$$\hbar\omega_p = E_f - E_i$$

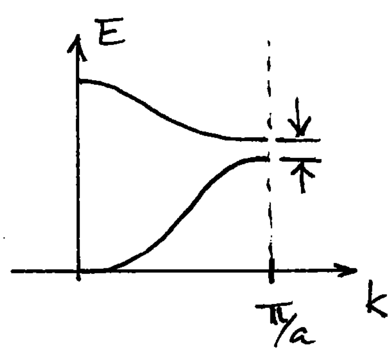
$$\hbar\bar{k}_p = \bar{P}_f - \bar{P}_i + \bar{G}$$

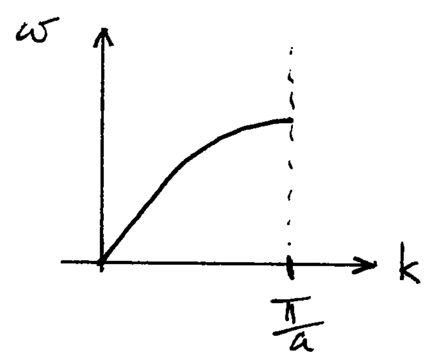
$$\text{ATOM}\left\{\begin{array}{l}\text{TRANSLATIONAL ENERGY}\\ \text{ELECTRONIC ENERGY}\end{array}\right.$$

$$\text{MOLECULES}\left\{\begin{array}{l}\text{VIBRATION ENERGY}\\ \text{ROTATION ''}\end{array}\right.$$

$$\text{SOLID}\left\{\begin{array}{l}\text{ELECTRONIC}\\ \text{PHONON}\end{array}\right.$$

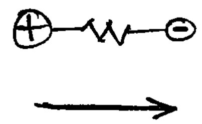
* IN GENERAL, DIELECTRICS HAVE A FREQ. BAND IN WHICH THEY ARE TRANSPARENT





de Broglie: $|\vec{P}| = p = \dfrac{h}{\lambda}$

OBTAINING OPTICAL CONSTANTS



$E_0 e^{-i\omega t}$

$$m\ddot{x} = -eE - K(x - x_0) - \beta\frac{dx}{dt}$$

$$\Delta x = x - x_0 = Ae^{-i\omega t}$$

$$\frac{d^2\Delta x}{dt^2} + \gamma\frac{d\Delta x}{dt} + \omega^2\Delta x = \frac{-eE_0}{m}e^{-i\omega t} \qquad \Delta x = \frac{-eE_0}{m}e^{-i\omega t}$$

$$A = \frac{-eE_0/m}{-\omega^2 + \omega_0^2 - i\gamma\omega}$$

$$\overline{D} = \varepsilon_0\overline{E} + \overline{P} = \varepsilon_0\underbrace{(1 + \chi)}_{\varepsilon_r}\overline{E}$$

$$\overline{P} = e(X - X_0)N$$

$$= \frac{e^2N/m}{\omega^2 - \omega_0^2 + i\gamma\omega}\overline{E}$$

$$= \varepsilon_0\chi\overline{E}$$

$$\varepsilon_r = 1 + \frac{\omega_p^2}{\omega^2 - \omega_0^2 + i\gamma\omega}$$ LORENTZ MODEL

$$\bar{\nabla}\times\bar{B} = \frac{\partial \bar{D}}{\partial t} + \bar{J}$$

$$\bar{D} = \bar{D}_0 e^{-i\omega t}$$

$$\bar{J} = \sigma \bar{E}$$

$$\bar{\nabla}\times\bar{B} = \frac{\partial}{\partial t}\left\{ \varepsilon_0 \underbrace{\left(1 + \chi - \frac{\sigma}{i\omega\varepsilon_0}\right)}_{\varepsilon_r} \bar{E} \right\}$$

FREE ELECTRONS

$$A = \frac{-e\, E_0/m}{-\omega^2 - i\gamma\omega}$$

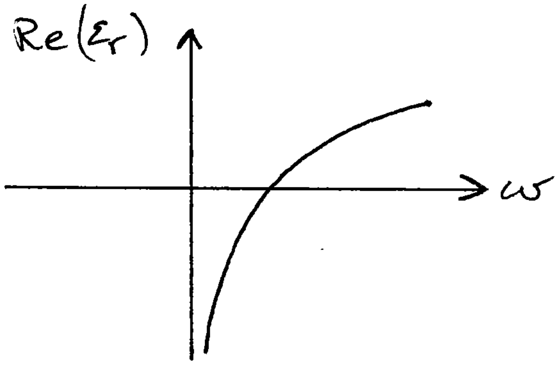
$$\bar{J} = -n_e e \bar{v} = -n_e e(-i\omega) A e^{-i\omega t} = (i\omega)\frac{e^2 n_e/m}{\omega^2 + i\gamma\omega}\bar{E}$$

"Drude Model"

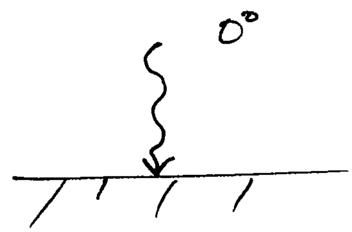
$$\varepsilon_r = 1 - \frac{\omega_p^2}{\omega^2 + i\gamma\omega} \quad ; \quad \omega_p^2 = \frac{e^2 n_e}{m\varepsilon_0}$$ "PLASMA FREQ."

$$N = n + i\kappa = \sqrt{\varepsilon_r}$$

$$n^2 - \kappa^2 = \mathrm{Re}(\varepsilon_r) \qquad 2n\kappa = \mathrm{Im}(\varepsilon_r)$$



