1 V_1^2} + 1} = \frac{A_1}{A_2}}$$



Max force is on the harbor gate →

① boat goes into Lock @ Low tide
from the river into the harbor

② tide Rises to high tide – now the
water in the chamber is 14' deep
harbor is 14' + 15' deep.



Pressure on wall (take bottom @ z = 0)

Left side

$P(z) = +\rho g(H_1 - z)$

$\text{Force} = L\int_0^{H_1} \rho g(H_1 - z)\,dz$

$= \rho g L\left(H_1 z - \frac{z^2}{2}\right)\Big|_0^{H_1}$

$F = \frac{1}{2}\rho g L H_1^2$

acts at $z = \frac{H_1}{3}$

R+ Side

$P(z) = \rho g(H_2 - z)$

$F = L\int_0^{H_2} \rho g(H_2 - z)\,dz$

$= \rho g L\left(H_2 z - \frac{z^2}{2}\right)\Big|_0^{H_2}$

$= \frac{1}{2}\rho g L H_2^2$

@ $z = H_2/3$

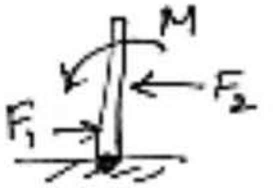
Resulting Force acts to Left:

$$\therefore F_R = (F_2 - F_1) = \frac{1}{2}\rho g L (H_2^2 - H_1^2) \left(\frac{kg\,m}{s^2}\right)$$

where $L = 22' \approx 6.7m$
$H_1 = 14' \approx 4.27m$
$H_2 = 29' \approx 8.84m$

$\boxed{\therefore F_R = 2007\ kN}$

Note there is a moment on the gate as well!



Total Force Acts to the left. There is a force F_1 acting to the right at $\bar{z}_1 = \frac{14'}{3}$ and Force F_2 acting to left @ $\bar{z}_2 = \frac{29'}{3}$.

These forces supply a moment to the gate as shown above.

$F_R \cdot \bar{z} = F_2 \cdot \bar{z}_2 - F_1 \bar{z}_1 \Rightarrow \boxed{\bar{z} = 3.4\,m \text{ from bottom}}$

The forces act at the center of the gate from side to side.

⇒ Design-wise any of the 3 will work.

- Design A will Require the most torque to move the gate as the force on each door in B is ½ of one whole gate in A. B requires Less space for doors to swing through.
- Design B requires 4 motors could be expensive though hydraulic system would not be too much different than in A
- Design C is easiest to open but hardest to seal