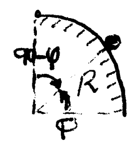
Session 2

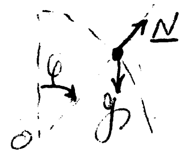
Example (1)

$\downarrow g$



Question: angle of departure
Point of departure is the point where the reaction force acting on particle becomes zero.

FBD



Linear momentum principle $\quad \dot{p} = F$

From geometry $\quad \underline{r} = (\cos\varphi\, \underline{e}_1 + \sin\varphi\, \underline{e}_2) R$

$$\underline{v} = \dot{\underline{r}} = (-\sin\varphi\,\dot{\varphi}\,\underline{e}_1 + \cos\varphi\,\dot{\varphi}\,\underline{e}_2) R$$

$$\underline{a} = \ddot{\underline{r}} = \{(-\cos\varphi\,\dot{\varphi}^2 - \sin\varphi\,\ddot{\varphi})\,\underline{e}_1 + (-\sin\varphi\,\dot{\varphi}^2 + \cos\varphi\,\ddot{\varphi})\,\underline{e}_2\} R$$

$$\Rightarrow \dot{p} = m\dot{\underline{v}} = m\underline{a} = mR(-\cos\varphi\,\dot{\varphi}^2 - \sin\varphi\,\ddot{\varphi})\,\underline{e}_1 + mR(-\sin\varphi\,\dot{\varphi}^2 + \cos\varphi\,\ddot{\varphi})\,\underline{e}_2$$

$$= \underline{N} + m\underline{g}$$

Project in the direction of $\underline{e}_N$
multiply by $\quad \underline{e}_N = \cos\varphi\,\underline{e}_1 + \sin\varphi\,\underline{e}_2$

$$-mR\dot{\varphi}^2 = N - mg\sin\varphi$$

At the point of departure $N = 0$

$$\Rightarrow R\dot{\varphi}_*^2 = g\sin\varphi_*$$

To obtain another equation for $(\varphi_*, \dot{\varphi}_*)$, use work-Energy principle

$$W_{12} = T_2 - T_1$$

$$W_{12} = \int_1^2 \underline{F}\, d\underline{r} = \int_1^2 \cancel{\underline{N}\, d\underline{r}}^{\,0} + \int_1^2 m\underline{g}\, d\underline{r}$$

$$\underline{F} = -\nabla V$$

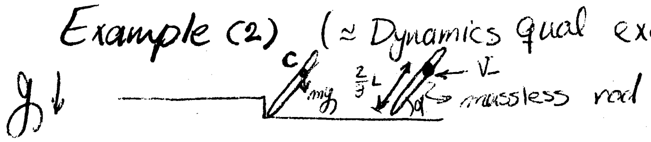
Since N doesn't do any work ($\underline{N} \perp \underline{v}$) $\Rightarrow$ System is Conservative $\quad T + V = \text{Const}$

$$0 + mgR = \frac{1}{2} m|v|^2 + mgR\sin\varphi$$

$$|\underline{v}| = \left|\frac{d}{dt}\left(R\left(\frac{\pi}{2} - \varphi\right)\right)\right| = R^2\dot{\varphi}^2$$

$$\Rightarrow \frac{1}{2}R^2\dot{\varphi}_*^2 = gR(1 - \sin\varphi_*) \quad \xrightarrow{\text{both}} \sin\varphi^* = \frac{2}{3} \Rightarrow \varphi^* = 41.81^\circ$$

Example (2) ($\simeq$ Dynamics qual exam, Spring 2004)



upon Collison rod pivots at point B
Question: minimum V for which rod tips over

part 1: Collision itself $(t_- \to t_+)$

Forces at B are unknown ⇒ Use angular momentum principle w.r.t B

$$\dot{H}_B + \cancel{v_B \times P}^{\,0} - M_B = r_{BC} \times (mg)$$

Integrate from t_- to t_+

$$H_B(t^+) - H_B(t^-) = \int_{t_-}^{t_+} r_{BC} \times (mg)\, dt \underset{t_+}{\overset{t_-}{\to}} 0$$

$$\Rightarrow H_B(t_-) \underset{t_+}{\overset{t_-}{\to}} H_B(t_+)$$

⇒ H_B is Conserved during Collision

$$H_B(t_-) = r_{BC} \times P(t_-) = r_{BC} \times (m v_-) = \left(\frac{2L}{3}\cos\alpha\right) m v_- \, e_3$$

$$H_B(t_+) = r_{BC} \times P(t_+) = r_{BC} \times (m v_+) = \frac{2L}{3} m v_+ \, e_3$$

$$\Rightarrow v_+ = v_- \cos\alpha$$

Part 2: Rotation

Work-Energy principle $W_{12} = T_2 - T_1$

$$W_{12} = W_{12}^{gravity} + \underset{0}{W_{12}^{reaction}}$$

(B does not move)

⇒ motion is Conservative $W_{12} = V_1 - V_2$

$T + V = Const$

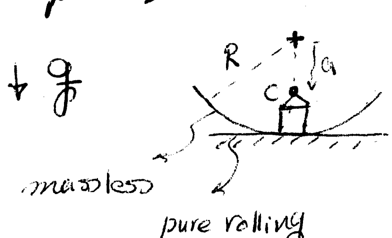
critical case

$$\frac{1}{2} m v_+^2 + mg\frac{2L}{3}\sin\alpha = 0 + mg\frac{2L}{3}$$

$$v_+^2 = \frac{4L}{3} g (1 - \sin\alpha)$$

$$v_- = \sqrt{\frac{4Lg}{3}\left(\frac{1}{\sin^2\alpha} - \frac{1}{\cos\alpha}\right)}$$

Example (3)



Question: equations of motion
- Constraint force
- frequency of small oscillations