FOR $\Rightarrow$
$Re(\varepsilon_r) < 0$

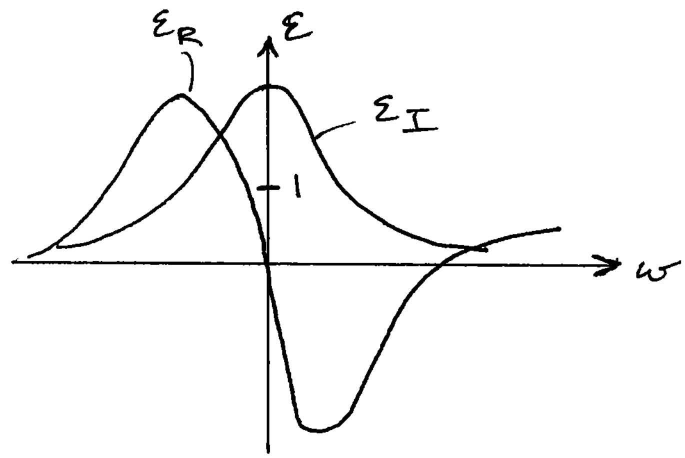


$$R = \left|\frac{N-1}{N+1}\right|^2 \approx \left|\frac{ai-1}{ai+1}\right|^2$$

* ABOVE ω_p METALS BECOME TRANSPARENT

$\omega \ll \omega_0$, $\varepsilon_r = 1 - \frac{\omega_p^2}{\omega_0^2}$

$\omega \gg \omega_0$, $\varepsilon_r = 1$



CAUSALITY $\Rightarrow$ KRAMER'S KRONIG RELATION

RELATES ε_R AND ε_I

* CAUSALITY / K-K RELATES REAL & IMAG. PARTS OF ANY FREQ. RESPONSE FUNCTION

$$\varepsilon_r = 1 + \sum_{j=1}^{N} \frac{\omega_{p_j}^2}{\omega^2 - \omega_{0j}^2 + i\gamma_j \omega} - \frac{\omega_p^2}{\omega^2 + i\gamma\omega}$$

$$= \varepsilon_0 + (\quad) - (\quad)$$

$$1 + \underbrace{\frac{\omega_{p01}^2}{\omega^2 - \omega_{01}^2}}_{\approx -\frac{\omega_{p01}^2}{\omega_{01}^2}} + \frac{\omega_{p02}^2}{\omega^2 - \omega_{02}^2}$$

$$\approx -\frac{\omega_{p01}^2}{\omega_{01}^2} \qquad \text{WHEN} \quad \omega \approx \omega_{02}^2$$

$$\text{AND} \quad \omega_{01} \gg \omega_{02}$$

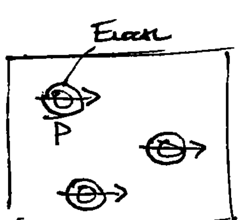
$$\Downarrow$$

$$1 - \left(\frac{\omega_{p01}}{\omega_{01}}\right)^2 = \varepsilon_0$$

LOCAL ELECTRIC FIELD

$$\overline{D} = \varepsilon_0 \overline{E} + \overline{P}$$

$\uparrow$ external



$$\overline{P} \sim \overline{E}_{LOCAL}$$

$$\overline{E}_{LOCAL} = \overline{E}_{ex} + \overline{E}_{INDUCED}$$

$$\overline{E}_{IND} = \overline{P}/3\varepsilon_0$$

$$\bar{E}_{LOCAL} = \frac{\varepsilon_0\varepsilon_r + 2\varepsilon_0}{3\varepsilon_0}\bar{E}_{external}$$

$$\Downarrow$$

$$\frac{\varepsilon_r - 1}{\varepsilon_r + 2} = \frac{1}{3}\sum_j \frac{\omega_{pj}^2}{\omega^2 - \omega_{oj}^2 + i\gamma_j\omega}$$

"CLAUSIUS-MOSSOTTI RELATION"

GAS PROPERTIES

ATOMS
- TRANSLATIONAL $kT \sim \hbar\omega_p$
- ELECTRONIC $-\frac{13.6\,eV}{n^2}$

MOLECULES
- VIBRATION $E_n = \hbar\omega_v(n+\frac{1}{2})$, $\omega_v = \sqrt{\frac{k}{m_{eff}}}$
- ROTATION $E_\ell = \frac{\hbar^2}{2I}\ell(\ell+1)$ $|m| \leq \ell$

FOR RIGID ROTATER

WHAT IF NOT "RIGID"

$\Longrightarrow$ COUPLED ROTATION | VIBRATION

$$E_{n\ell} = \hbar\omega_v\left(n+\frac{1}{2}\right) + \frac{\hbar^2}{2I_n}\ell(\ell+1)$$

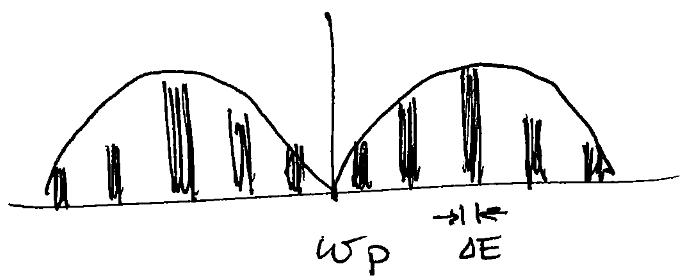
$O_2 \Rightarrow O=O$

$CO_2 \Rightarrow O=C=O$

$$\hbar\omega_p = E_{n'\ell'} - E_{n\ell}$$

$$\Delta n = \pm 1, 0$$
$$\Delta \ell = \pm 1, 0$$

* NOW WE HAVE MORE CHOICES FOR ENERGY DUE TO n, ℓ COMBINATIONS



$$\Delta E \cdot \Delta t \geq \frac{\hbar}{2}$$