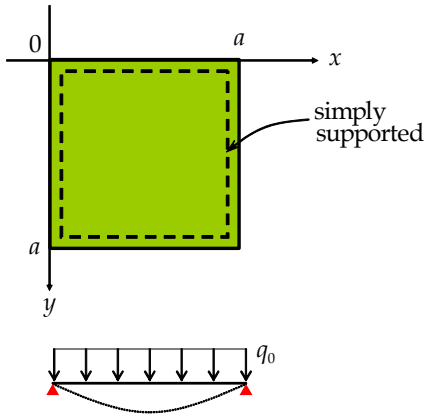
2.081J/16.230J Plates and Shells

Homework #2
Due date: class on Monday February 27

***PROBLEM* 1**

(a) Consider a simply supported square ($a \times a$) plate loaded by a uniform pressure q_0. Find an approximate solution, i.e. *the relation between the load intensity* q_0 *and the central deflection of the plate* w_0. Use the Raleigh-Ritz method ($\delta\Pi = 0$) and try sinusoidal shape function. Moreover, the Gaussian curvature vanishes if the edge of the plate are straight.

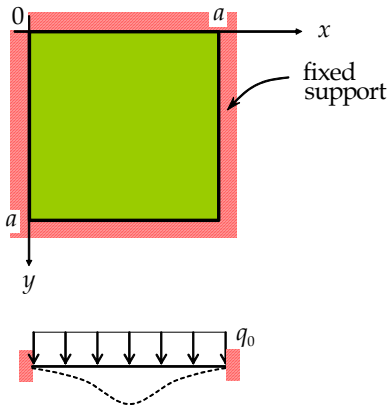


$$\Pi = \frac{D}{2}\int_S (w_{,xx} + w_{,yy})^2 \; dS - \int_S q \; w \; dS$$

$$w(x,y) = w_0 \sin\left(\frac{\pi \; x}{a}\right) \sin\left(\frac{\pi \; y}{a}\right)$$

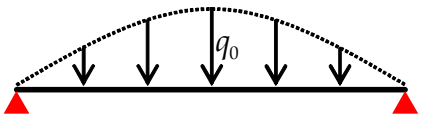
(b) [*Extra Credit*] Derive an approximate solution for the clamped plate. Try this shape:

$$w(x,y) = \frac{w_0}{4}\left[1 - \cos\left(\frac{2\pi \; x}{a}\right)\right]\left[1 - \cos\left(\frac{2\pi \; y}{a}\right)\right]$$



PROBLEM 2

(a) Find the location and magnitude of the maximum in-plane stress components $\sigma_{\alpha\beta}$ in the problem of simply supported square plate loaded by a sinusoidal pressures, solved in the class.



$$q(x,y) = q_0 \sin\left(\frac{\pi x}{a}\right)\sin\left(\frac{\pi y}{a}\right)$$

[*Hint*] The stress formula for plate is:

$$\sigma_{\alpha\beta} = \frac{z\ M_{\alpha\beta}}{h^3/12}$$

(b) Find the magnitude of the maximum out-of-plane average shear stress $(\sigma_{zx})_{av}$ and $(\sigma_{zy})_{av}$.

[*Hint*] The average stress formula for plate is:

$$(\sigma_{\alpha z})_{av} = \frac{Q_{\alpha z}}{h}$$

(c) Assuming that out-of-plane shear is distributed in a parabolic way over the plate thickness (similarly to beams), what is the maximum shear stress at the plate middle surface?