This quiz is open-book. There are four problems, and each is worth 25 points.

Problem 1 (25 points): This problem considers six transfer functions. These are

$$H_1(z) = \frac{20(z - 0.95)}{z - 1} \quad (1)$$

$$H_2(z) = \frac{z - 1}{z - 0.9} \quad (2)$$

$$H_3(z) = \frac{z - 0.9}{10(z - 0.99)} \quad (3)$$

$$H_4(z) = \frac{10(z - 0.99)}{z - 0.9} \quad (4)$$

$$H_5(z) = \frac{1 - 2r_1 \cos\Omega_1 + r_1^2}{z^2 - z2r_1 \cos\Omega_1 + r_1^2} \quad (5)$$

where $r_1 = 0.995$ and $\Omega_1 = 0.1$.

$$H_6(z) = \frac{(1 - 2r_1 \cos\Omega_1 + r_1^2)(z^2 - z2r_2 \cos\Omega_2 + r_2^2)}{(z^2 - z2r_1 \cos\Omega_1 + r_1^2)(1 - 2r_2 \cos\Omega_2 + r_2^2)} \quad (6)$$

where $r_1 = 0.995$ and $\Omega_1 = 0.1$, as before, and $r_2 = 0.9995$ and $\Omega_2 = 0.01$.

On two pages attached to the end of this exam are six step responses and six frequency response (Bode) plots. These plots are labeled I, II, III, IV, V, VI; and A, B, C, D, E, F; respectively. For each of the transfer functions above, indicate which are the corresponding step and frequency responses. Your answer should take the form of a number from 1-6 for each transfer function followed by a capital letter indicating the corresponding step response, followed by a Roman numeral indicating the corresponding frequency response. Wrong answers will count as zero; no partial credit will be given in this problem.

Problem 2 (25 points): Consider a continuous time filter:

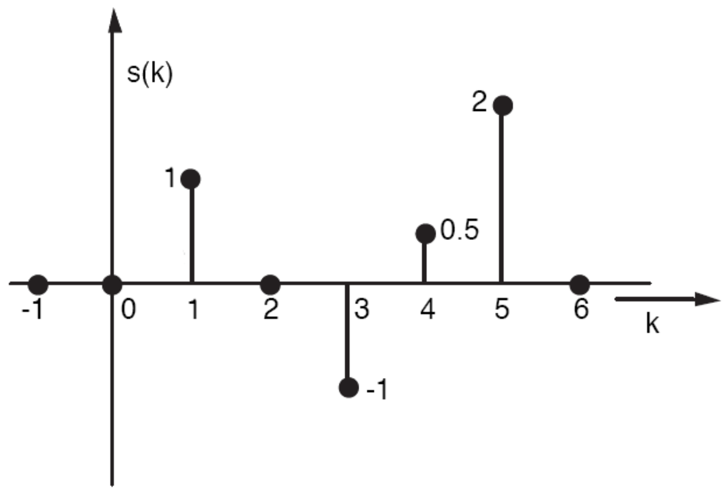
$$H(s) = \frac{10(s+1)}{(0.5s+1)(0.05s+1)}$$

a) Use the forward Euler approximation to map this to an approximating discrete-time system for T = 0.1sec.

b) What is the approximating difference equation?

c) Plot the poles in the z-plane. Comment on the quality of the approximation.

Problem 3 (25 points): A given linear, time-invariant (LTI) system H has an impulse response s(k) which is shown below:



Note that the only non-zero values of s(k) are at k = 1, 3, 4, 5.

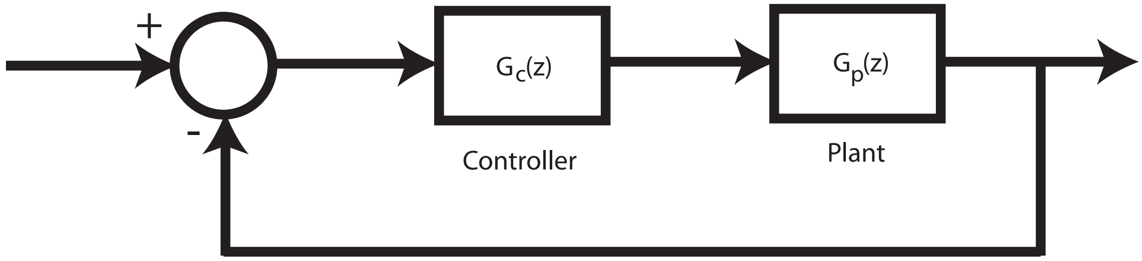
a) Use any convenient method to determine the difference equation for this system in terms of an input e(k) and an output y(k). Please explain your approach.

