13.42 Design Principles for Ocean Vehicles
Homework #2

Out: Thursday, 12 February 2004
Due: Thursday, 19 February 2004

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1. Determine whether the following systems are linear and/or time invariant.

a. $y(t) = \int_0^{t+\alpha} u(s)ds$

b. $y(t) = \int_{t-\alpha}^{t+\alpha} [u(s)]^2 ds$

c. $y(t) = \alpha \frac{du(t)}{dt} \left| \frac{du(t)}{dt} \right|$

d. $\alpha \ddot{y}(t) + \beta \dot{y}(t) + \gamma y(t) = u(t)$

2. Determine whether the following systems are LTI systems.

a. $a_1 \cos \omega t \rightarrow \boxed{} \rightarrow a_2 \cos(\omega t + \phi)$

b. $\sin 5t \rightarrow \boxed{} \rightarrow 2\cos(10t + \pi)$

3. Fourier Transform

a. Find the Fourier Transform of $f(t) = u_0(t - \tau)$.

b. Given that $f(x) \rightarrow 0$ as $|x| \rightarrow \infty$, and the Fourier Transform of $f(x)$ is $\tilde{f}(\alpha)$, what is the Fourier Transform of $\frac{df}{dx}$? (Hint: Use partial integration.)

c. Given that $\frac{df}{dx} \rightarrow 0$ as $|x| \rightarrow \infty$, what is the Fourier Transform of $\frac{d^2 f}{dx^2}$?

4. Transfer Function:
   a. Given the following linear system:

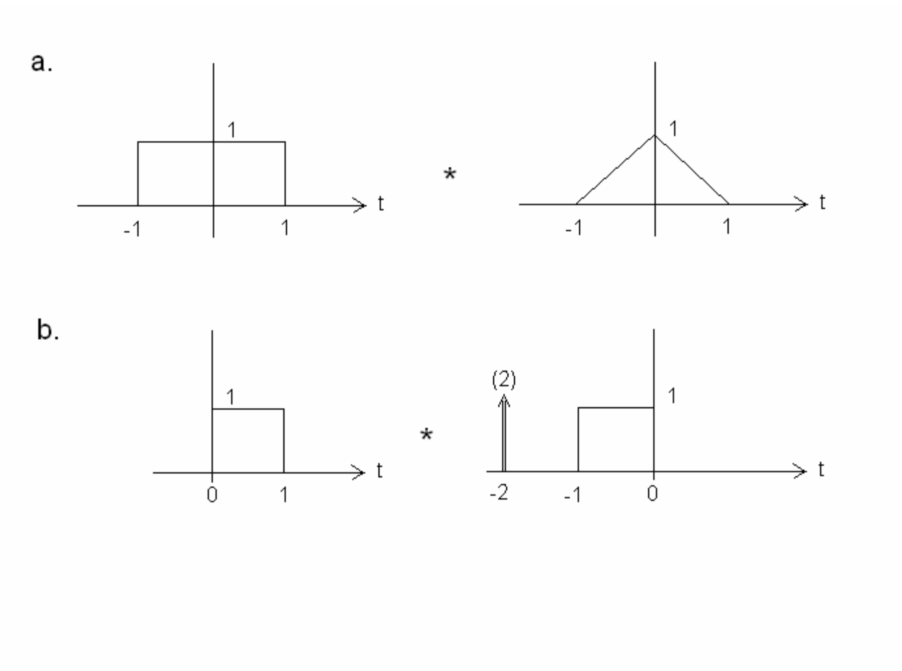
$$m\ddot{x} + c\dot{x} + kx = f(t)$$

where input $f(t) = \mathrm{Re}\{\mathbf{F}e^{i\omega t}\}$ and response $x(t) = \mathrm{Re}\{\mathbf{X}e^{i\omega t}\}$, and **X** and **F** are both complex quantities, find the transfer function $H(\omega)$.

b. Using the same system, for which you have just found the transfer function, if the input is $\alpha f_1(t) + \beta f_2(t)$ determine the system output, *x(t)*.

5. Convolution

Perform the following convolutions (from page 2.11 in Triantafyllou and Chryssostomidis, *Environmental Description, Force Prediction and Statistics for Design Applications in Ocean Engineering*):



6. A linear time-invariant system has a transfer function *H(ω)*, show that when the input is sinusoidal with frequency ω_o, i.e.

$$f(t) = f_o \cos(\omega_o t + \psi)$$

the output is also a sinusoidal function with the same frequency.