Supplementary note on the contact lens example

Why we need to slide, not to pull, contact lens.
Here, we assume a contact lens as a circular disc, as shown in the figure.


We are interested in calculating the force, $F$, which enables to hold the lens at its height, $h$, from an eye.
First, we consider the free body diagram for the lens, shown right.


Force balance for the lens gives

$$
F+\left(P_{i}-P_{a}\right) \pi a^{2}=2 \pi a \sigma \sin \theta
$$

or

$$
\begin{equation*}
F=\left(P_{a}-P_{i}\right) \pi a^{2}+2 \pi a \sigma \sin \theta \tag{1}
\end{equation*}
$$

The pressure of the inside liquid $P_{i}$ is determined by the Young-Laplace equation.


$$
\begin{align*}
& P_{a}-P_{i}=\sigma\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right) \\
& R_{1}=\frac{h / 2}{\cos \theta} \& R_{2}=-a \\
& \text { Thus, } \quad P_{a}-P_{i}=\sigma\left(\frac{2 \cos \theta}{h}-\frac{1}{a}\right) \tag{2}
\end{align*}
$$

By combining the equation (1) and (2),

$$
\begin{aligned}
F & =\sigma\left(\frac{2 \cos \theta}{h}-\frac{1}{a}\right) \pi a^{2}+2 \pi a \sigma \sin \theta \\
& =\frac{2 \pi a^{2} \sigma}{h}\left[\cos \theta-\frac{h}{a}\left(\frac{1}{2}-\sin \theta\right)\right]
\end{aligned}
$$

For small $h\left(\frac{h}{a} \ll 1\right), \quad F \approx \frac{2 \pi a^{2} \sigma \cos \theta}{h}$
How large is $F$ ?
e.g., for $h=1 \mu \mathrm{~m} \& \theta=0$
$\frac{F}{\pi a^{2}}=\frac{2 \sigma}{h}=1.5 \mathrm{bar}$

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### 2.06 Fluid Dynamics

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