13.42 HW # 5 SOL'NS

1a. $\sigma_\zeta^2 = \sum_{i=1}^{4} S_\zeta^+(\omega_i)\,\delta\omega$

$= 10(0.5) + 15(0.5) + 10(0.5) + 5(0.5)$

$= 20\ m^2$

b. • FROM EQ. (6.6) IN 13.42 READING # 2:

$$\overline{\frac{dE}{dt}} = \bar{E}\cdot C_g = \frac{1}{2}\rho g A^2 \cdot \frac{1}{2}\frac{g}{\omega}$$

• $\frac{1}{2} A_i^2 = S_\zeta^+(\omega_i)\,\delta\omega$

$$A_i = \sqrt{2 S_\zeta^+(\omega_i)\,\delta\omega}$$

$$\begin{cases} A_1 = 3.16\ m \\ A_2 = 3.87\ m \\ A_3 = 3.16\ m \\ A_4 = 2.24\ m \end{cases}$$

• $\overline{\frac{dE}{dt}} = \sum_{i=1}^{4} \frac{1}{2}\rho g A_i^2 \cdot \frac{1}{2} g/\omega_i$

$= 1.09 \times 10^6\ J/s$ per m

c. LINEAR H'DYNAMIC PRESSURE:

$$p = -\rho \frac{\partial \phi}{\partial t}\bigg|_{x=0} = \sum_{i=1}^{4} \underbrace{\rho g A_i e^{k_i z}}_{P_i} \cos(\omega_i t + \phi_i)$$

ITS SPECTRUM, $S_p(\omega)$, MAY BE FOUND AS FOLLOWS:

$$\frac{1}{2} P_i^2 = S_p(\omega_i)\,\delta\omega$$

$$\begin{cases} P_1 = \rho g A_1 e^{-10k_1} = 2.46 \times 10^4 \\ P_2 = \rho g A_2 e^{-10k_2} = 1.41 \quad '' \\ P_3 = \rho g A_3 e^{-10k_3} = 0.32 \quad '' \\ P_4 = \rho g A_4 e^{-10k_4} = 0.04 \quad '' \end{cases}$$

- $$\sigma_p^2 = \sum_{i=1}^{4} S_p(\omega_i)\,\delta\omega = \sum_{i=1}^{4} \frac{1}{2} P_i^2 = 4.08 \times 10^8$$

d. $$\bar{\eta}(3.5) = \frac{1}{2\pi}\sqrt{\frac{M_2}{M_0}}\, e^{-\frac{3.5^2}{2M_0}}$$

$$\left[\begin{array}{l} M_2 = \sum_{i=1}^{4} \omega_i^2 S_\zeta^+(\omega_i)\,\delta\omega = 30 \\ \\ M_0 = \sigma_\zeta^2 = 20\ m^2 \end{array}\right.$$

$$= 0.1435\ \frac{\text{UPCROSSINGS}}{s}$$

e. FOR A WAVE OF FREQUENCY ω, THE PROBE WILL ENCOUNTER A CREST EVERY

$$T_e = \frac{\lambda}{C_p - U} \quad \text{seconds.}$$

THE CORRESPONDING "ENCOUNTER FREQ." IS THEN

$$\omega_e = \frac{2\pi}{T_e} = \frac{2\pi}{\lambda}(C_p - U)$$

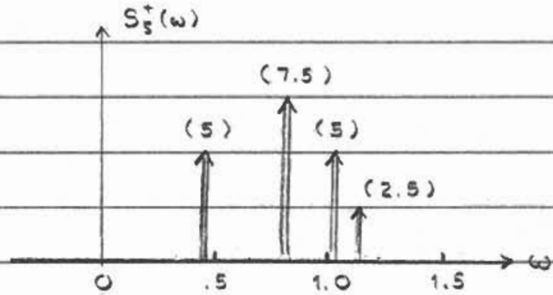
$$= \omega - Uk.$$

THUS $\omega_{1e} = \omega_1 - Uk_1 = 0.5 - 2(.0255) = .4490\ ^{rad}/_s$

$$\omega_{2e} = \omega_2 - Uk_2 = 1.0 - 2(.1019) = .7961\ ^{rad}/_s$$

$$\omega_{3e} = \omega_3 - U k_3 = 1.5 - 2(.2294) = 1.0413 \ \text{rad/s}$$

$$\omega_{4e} = \omega_4 - U k_4 = 2 - 2(.4077) = 1.1845 \ \text{rad/s}$$



2 a. FROM HW#2, WE KNOW THAT

$$x_i(t) = |H(\omega_i)| f_i \cos(\omega_i t + \phi_i + \varphi_i)$$

WHERE $\varphi_i = \angle H(\omega_i)$.

FURTHERMORE, $H(\omega_i) = \dfrac{1}{(k - \omega_i^2 m) + i(\omega_i c)}$.

