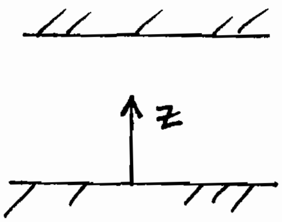


$$\mu \frac{dI}{d\xi} + I = I_b$$

$$I = I(z, \theta, \phi)$$

$$q'' = \int_{4\pi} I \cos\theta \, d\Omega = 2\pi \left[\int_0^1 I^+ \mu \, d\mu - \int_0^1 I^-(-\mu) \mu \, d\mu \right]$$

RECALL $\displaystyle\int_a^b f(x)\,dx \approx \sum_{i=1}^{N} f(x_i)\, w_i$

$$\therefore \quad \sum I(\mu_i)\, \mu_i \, w_i$$

THUS WE CAN GET A BUNCH OF 1^{st}-ORD COUPLED ODES

$$\longrightarrow \quad \mu_i \frac{dI}{d\xi} + I = I_b(T)$$

$\Longrightarrow$ DISCRETE ORDINATE METHOD

$$\cancel{\frac{dq}{d\xi}} + \int I \, d\Omega = \int I_b \, d\Omega$$

$\bar{\nabla} \cdot \bar{q}_{RAD} = 0$

1^{st} LAW

ENERGY BAL

FOR RAD. ONLY

$$I_b = \frac{1}{4\pi} \int I \, d\Omega = -\frac{1}{2} \int_1^{-1} I \, d\mu$$

$$= \frac{1}{2} \left[\int_0^1 I^* d\mu + \int_0^1 I^-(-\mu)\, d\mu \right]$$

$$\Downarrow$$

$I = I(\mu_i, z_j)$ $\qquad \displaystyle\sum_{i=1}^{N} I(\mu_i)\, w_i$ $\qquad$ N points for μ

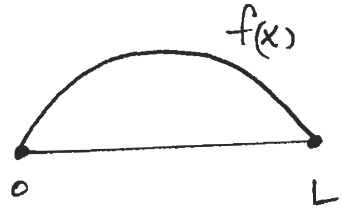
5/9/06 2.58

METHOD OF SPHERICAL HARMONICS

WANT TO SOLVE $\nabla^2 T = 0$

IN CARTESIAN COORDS, SOLVING THE BOUNDARY VALUE PROBLEM WE CAN HAVE/USE FOURIER SERIES AS AN EXPANSION BASIS AND MAKE USE OF ORTHOGONALITY

$$f(x) = \sum a_m \sin\left(\frac{2\pi m x}{L}\right)$$



... sine series sol'n

BUT IN SPHERICAL COORDS —

$$T = R(r)\,\Theta(\theta)\,\Phi(\phi)$$

$$\Theta = P_\ell^m(\mu) = \frac{1}{2^n n!}(1-\mu^2)^{m/2}\frac{d^{m+\ell}}{d\mu^{m+\ell}}(\mu^2-1)^\ell$$

$-\ell \le m \le \ell$

~~$-m \le \ell \le m$~~

$$\Phi = e^{im\phi}$$

ANALOGOUS TO

$\ell \equiv$ ANG. MOM. QUANT. #

$m =$ MAG. QUANT. #

SPHERICAL HARMONICS:

$$Y_\ell^m = (-1)^{(m+|m|/2)}\sqrt{\frac{(\ell-|m|)!}{(\ell+|m|)!}}\,e^{im\phi}\cdot P_\ell^{|m|}(\mu)$$

$$\int_{-1}^{1} P_\ell P_m \, d\mu = \begin{cases} 0 & \ell \neq m \\ \dfrac{2}{2\ell+1} & \ell = m \end{cases}$$

$$I(\bar{r}, \theta, \phi)$$

$$I(\bar{r}, \hat{e}_\Omega) = \sum_{\ell=0}^{\infty} \sum_{m=-\ell}^{\ell} I_\ell^m(\bar{r}) \, Y_\ell^m(\hat{e}_\Omega)$$

P_1 ; $\ell = 1$

P_3 ; $\ell = 3$

$P_5 \Rightarrow$ DIFF. EQ BECOMES MESSY

<u>P_1 -APPROX.</u>

$$I(\bar{r}, \hat{e}_\Omega) = I_0^0 Y_0^0 + I_1^{-1} Y_1^{-1} + I_1^0 Y_1^0 + I_1^1 Y_1^1 \quad (15.23)$$

⋮

(15.24)

⋮

$$I(\bar{r}, \hat{e}_\Omega) = \bar{a}(\bar{r}) + \bar{b} \cdot \hat{e}_\Omega \quad (15.25)$$

