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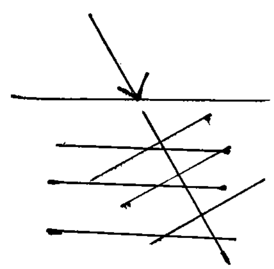
N_2 IS COMPLEX

$$n_1 \sin\theta_i = N_2 \sin\theta_t$$

$$\sin\theta_t = \frac{e^{i\theta_t} - e^{-i\theta_t}}{2i} = a + bi$$

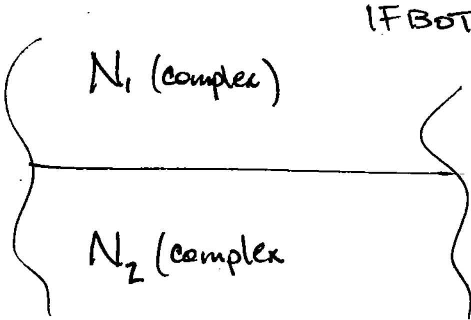
$$\cos\theta_t = a_1 + b_1 i$$

$$\vec{E}_{t\parallel} = \vec{E}_{t\parallel_0} \exp\left[-i\left(\omega t - k_{tx}x - a_1 z\right)\right] e^{-b_1 z}$$



INHOMOGENEOUS WAVE

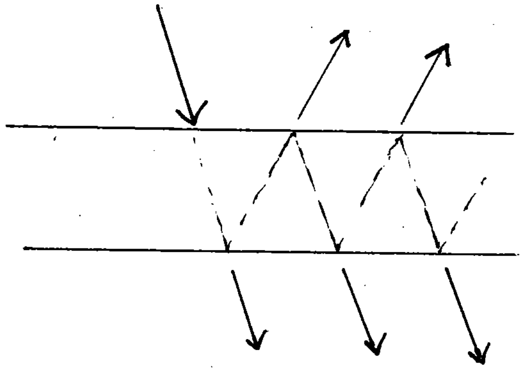
IF BOTH SIDES COMPLEX



$$R_\parallel + T_\parallel \neq 1$$

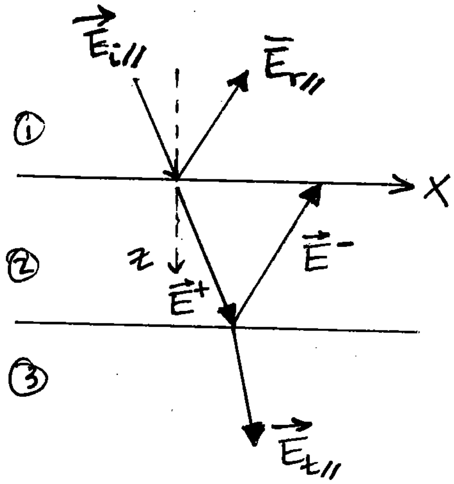
NON LINEAR OPTICS

$$\bar{D} = \varepsilon\bar{E} + \bar{\bar{\sigma}} : \bar{E}\cdot\bar{E} + (\quad)\bar{E}^3$$



RAY TRACING IS PHYSICALLY UNDERSTANDABLE, BUT MATHEMATICALLY NOT ELEGANT

SO,



$$\bar{E}^+ = \bar{E}_{\parallel}^+ \exp\left[-i\left(\omega t - k_{2x}x - k_{2z}z\right)\right]$$

$$\bar{E}^- = \bar{E}_{\parallel}^- \exp\left[-i\left(\omega t - k_{2x}x + k_{2z}\cdot z\right)\right]$$

INTERFACE 1: $E_{i\parallel}\cos\theta_i + E_{r\parallel}\cos\theta_i = E_{\parallel}^+\cos\theta_2 + E_{\parallel}^-\cos\theta_2$

$$n_1 E_{i\parallel} - n_1 E_{r\parallel} = n_2 E_{\parallel}^+ - n_2 E_{\parallel}^-$$

INTERFACE 2: $E_{\parallel}^+\cos\theta_2 \exp\left[i\,\frac{\omega n_2\cos\theta_2}{c_0}\,d\right] + E_{\parallel}^-\cos\theta_2 \exp\left[\frac{-i\omega n_2\cos\theta_2}{c_0}\right.$ d

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$$= E_{t\parallel} \cos\theta_t \exp\Big[i \underbrace{\frac{\omega n_3 \cos\theta_3 d}{c_0}}_{\equiv \phi_3}\Big]$$

$$n_2 E_\parallel^+ \exp[i\phi_2] - n_2 E_\parallel^- \exp[-i\phi_2] = n_3 \exp[i\phi_3] E_{t\parallel}$$

... WILL FIND

$$r_\parallel = \frac{E_{r\parallel}}{E_{i\parallel}} = \frac{r_{12\parallel} + r_{23\parallel} e^{2i\phi_2}}{1 + r_{12\parallel} r_{23\parallel} e^{2i\phi_2}}$$

* FACTOR OF 2 IN REFLECTION due to "ROUND TRIP"

$$t_\parallel = \frac{E_{t\parallel}(d)}{E_{i\parallel}} = \frac{t_{12\parallel} t_{23\parallel} e^{i\phi_2}}{1 + r_{12\parallel} r_{23\parallel} e^{2i\phi_2}}$$

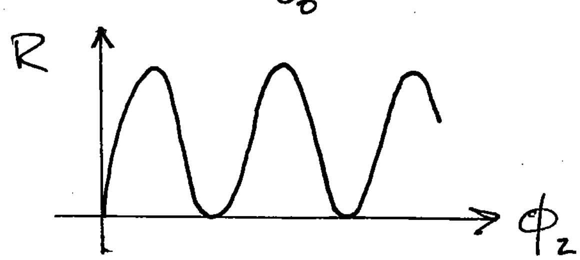
$$R_\parallel = |r_\parallel|^2, \quad T_\parallel = \frac{\mathrm{Re}(n_3 \cos\theta_3)}{\mathrm{Re}(n_1 \cos\theta_1)} |t_\parallel|^2$$

