MASSACHUSETTS INSTITUTE OF TECHNOLOGY
DEPARTMENT OF MECHANICAL ENGINEERING
# 2.06 Fluid Dynamics

**PROBLEM SET #2, Spring Term 2013**
**Issued: Thursday, April 11, 2013** **Due: Thursday, April 18, 2013, 1:05 PM**

**Objective:** The goal of this Problem Set is to utilize concepts of fluid hydrostatics, pressure forces, buoyancy forces and surface tension in fluid engineering systems.

**Problem 0:** Please read chapter 2 in White.

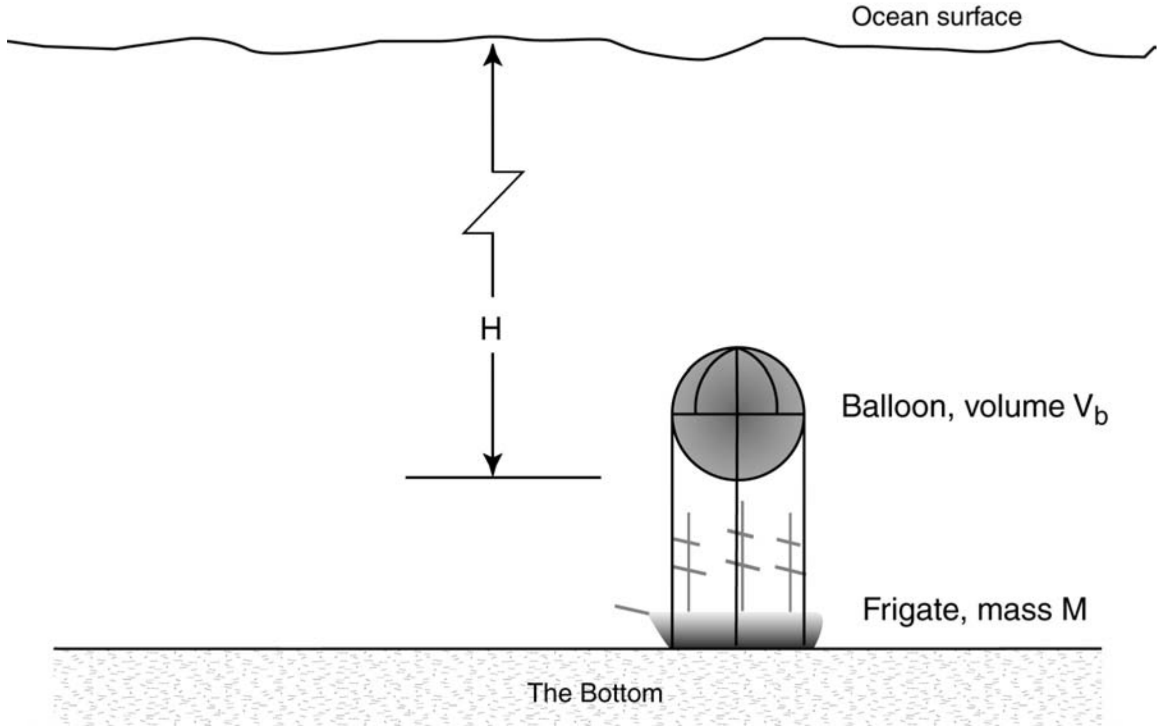
**Problem 1: Shorter Concept Questions**

i. A balloon filled with helium at a pressure of $P_{he} = 1.3 \times 10^5$ Pa is attached by a string to the bottom of the MBTA train at the Kendall station. The diameter of the balloon is d = 40 cm. The train now accelerates to the right with a constant acceleration of a = 5 m/s$^2$. The density of the air at ambient temperature in the train is measured to be 1.2 kg/m$^3$. You can neglect the mass of the balloon and that of the string.
   a. Please determine the horizontal and vertical forces acting on the balloon once the train has just accelerated and resulting air pressure gradients are established.
   b. What is the angle of inclination of the balloon with respect to the vertical at that time?
   c. Does the balloon lean to the left or to the right?

ii. Consider a heavy van submerged in water in a lake with a perfectly flat bottom. The driver's side door of the van is 2 m high and 1 m wide. The top edge of the door is 5 m below the water surface. Determine the net force acting on the door (normal to its surface) in two cases: (i) the van is well-sealed and contains air at atmospheric pressure ($10^5$ Pa), and ii), the van is filled with water. Assume the density of water is 1000 kg/m$^3$.

iii. Consider an air bubble of diameter 0.01 mm that is trapped at a depth of 1m in a water column. The density of water is 1000 kg/m$^3$. If the interfacial tension of the air-water interface is 0.072 N/m what is the pressure inside the air bubble. Will the pressure inside the bubble be greater or less than outside the bubble. The ambient pressure is 101.325 kPa. What is the maximum radius of a bubble that can be stable at this depth without breaking into smaller bubbles?