b) Draw a minimum block diagram realization of the system from the input to the output.

c) Now suppose that the system input is a unit step, i.e., e(k)=u_s(k). Using whatever approach you find most efficient, write an expression for the resulting output y(k) which is valid for all k.

d) Find the z-transform S(z) of s(k) as given in the figure above. How is this transform related to the the system transfer function H(z)?

Problem 4 (25 points): A block diagram for a system with a varying plant model and a fixed lead compensator is shown below:



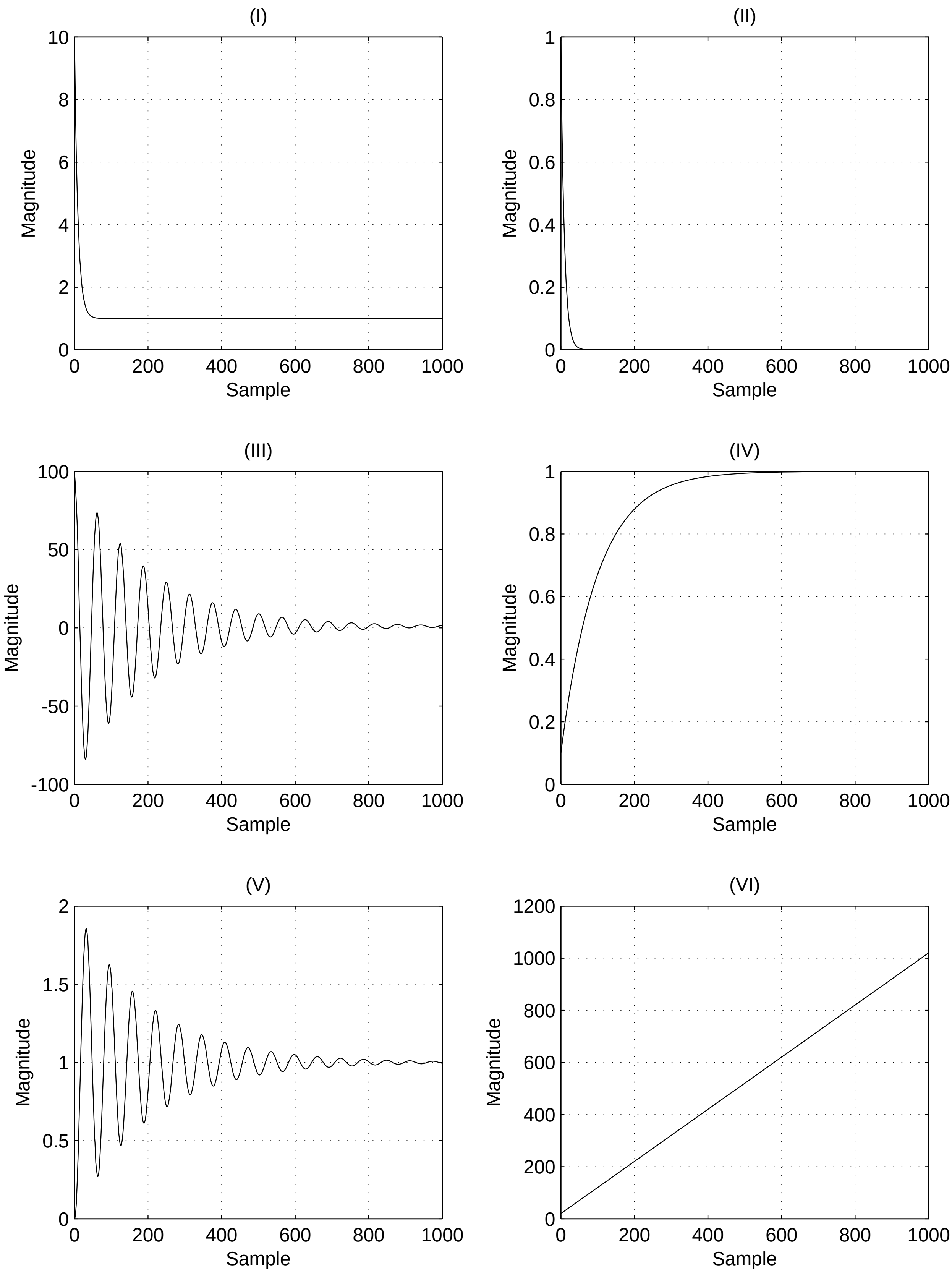
