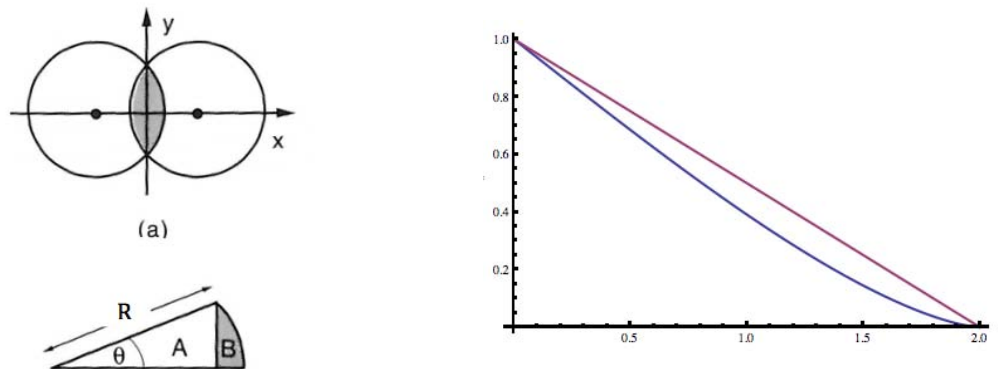Here I want to clarify on an approximation we made in solution one of the problem sets. We assumed the convolution of two circ functions is a triangular function in the polar coordinate. The first question is what the exact convolution of two circ functions is. Direct evaluation of the integral is somewhat challenging, nevertheless feasible! Goodman suggests a graphical way to evaluate the convolution that is to calculate the area of the overlap for two circles (of radius R) as their center moves relative to one another.

$$Area\left(A+B\right)=\left[\frac{\theta}{2\pi}\right]\left(\pi R^{2}\right)=\left[\frac{\cos^{-1}\left(r/2R\right)}{2\pi}\right]\left(\pi R^{2}\right)$$

$$Area\left(A\right)=\frac{1}{2}\left(r/2R\right)\sqrt{R^{2}-\left(r/2R\right)^{2}}\qquad (1)$$

$$circ\otimes circ=\frac{4\left[Area\left(A+B\right)-Area\left(A\right)\right]}{\pi R^{2}}$$

where r is the distance between the two circles. Substituting the areas, the end result for the convolution is,

$$\frac{2}{\pi}\left[\cos^{-1}\left(r/2R\right)-\left(r/2R\right)\sqrt{1-\left(r/2R\right)^{2}}\right]\qquad (2)$$

That is not clearly a line! The question is now how much this profile deviates from that of a straight line. We plot both of these profiles as seen in the figure below, the highest difference may be calculated and turns out to occur at $r/2R = 0.61$ that is about 10%. This is the error that is normally associated with the field. The rule of thumb for the error associated with the diffraction pattern that is proportional to the intensity is the square of this error that is approximately 1.25% at the worst. The error associated with both the field and intensity pattern is well within the range of engineering approximations. So you are allowed to make such an approximation as long as you are mindful of the scale of the error associated with it.

(a)

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