Massachusetts Institute of Technology
Department of Mechanical Engineering

2.12 Introduction to Robotics

Problem Set No.7

Out: November 9, 2005   Due: November 16, 2005

Problem 1

A two degree-of-freedom robot arm with one prismatic joint is shown below. The direction of the prismatic joint is perpendicular to the centerline of the first link. As shown in the figure, joint angle θ and distance z between the tip of the first link and the mass centroid of the second link are used as generalized coordinates. The first actuator fixed to the base link produces torque τ about the first joint, while the second actuator located at the tip of the first link generates linear force f acting on the second link. Using the parameters shown in the figure, answer the following questions.

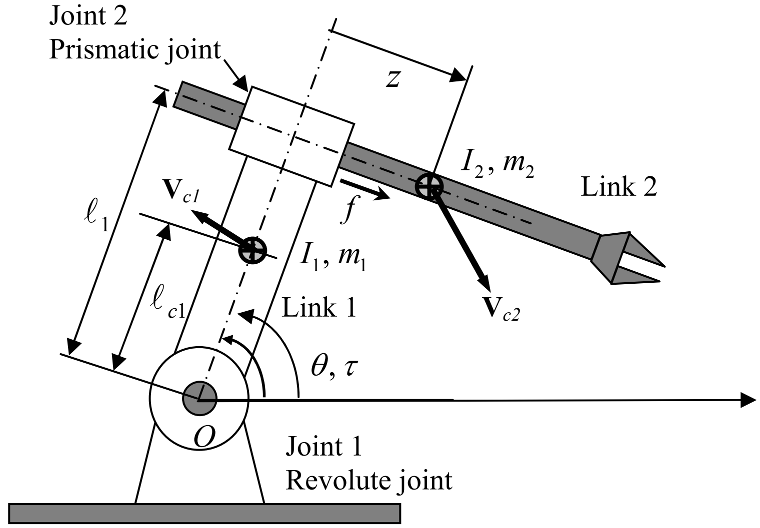

Figure 1 Mass properties and link parameters of a two d.o.f. arm

a). Obtain the moment of inertia reflected to Joint 1 when the second joint is fixed at $z = z_0$. At which arm configuration does the moment of inertia become minimal?
b). Obtain the centrifugal force acting on Link 2 when the first joint is rotating at a constant angular velocity $\dot{\theta}$? Also obtain the torque induced by the centrifugal force upon Joint 1, i.e. the joint torque τ needed for canceling out the centrifugal effect.
c). Obtain the Coriolis force acting on Link 2 when the first joint is rotating at a constant angular velocity $\dot{\theta}$ and the second joint is moving at a constant linear velocity $\dot{z}$. Also obtain the torque induced by the Coriolis force upon Joint 1.
d). Obtain the linear velocity vector of each mass centroid, $\mathbf{V}_{ci}$, as functions of generalized coordinates and their time derivatives.
e). Obtain the linear acceleration vector of each mass centroid, $\boldsymbol{a}_{ci}$.
f). Obtain Newton-Euler's equations of motion by drawing Free-body-diagrams of the individual links.
g). Eliminate constraint forces involved in the Newton-Euler equations, and obtain closed-form dynamic equations relating actuator torques, τ and f, to $\theta, \dot{\theta}, \ddot{\theta}$ and $z, \dot{z}, \ddot{z}$.

Problem 2

Figure 2 shows the schematic of a three degree-of-freedom rehabilitation bed/chair system. The seat is tilted with Actuator 1 fixed to the base frame. The back leaf and the footrest are driven together by Actuator 2 fixed to the seat. Note that the motor shaft of Actuator 2 is connected to a belt-pulley mechanism to move the footrest together with the back leaf. The headrest is moved with Actuator 3 fixed to the seat through another belt-pulley mechanism as shown in the figure. Figure 3 shows the kinematic structure and joint variables along with geometric and mass parameters of the individual links. Note that joint angles θ_2 and θ_3 are measured from the seat, while angle θ_1 is from the base frame.