ASSUME ALL REAL

$$R = \frac{r_{12}^2 + r_{23}^2 + 2 r_{12} r_{23} \cos 2\phi_2}{1 + 2 r_{12} r_{23} \cos 2\phi_2 + r_{12}^2 r_{23}^2}$$

PERIODIC W.R.T. ϕ_2

$$\phi_2 = \frac{d\omega n_2 \cos\theta_2}{c_0} = \frac{2\pi n_2 d \cos\theta_2}{\lambda_0}$$



INTERFERENCE

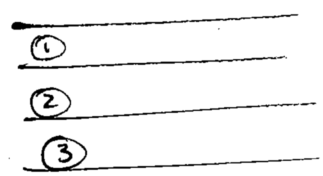
$$\frac{4\pi n_2 d\cos\theta_2}{\lambda_0} = m\pi$$

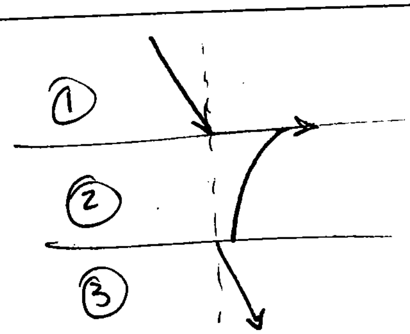
$$d\cos\theta_2 = \frac{m\lambda_0}{4n_2}$$

m is odd

$$R = \left(\frac{r_{12}-r_{23}}{1-r_{12}r_{23}}\right)^2 = \left(\frac{n_1 n_3 - n_2^2}{n_1 n_3 + n_2^2}\right)^2 \Longrightarrow$$

(USEFUL FOR ANALYZING ANTI REFLECTION COATINGS USING VARIOUS LAYERS)





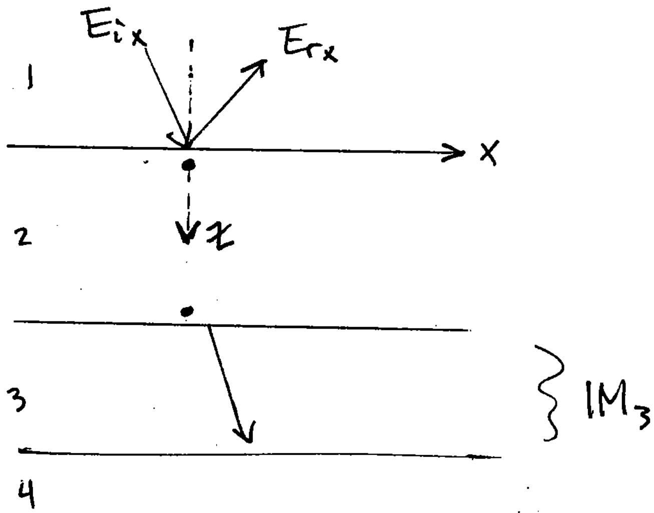
$$n_1\sin\theta_1 = n_2\sin\theta_2 = n_3\sin\theta_3$$

$n_1 > n_2$; BUT θ_i MUST BE SUCH THAT $\sin\theta_3$ IS REAL

POYNTING VECTOR IN A MATERIAL

CHECK THESE SIGNS !

$$\langle \bar{S}\rangle = \frac{1}{2}\mathrm{Re}\left[(\bar{E}^{+} + \bar{E}^{-}) \times (\bar{H}^{+*} + \bar{H}^{-*})\right] \neq S^{+} - S^{-}$$



$$\begin{pmatrix} E_x(z) \\ H_y(z) \end{pmatrix} = \begin{pmatrix} \cos\phi(z) & i p_2 \sin\phi(z) \\ \frac{i}{p_2}\sin\phi(z) & \cos\phi(z) \end{pmatrix} \begin{pmatrix} E_x(0) \\ H_y(0) \end{pmatrix}$$

$$\begin{pmatrix} E_x(d) \\ H_y(d) \end{pmatrix} = \mathbb{M}_2 \begin{pmatrix} E_x(0) \\ H_y(0) \end{pmatrix}$$

$$\begin{pmatrix} E_{x3}(d_3) \\ H_{y3}(d_3) \end{pmatrix} = \mathbb{M}_3\mathbb{M}_2 \begin{pmatrix} E_x(0) \\ H_y(0) \end{pmatrix}$$

p = Surface Impedance

$$p_2 = \frac{\cos\theta_2}{n_2/\mu c_0} \quad \text{TM}$$

$$\left\{ = -\frac{n_2\cos\theta_2}{\mu c_0} \right\} \quad \text{TE}$$

$$\begin{pmatrix} E_x(0) \\ H_y(0) \end{pmatrix} = \begin{pmatrix} 1 & 1 \\ \frac{1}{P_1} & -\frac{1}{P_1} \end{pmatrix} \begin{pmatrix} E_{ix} \\ E_{rx} \end{pmatrix}$$

~~* Correction~~

$$\begin{pmatrix} E_x(d) \\ H_y(d) \end{pmatrix} = \begin{pmatrix} 1 \\ \frac{1}{P_3} \end{pmatrix} E_{tx}$$

* correction

$$\begin{pmatrix} E_x(0) \\ H_y(0) \end{pmatrix} = \mathbb{M}_2 \mathbb{M}_3 \mathbb{M}_4 \cdots \mathbb{M}_N \begin{pmatrix} E_x d_n \\ H_y d_n \end{pmatrix}$$