**Problem 2**



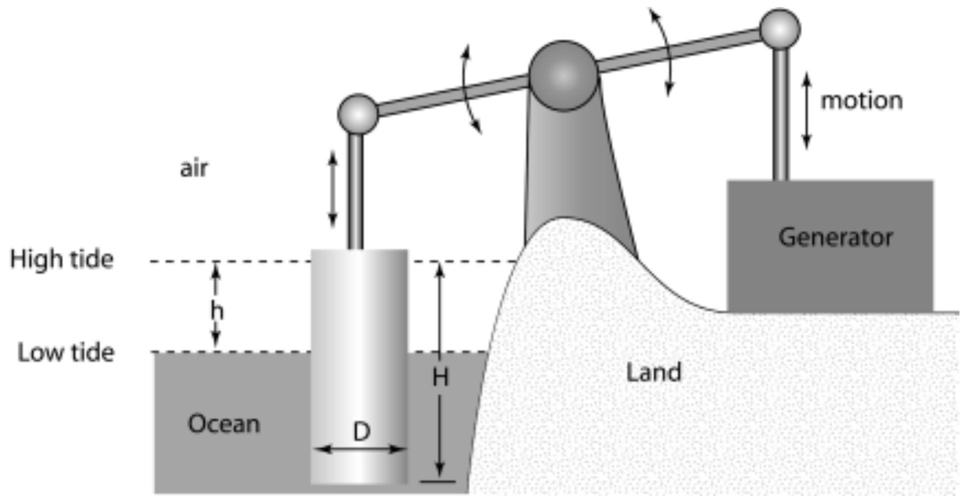

A salvage operation is underway to raise a sunken frigate of mass M from a depth H on the ocean floor. This is to be accomplished by attaching a floatation device (an inflatable balloon in this case) to the frigate and filling it with air pumped down from the surface. The density of the ocean water can be assumed constant and equal to $\rho_w$ and the average density of the materials of which the frigate is composed is $\rho_f$ ($>\rho_w$). The air in the balloon can be treated as an ideal gas (gas constant R) and is in good thermal contact with the ocean water at temperature T. The mass of the balloon is $M_b$ and the volume of the materials from which it is made is $V_{mb}$. You may assume that even when fully inflated, the vertical dimension of the balloon is small compared to H, and that the air pressure equals the water pressure at the same depth.

a) What is the volume of the balloon, $V_b$ at the instant the frigate begins to rise from the ocean floor? You may neglect the gravitational forces on the gas in the balloon.

b) Find an expression for the mass of air, $M_{air}$, that has been pumped into the balloon for the conditions in part a). (Hint: Ideal gas law is $PV = mRT$ where $m$ is the mass of an ideal gas)

c) Will it be necessary to add air to the balloon as it rises, in order to bring the frigate to the surface? Why or why not?

**Problem 3**

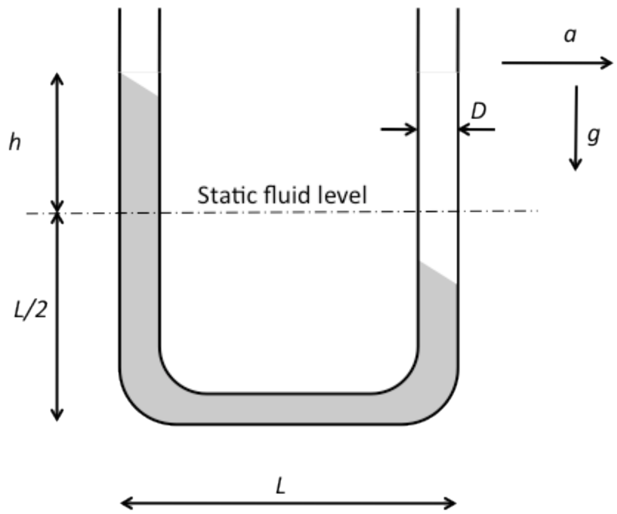



An inventor proposes to generate power from the tidal rise and fall of the ocean surface by linking a float to an electric generator. The proposed device is sketched in the figure above. A cylindrical tank of diameter D = 20 m and height H = 15 m is attached to a lever that causes the armature of an electric generator to rotate as the tank rises and falls with the level of the ocean. The density of ocean water is $\rho = 1030$ kg/m$^3$.