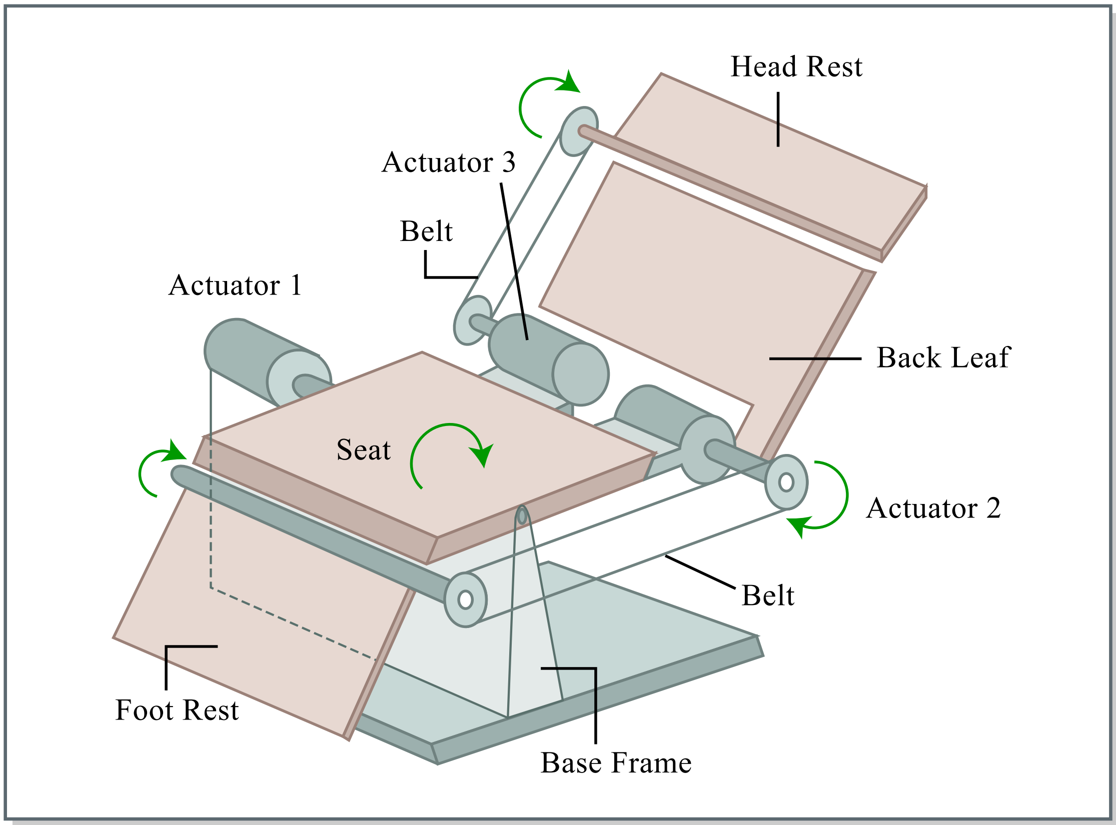

Figure 2 Powered rehabilitation bed-chair

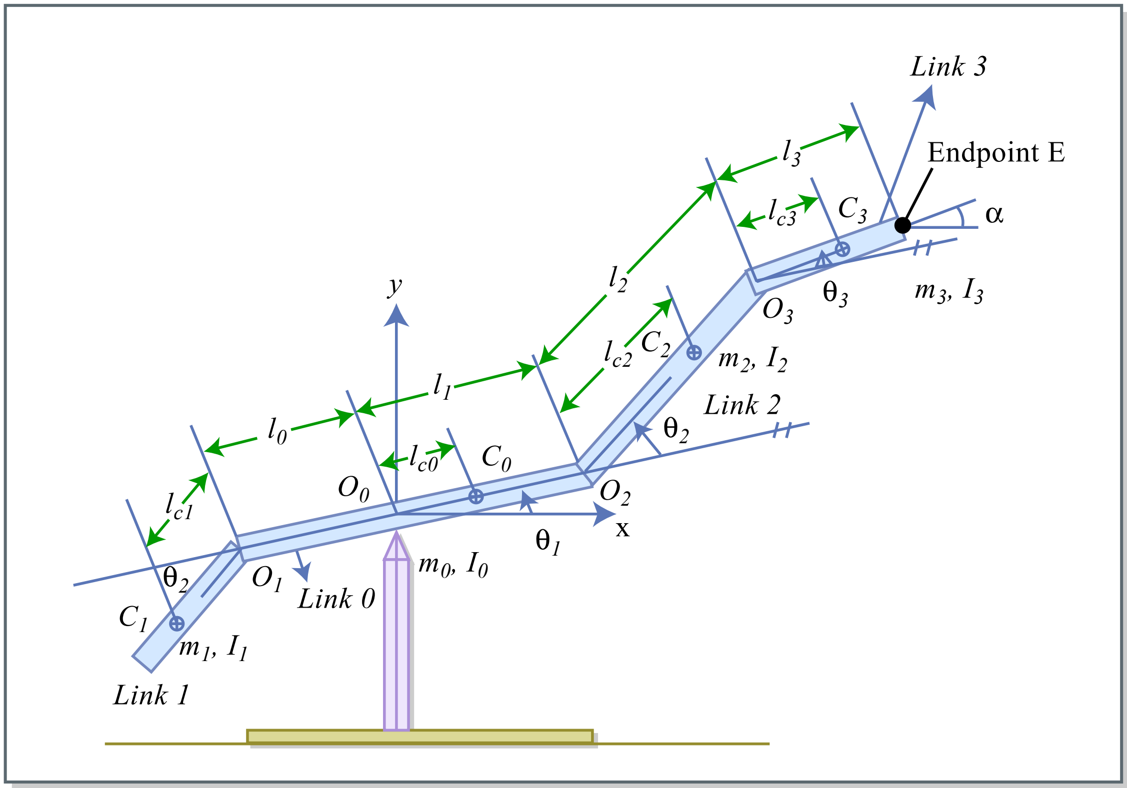

Figure 3 Mass parameters of the bed-chair system Figures by MIT OCW.

The closed-form equations of motion are in form:

$$
\begin{aligned}
\tau_1 &= H_{11}\ddot{\theta}_1 + H_{12}\ddot{\theta}_2 + H_{13}\ddot{\theta}_3 + \cdots \\
\tau_2 &= H_{21}\ddot{\theta}_1 + H_{22}\ddot{\theta}_2 + H_{23}\ddot{\theta}_3 + \cdots \\
\tau_3 &= H_{13}\ddot{\theta}_1 + H_{23}\ddot{\theta}_2 + H_{33}\ddot{\theta}_3 + \cdots
\end{aligned} \tag{1}
$$

where H_{ij} is the *i-j* element of the 3x3 inertia matrix $\mathbf{H} = \{H_{ij}\}$ associated with the joint coordinates. Answer the following questions.

a) Explain the physical meaning of the inertia matrix elements H_{11} and H_{22}, respectively. Show which part of the link inertia is associated with each of H_{11} and H_{22}. Be sure which type of motion, translation and/or rotation, is involved in H_{11} and H_{22}.

b) Based on the physical interpretation in part a), obtain H_{11} and H_{22}, respectively. Use the mass parameters shown in the figure: m_i is mass, I_i the moment of inertia at the centroid C_i; ℓ_{ci} the distance between *i*-th joint axis and the mass centroid of the link.