Supplementary note on surface tension
The following figure is from White (e7), Fig. 11 in page 33
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(a)

(b)

(c)

In this figure, the surface tension is notated by $\Upsilon$ which is $\sigma$ in the lecture.
(b) is the same figure for a drop discussed in the lecture

The net force balance is

$$
\begin{aligned}
& P_{i} A-P_{a} A=\sigma 2 \pi R \\
& \Delta P=P_{i}-P_{a}=\frac{2 \sigma}{R}
\end{aligned}
$$

(c) is a more a general case

The force by pressure differences $\Delta P\left(=P_{i}-P_{a}\right)$ is

$$
F_{\Delta P}=\left(P_{i}-P_{a}\right) d L_{1} d L_{2}
$$

Note, here the area, $d L_{1} d L_{2}$ is not exactly the projected area, perpendicular to the direction in which we consider the force balance. But, as we consider a very small element, the area would be very close to $d L_{1} d L_{2}$.

And, $d L_{1}=R_{1} 2 d \phi_{1}$ and $d L_{2}=R_{2} 2 d \phi_{2}$
Now,

$$
F_{\Delta P}=\left(P_{i}-P_{a}\right) d L_{1} d L_{2}=\left(P_{i}-P_{a}\right)\left(R_{1} 2 d \phi_{1}\right)\left(R_{2} 2 d \phi_{2}\right)
$$

The force by the surface tension is

$$
F_{\sigma}=2\left(\sigma d L_{1} \sin d \phi_{2}\right)+2\left(\sigma d L_{2} \sin d \phi_{1}\right)
$$



Since $d \phi_{1}$ is very small $\left(d \phi_{1} \ll 1\right), \sin d \phi_{1} \approx d \phi_{1}$
Similarly, $\sin d \phi_{2} \approx d \phi_{2}$
Then, the force by surface tension becomes

$$
F_{\sigma}=2\left(\sigma d L_{1} \sin d \phi_{2}\right)+2\left(\sigma d L_{2} \sin d \phi_{1}\right)=4 \sigma R_{1} d \phi_{1} d \phi_{2}+4 \sigma R_{2} d \phi_{2} d \phi_{1}
$$

Net force balance gives

$$
\begin{aligned}
& F_{\Delta P}=F_{\sigma} \\
& \left(P_{i}-P_{a}\right) 4 R_{1} R_{2} d \phi_{1} d \phi_{2}=4 \sigma d \phi_{1} d \phi_{2}\left(R_{1}+R_{2}\right) \\
& P_{i}-P_{a}=\sigma\left(\frac{1}{R_{1}}+\frac{1}{R_{2}}\right)
\end{aligned}
$$

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### 2.06 Fluid Dynamics

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