$$G_c(z) = K\frac{z-0.5}{z-0.2}$$

Make reasonably accurate root locus plots for $K \geq 0$.

a) $G_p(z) = \frac{1}{(z-1)^2}$

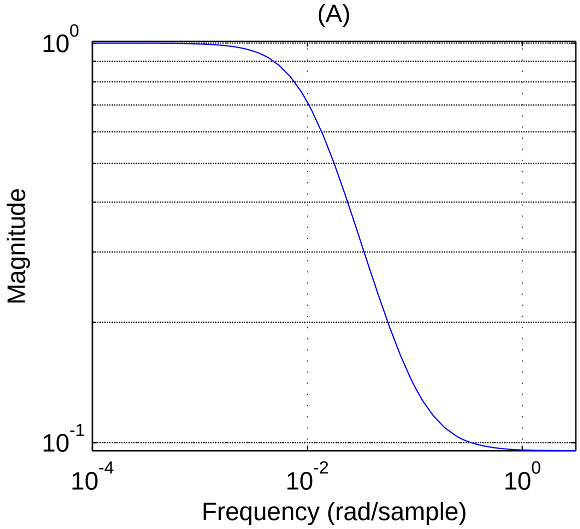
b) $G_p(z) = \frac{1}{(z^2-2R_0cos(\Omega_0)z+R_0^2)(z-1)}$ where $\Omega_0 = \frac{\pi}{4}$, $R_0 = 0.9$

c) $G_p(z) = \frac{z^2-2R_1cos(\Omega_1)z+R_1^2}{(z^2-2R_2cos(\Omega_2)z+R_2^2)(z^2-1)}$ where $R_1 = 0.99$, $\Omega_1 = \frac{\pi}{3}$, $\Omega_2 = \frac{32}{100}\pi$, $R_2 = R_1\frac{cos(\Omega_1)}{cos(\Omega_2)}$



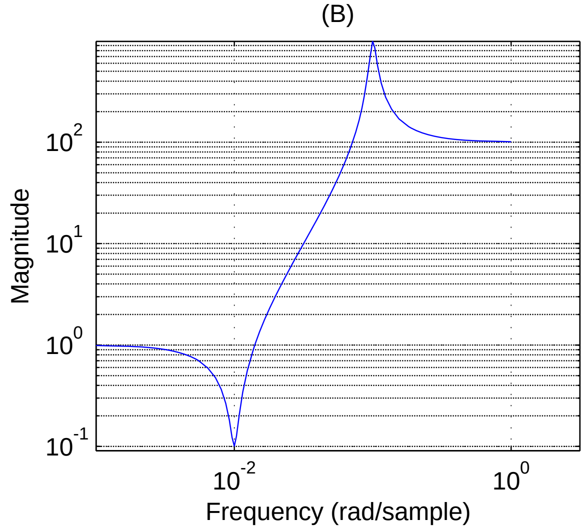
(I)
(II)
(III)
(IV)
(V)
(VI)
Magnitude
Sample

(A)



