1.138J/2.062J/18.376J, WAVE PROPAGATION

Fall, 2006 MIT

Midterm, Oct 24-31, 2006

Please abide by the following rules of this exam strictly

- You can use handouts from this class, your own notes, homework and mathematical handbooks. Do not use any other references, printed or handwritten.
- If you have any question regarding the questions and the exam, ask me only . Do not ask others even just for clarification of the exam. In case the questions are of general interest, I will inform all of you by E-mail.

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1.(20%). A straight tube of radius a and infinite length $-\infty < x < \infty$ containing air communicates with a branch tube of equal radius. The branch has the same radius a and finite length $L \gg a$, and is closed at the end. A monochromatic incident sound wave of frequency ω approaches from $x \sim -\infty$. Find the reflected and transmitted waves in the main tube and the waves in the branch. Examine and explain the physical effects of different frequencies of branch length L. Ignore the possible three dimensionality at the junction and assume boundary conditions that make sense on the average.

2. (20%). Elastic wave guide: Consider the propagation of SH waves in a thick slab of elastic material: $-\infty < x, z < \infty, -b < y < b$. A monochromatic wave propagates in the plane of x, y. The displacement is strictly in the z direction. The surfaces $y = \pm b$ are stress-free. Find the dispersion relation of plane waves

$$u_z(x, y, t) = \Re(F(y)e^{ikx-i\omega t}) \tag{1}$$

Show that the dispersion relation has many branches corresponding to many modes with different $y-$dependencies. Make a sketch or plot of the dispersion relation and discuss the $y-$profiles of the first few modes.

3. (20%). Radiation of SH waves from a circular cylinder in an elastic surrounding: Consider waves radiated from an oscillating cylinder due to its out-of-plane motion :

$$u_z(a, \theta) = U \cos\theta, \quad r = a \tag{2}$$

Solve for the elastic wave field in $\infty > r > a, 0 < \theta < 2\pi$. Use known properties of Hankel functions to find the asymptotic wave form for $kr \gg 1$.

Note that the displacment is related to the (stream-function like) potential ψ by

$$u_z = \nabla^2 \psi \tag{3}$$

In polar coordinates, the two-dimensional Laplacian of F is

$$\nabla^2 F(r, \theta) = \frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial F}{\partial r}\right) + \frac{1}{r^2}\frac{\partial^2 F}{\partial \theta^2} \tag{4}$$

4. (20%). 2D loudspeaker on a wall: On an infinite plane wall $y = 0$, there is a long semi-circular cylindrical loudspaeaker of radius a. The surface of the cylinder executes sinusoidal motion radially:

$$\frac{\partial \phi}{\partial r} = Ae^{-i\omega t}, \quad r = a, \quad 0 < \theta < \pi \tag{5}$$

where A is a constant. The wall is perfectly rigid, so that there is no normal velocity

$$\frac{\partial \phi(r, \theta)}{\partial \theta} = 0, \quad r > a, \quad \theta = 0, \pi \tag{6}$$

Find the sound potential for all $r > a$. Find the total period-averaged power outflux across a semi circle at (i) along the cyulinder surface $r = a$, and (ii) along a very large semi circle in the far field $kr \gg 1$.

5. (20%). Consider two scattering problems for the same cylindrical cavity of some given cross-section. In both problems, the incident waves are plane sinusoidal waves of the same frequency and amplitude A_0. The two are distinguished by the incident angle. In problem 1 for ϕ_1, the incidence angle is θ_1; in problem 2 for ϕ_2, the incidence angle is θ_2.

Show by using Green's theorem the following reciprocity property of the scattered wave amplitude factor:

$$\mathcal{A}_1(\theta_2) = \mathcal{A}_2(\theta_1) \tag{7}$$

Remark: In your answers please discuss as much as possible physical implications, in addition to formulas, sketches and plots.