13.42 Homework #8

Spring 2005

Out: Thursday, April 14, 2005

Due: Thursday, April 21, 2005

Problem 1: Consider the naval vessel in Figure 1 which is transiting in head seas at a speed of $U = 10\ m/s$.

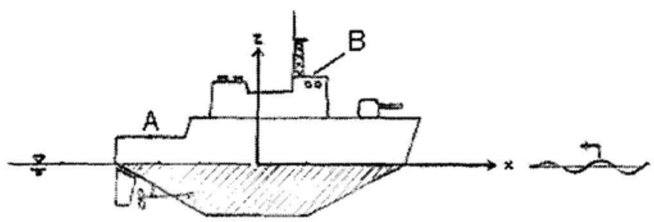

Figure 1 – Naval vessel

Its waterline (intersection of the hull with the calm free surface) is described in Figure 2.

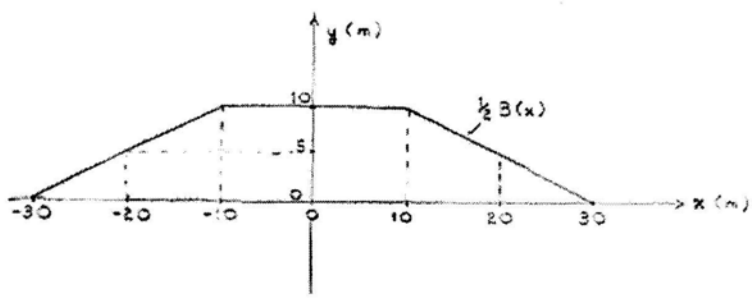

Figure 2 – where the breadth, $B(x)$, refers to the width of the ship at the waterline

The cross-section of the ship at any given location along the x-axis is a semi-circle of radius $R(x) = B(x)/2$. Refer to Figure 3. Assume that the ship is wall-sided above the waterline.

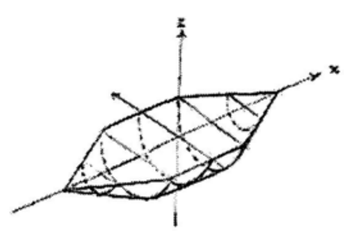

Figure 3 – Semi-circular ship sections

The non-dimensional added mass and damping coefficients in heave for semi-circular ship sections are given in Figure 4.

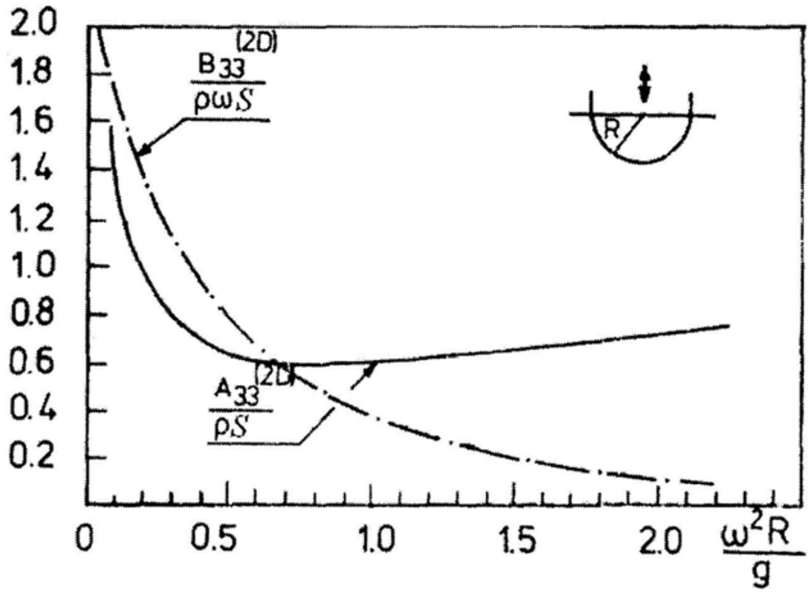

Figure 4 – Two-dimensional added mass and damping coefficients in heave for semi-circular ship sections, where ρ is the density of water, *S* is the area of the section (up to the waterline), and *R* is the radius of the section – from Faltinsen (1990)

For a ship operating in head seas, the important modes of motion are heave (3) and pitch (5). Surge is non-zero but generally small for slender ships and in ambient waves of small steepness.

a. For incident waves of frequency $\omega_o = 0.5\ rad/s$, compute the added mass and damping coefficients – A_{ij} and B_{ij} – where $i, j = 3,5$. Assume deep water. It is suggested that you compute the coefficients at sections spaced 5 meters apart and that you create a table in *Excel* for your calculations (this will save you time in Part f).

b. Given that the center of gravity of the ship is located at $(0,0,z_G)$ where $z_G = -3/4 * R(0)$, find the restoring coefficients C_{ij} for $i, j = 3,5$.

c. Determine the magnitudes of the Froude-Krylov heave excitation force $|F_3|$ and pitch excitation moment $|F_5|$, where $F_3(t)=\text{Re}\{|F_3|e^{i\omega t}\}$ and $F_5(t)=\text{Re}\{|F_5|e^{i(\omega t-\pi/2)}\}$. (Hint: Recall that the ambient wave elevation in the ship-fixed coordinate system is $\zeta(x,t)=A\text{Re}\{e^{ikx+i\omega t}\}$.)

d. Find the transfer functions for the following linear systems:

$$\zeta \rightarrow \boxed{\quad L \quad} \rightarrow F_i \;,\; i=3,5$$

e. Given that $M_{55} = 1.5 * 10^9\ kg*m^2$, find the transfer functions for the *uncoupled* heave-pitch equations of motion.

f. Repeat the above (a-e) for $\omega_o = 0.75\ rad/s$.

g. Now given the ambient wave spectrum in Figure 5, plot the spectra of heave and pitch response.