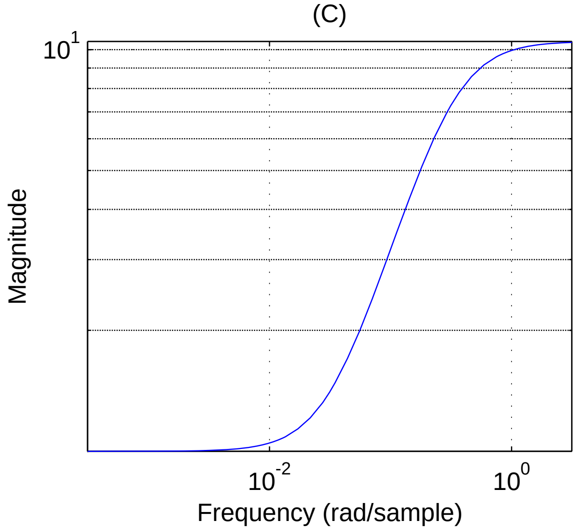
10^0
10^-1
Magnitude
10^-4
10^-2
10^0
Frequency (rad/sample)

(B)



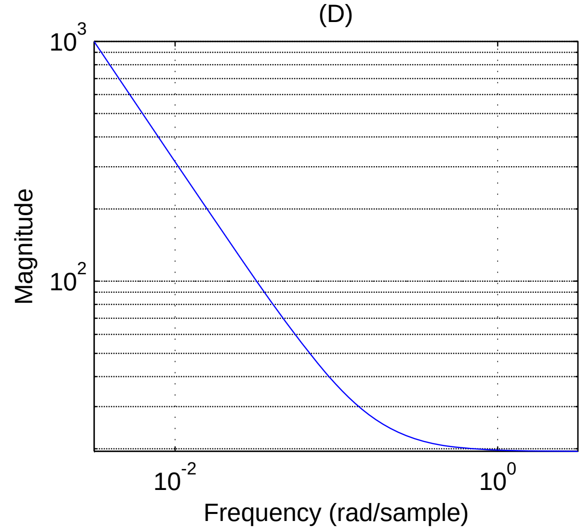
10^2
10^1
10^0
10^-1
Magnitude
10^-2
10^0
Frequency (rad/sample)

(C)



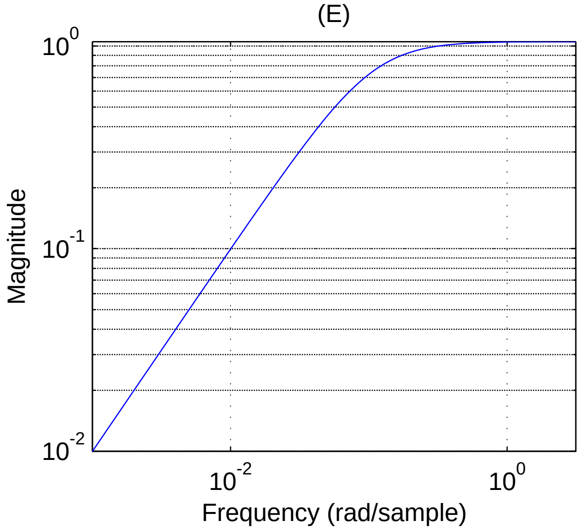
10^1
Magnitude
10^-2
10^0
Frequency (rad/sample)

(D)



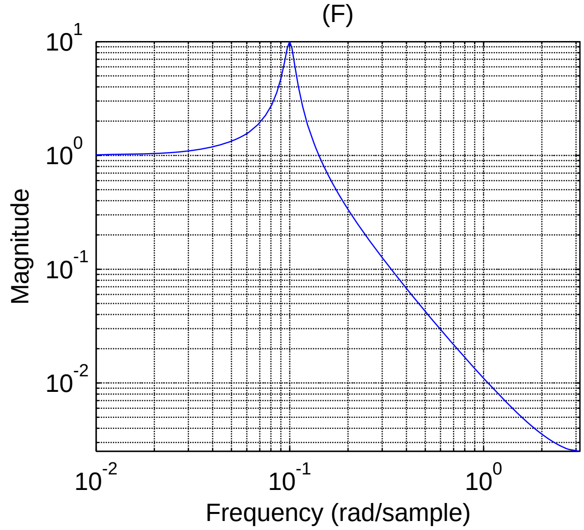
10^3
10^2
Magnitude
10^-2
10^0
Frequency (rad/sample)

(E)



10^0
10^-1
10^-2
Magnitude
10^-2
10^0
Frequency (rad/sample)

(F)



10^1
10^0
10^-1
10^-2
Magnitude
10^-2
10^-1
10^0
Frequency (rad/sample)