It is proposed that at low tide the float and lever will be locked at its equilibrium position where the buoyant force of the water is balanced by the gravity force on the float (neglecting the gravity force on the lever). At this point, the float is half submerged as shown in the sketch above. The tide rises h = 5 m over the next six hours and at high tide the device is unlocked. The extra force on the float pushes it upward actuating the generating station and causing the generator to generate electricity. The electric generator and the float have been matched so that the float rises slowly when the float is unlocked. (Why?) When the float has risen a distance h, the extra buoyant force will have returned to zero and no more power can be generated. At this point, the apparatus is locked in place until the tide has returned to its low tide level when the second half of the power generation cycle is completed.

a) Please calculate the force $F_0$ exerted by the float on the lever at high tide just before the lever is released.
b) Please derive an expression for the force F($z$) exerted by the float when it has risen slowly a distance $z$ from its initial position.
c) Calculate the energy E generated by the electric generator (assumed 100% efficient) during the process of the float rising through the distance h.
d) Derive an expression for the total energy $E_t$ produced in one tidal cycle.
e) What is the average daily power output of this plant? How does this compare to a 21MW power plant (a combined cycle, i.e. gas + steam, with co-generation, see http://cogen.mit.edu/ and http://cogen.mit.edu/ctg.cfm)
f) Studying how your answer in d) varies with the variables of the problem, provide two ways by which you could increase the power of the inventor's tide-based plant.

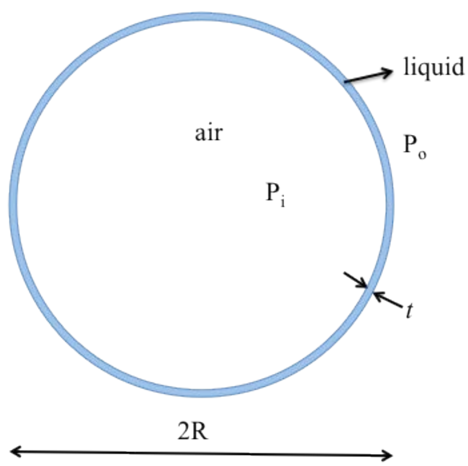
**Problem 4: U-tube Accelerometer (Adapted from F. White)**




(a) Here we will show how a U-tube filled with liquid can be used to make a cheap accelerometer. Consider a U-tube filled with a liquid density $\rho$ as shown in the above figure. The U-tube is being accelerated to the right at $a$ m/s$^2$. Express the acceleration $a$ in terms of the height $h$ the fluid reaches above static level in the left leg and other geometric parameters of the tube.

(b) Suppose the U-tube of the above figure is not translated but rotated about the left leg at an angular velocity $\omega$ rad/s. Express the angular velocity $\omega$ in terms of the height $h$ (above the static fluid level) of the right leg and other geometric parameters of the tube.

**Problem 5: Soap bubble**

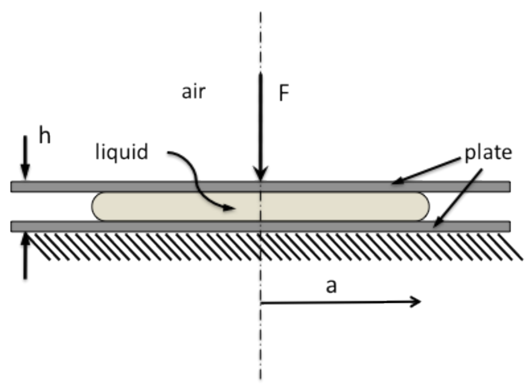



Consider a soap bubble of radius R and film thickness $t$ as shown above. The liquid-air surface tension is $\sigma$ and density of the liquid is $\rho$. What is the pressure $P_i$ inside the soap bubble if the outside pressure is $P_o$?

**Problem 6: (from White)**

A soap bubble of diameter $D_1$ coalesces with another bubble of diameter $D_2$ to form a single bubble of diameter $D_3$ with the same volume of air. The ambient pressure is $P_o$ and the liquid-air surface tension is $\sigma$. Assume an isothermal process (i.e., temperature is constant), derive an expression for finding $D_3$ as a function of $D_1$, $D_2$, $P_o$, and $\sigma$.

**Problem 7: Thin liquid film**




A drop of liquid of volume $V$ is squeezed between two parallel smooth surfaces. These surfaces are non-wetting to the liquid (for example consider mercury being squeezed between two glass slides). The liquid is squeezed until the thickness of the liquid layer $t$ is very small compared to the radius $a$ of the liquid. The three phase contact angle is $\theta$ and liquid-air surface tension is $\sigma$. Gravity effects can be neglected.

(a) Derive an expression for the force F required to hold the plates in position

(b) For case of complete non-wetting (i.e., $\theta = 180$ degrees), what would be a force required to hold a 2 $mm^3$ drop of mercury (surface tension 0.48 N/m) squeezed into a disc of radius $a$ =2cm?

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