- $$|H(\omega_i)|^2 = H(\omega_i) H^*(\omega_i)$$

$$= \frac{1}{(k - \omega_i^2 m)^2 + (\omega_i c)^2}$$

- $$\sigma_x^2 = \sum_{i=1}^{N} S_x(\omega_i)\, \delta\omega = \sum_{i=1}^{N} |H(\omega_i)|^2 S_f(\omega_i)\, \delta\omega$$

$$= \sum_{i=1}^{N} \frac{S_f(\omega_i)}{(k - \omega_i^2 m)^2 + (\omega_i c)^2}\, \delta\omega$$

TO FIND THE VARIANCE OF $\ddot{x}(t)$, CONSIDER THAT

$$\ddot{x}(t) = -\omega_i^2 |H(\omega_i)| f_i \cos(\omega_i t + \phi_i + \varphi_i)$$

LET $y(t) = \ddot{x}(t)$. THEN FOR THE LINEAR SYSTEM

$$x(t) \longrightarrow \boxed{L} \longrightarrow y(t) ,$$

$H_1(\omega_i) = -\omega_i^2$.

- $$\sigma_y^2 = \sum_{i=1}^{N} S_y(\omega_i)\,\delta\omega = \sum_{i=1}^{N} |H_1(\omega_i)|^2 S_x(\omega_i)\,\delta\omega$$

$$= \sum_{i=1}^{N} \frac{-\omega_i^2 S_f(\omega_i)}{(k-\omega_i^2 m)^2 + (\omega_i c)^2}\,\delta\omega$$

b. LET $y(t) = \dot{x}(t)$. THEN

$$x(t) \longrightarrow \boxed{L} \longrightarrow y(t)$$

RECALL THAT FOR A LINEAR SYSTEM, IF THE INPUT PROCESS IS GAUSSIAN, THE OUTPUT PROCESS WILL ALSO BE GAUSSIAN — i.e., $f(t)$ IS GAUSSIAN, $\therefore$ $x(t)$ IS GAUSSIAN, AND $\therefore$ $y(t)$ IS GAUSSIAN.

THEN $\mu_{|\dot{x}|\dot{x}} = E\{|\dot{x}|\dot{x}\} = E\{|y|y\}$

$$= \int_{-\infty}^{\infty} |y|\,y\,f_y(y)\,dy$$

RECALL FROM HW#4, Prob. 3a THAT IF $\mu_f = 0$, THEN $\mu_x = 0$. IF $\mu_x = 0$, THEN $\mu_y = 0$, AND $f_y(y)$ IS AN EVEN FCN. $|y|\,y\,f_y(y)$ IS THEN ODD AND $\underline{\mu_{|\dot{x}|\dot{x}} = 0}$.

$$\underline{\sigma^2_{|\dot{x}|\dot{x}}} = E\{(|y|y - \cancelto{0}{\mu_{|y|y}})^2\} = \underline{\int_{-\infty}^{\infty} y^4 f_y(y)\,dy}$$

3. • $M_0 = \int_0^1 10\omega \, d\omega + \int_1^2 10 \, d\omega + \int_2^3 (30 - 10\omega) \, d\omega$

$= 10 \frac{1}{2}\omega^2 \Big|_0^1 + 10\omega \Big|_1^2 + (30\omega - 10 \cdot \frac{1}{2}\omega^2)_2^3$

$= 5 + 10 + 5$

$= 20.$

• $M_2 = \int_0^1 10\omega^3 \, d\omega + \int_1^2 10\omega^2 \, d\omega + \int_2^3 (30\omega^2 - 10\omega^3) \, d\omega$

$= 10 \cdot \frac{1}{4}\omega^4 \Big|_0^1 + 10 \cdot \frac{1}{3}\omega^3 \Big|_1^2 + (30 \cdot \frac{1}{3}\omega^3 - 10 \cdot \frac{1}{4}\omega^4)_2^3$

$= \frac{10}{4} + \frac{70}{3} + \frac{570}{3} - \frac{650}{4}$

$= 53.33$

• $\bar{\zeta}(h) \leq 20 \ \text{UPCROSSINGS}/\cancel{\text{hr}} \left(\frac{1 \ \cancel{\text{hr}}}{3600 \ \text{s}} \right)$

$\leq .0056 \ \frac{\text{UPCROSSINGS}}{\text{s}}$

• $\bar{\zeta}(h) = \frac{1}{2\pi} \sqrt{\frac{M_2}{M_0}} \, e^{-\frac{h^2}{2M_0}} \leq .0056$

$\underline{h \geq 12.39 \ \text{m}}$

4a, b. SEE p. 4, 5 ON 13.42 READING # 7

c. THIS VALUE IS VERY CLOSE TO THAT WHICH A CASUAL OBSERVER WOULD ESTIMATE AS THE WAVEHEIGHT WHEN WATCHING THE SEA.