NOW, THE TRICK IS TO SOLVE FOR $a(\bar{r})$ AND $\vec{b}(\bar{r})$

INSERT INTO E.R.T.

$$\frac{1}{\chi_e}\hat{e}_\Omega \cdot \bar{\nabla}_r I_\eta = -I_\eta + S_\eta$$

$$I(\bar{r},\hat{e}_\Omega) = a + \bar{b}\cdot\hat{e}_\Omega = \frac{1}{4\pi}\left[G + 3\bar{q}\cdot\hat{e}_\Omega\right]$$

$$G(\bar{r}) = \int I\, d\Omega = 4\pi a \qquad (15.26)$$

$$\bar{q}'' = \int I\,\hat{e}_\Omega\, d\Omega = \frac{4\pi}{3}\vec{b}$$

a IS RELATED TO LOCAL ENERGY DENSITY

$G = cu$ ← u: NRG DENSITY

$\vec{b}$ IS RELATED TO LOCAL HEAT FLUX

PHASE FUNCTION: $\Phi = 1 + A_1\,\hat{e}_{\Omega'}\cdot\hat{e}_\Omega$

ERT:

$$\frac{1}{4\pi\chi_e}\bar{\nabla}_r\cdot\left[\hat{e}_\Omega\left(G + 3\bar{q}\cdot\hat{e}_\Omega\right)\right] + \frac{1}{4\pi}\left[G + 3\bar{q}\cdot\hat{e}_\Omega\right]$$

$$= \left[1-\omega\right]I_b + \frac{\omega}{4\pi}\left[G + A_1\bar{q}\cdot\hat{e}_\Omega\right]$$

$\int Y_0^0\, d\Omega \times$ see above eqn. (15.34)

use orthogonality

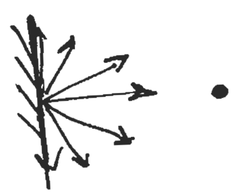
2.58 5/9/06

$$\overline{\nabla}_r \cdot \overline{q} = (1-\omega)\left[4\pi I_b - G\right]$$

$$\int (\text{Eqn.}) \cdot Y_1^m \, d\Omega \Rightarrow \overline{\nabla}_r G = -(3 - A_1\omega)\overline{q} \quad (15.36)$$

ISOTROPIC $A_1 = 0 \Rightarrow \underbrace{\overline{q} = -\frac{1}{3K_e}\overline{\nabla}_r G}_{\text{"FOURIER FORM"}} \quad (15.41,\ \omega/A=0)$

BOUNDARY COND'S



→ DIFFICULT TO SATISFY LOCAL ?? IS IT?

→ SATISFY GLOBAL ENERGY CONSERVATION

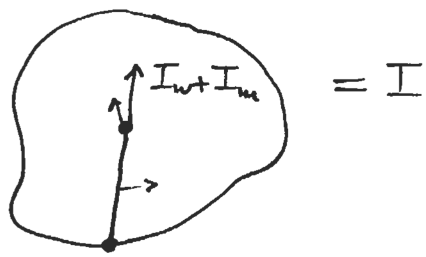
$$\int_{\hat{n}\cdot\hat{e}_\Omega} I_w \hat{e}_\Omega \cdot \hat{n} \, d\Omega = \frac{1}{4\pi}\int_{\hat{n}\cdot\hat{e}_\Omega > 0} (G + 3\overline{q}\cdot\hat{e}_\Omega)\hat{e}_\Omega \cdot \hat{n} \, d\Omega$$

$$\varepsilon I_{bw} + (1-\varepsilon)\frac{1}{4\pi}\left[G + 3\overline{q}\,\hat{e}_\Omega\right]_{\hat{e}_n \cdot \hat{n} < 0}$$

$$-\frac{2-\varepsilon}{\varepsilon} \cdot \frac{2}{3 - A_1\omega} \frac{1}{K_e} \hat{n} \cdot \overline{\nabla}_r G + G = 4\pi I_{bw}$$

ENHANCEMENTS TO THE P_1-APPROXIMATION

$\Longrightarrow$ MODIFIED DIFFUSION EQN.



E.R.T.

$$\frac{dI}{d\tau} = -I + S$$

$$\frac{dI_w}{d\tau} = -I_w$$

$$I_w = I_w(0)e^{-\tau}$$

$$\frac{dI_m}{d\tau} = S - I_m$$

$$I_m = \frac{1}{4\pi}\left[G_m + 3\bar{q}_m \cdot \hat{e}_\Omega\right]$$

$\uparrow$

$$G = G_m + G_w$$

$$G = \int I \, d\Omega$$

$$\bar{q} = \bar{q}_w + \bar{q}_m$$