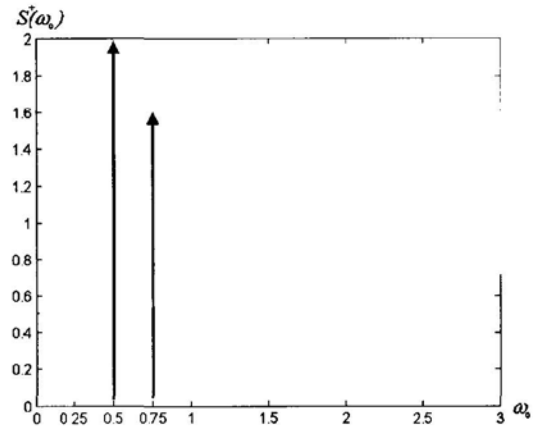

Figure 5 – Ambient wave spectrum

Problem 2: Consider the cylindrical buoy in Figure 6 in ambient plane progressive waves of amplitude A and frequency ω. Let the mass of the buoy be m and the center of gravity be located at $z = -\frac{3}{4}T$. Assume deep water.

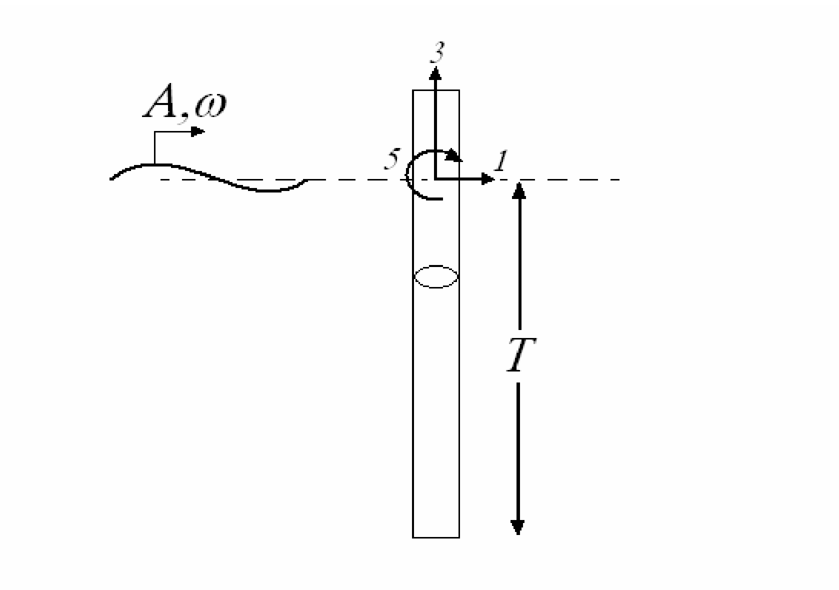

Figure 6 – Cylindrical buoy with circular cross-section

a. Given that $\lambda >> d$ (the diameter of the buoy), find the surge excitation force and the pitch exciting moment, neglecting viscous effects.

b. Find the added mass and linear restoring coefficients A_{ij} and C_{ij} – i,j = 1, 3, 5. The heave added mass may be approximated by the added mass of a sphere of equal diameter.

c. Determine the natural frequency of the buoy in heave.

d. State the equations of motion for the system.

Now consider the following cylindrical buoy tethered to the sea floor by a cable as depicted in Figure 7. Assume that the weight of the buoy is less than the buoyant force, and that the cable tension is P. The linear restoring matrix due to the cable is given as:

$$C_{CABLE} = \begin{pmatrix} k_{11} & 0 & k_{15} \\ 0 & k_{33} & 0 \\ k_{51} & 0 & k_{55} \end{pmatrix}$$

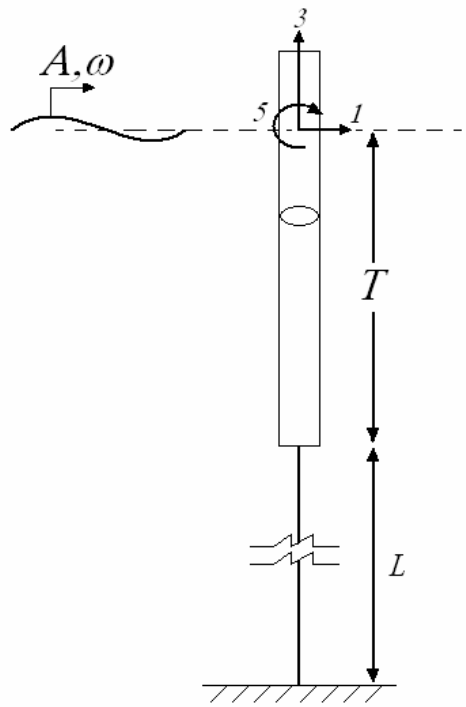

Figure 7 – Cylindrical buoy with circular cross-section tethered to the seafloor.

e. Find the added mass and linear restoring coefficients A_{ij} and C_{ij} – i,j = 1, 3, 5.

f. Find k_{11} and the natural frequency of the buoy in surge.