

Roll Motion

Coupled with sway & yaw.

Linear damping due to radiated waves is small
So Roll Motion can be LARGE.



- perfect circular shape w/ motion about its center
- No waves are created



- Non circular shape
- Wave making is relatively small

Where does additional damping come from?

- FRICTION
- Eddy-generation



- Surface friction \Rightarrow VISCOUS



- Eddy-generation




- Separation on appendages such as fins or keels
- Also Eddy-generation

FRICTION

$$F_f = C_f \frac{1}{2} \rho U^2 S_{\text{surface area}}$$

$$F \sim U^2 \leftarrow \text{NON-LINEAR}$$

Roll Force $\rightarrow F_R = \phi \dot{x}_4 |\dot{x}_4| \quad \phi = \text{constant}$



$x_4 = \phi_0 \cos \omega t \quad \phi_0 = \text{roll amplitude}$

CAN WE FIND AN EQUIVALENT LINEARIZATION OF ROLL FORCE?

~~Roll Force~~ $F_R^L = \epsilon \dot{x}_4$

Where coefficient ϵ is determined such that the energy per cycle spent by F_R is the same as from the linearized $F_R^L \rightarrow$

$$\int_0^T F_R \dot{x}_4 dt = \int_0^T F_R^L \dot{x}_4 dt$$

↑
integral
over one period
(cycle)

$$\dot{x}_4 = \phi_0 \omega \sin \omega t$$

$$\phi \omega^3 \phi_0^3 \int_0^{2\pi} \sin^2 \theta |\sin \theta| d\theta = \epsilon \omega^2 \phi_0^2 \int_0^{2\pi} \sin^2 \theta d\theta$$

$$\int_0^{2\pi} \sin^2 \theta |\sin \theta| d\theta = 2 \int_0^{\pi} \sin^3 \theta d\theta = \frac{4}{3} \cdot 2$$

$$\int_0^{2\pi} \sin^2 \theta d\theta = \frac{2\pi}{2}$$