2.72 Elements of Mechanical Design
Spring 2009

Reading and plans

Shigley-Mischke sections

- None

Today: Actuators: Hydraulic and Electromagnetic

- Energy transfer and scale
- Hydraulic / fluidic
- DC Permanent magnet motors

- Perhaps…. wrap up of MEMS

Friction-based machines

Purpose:

- Do work at a given rate, Energy - Power

- Physics: Energy and mass conservation/balances

Characteristics of import

- Load

- Speed

- Bandwidth

- Cost

And there can be other issues of import…

Image removed due to copyright restrictions. Please see
http://www.onefunsite.com/images/donkey.jpg

Consequences

Please see trigirl. “Crane Drops Steamroller on Car!” May 8, 2007. LiveVideo. Accessed November 25, 2009. http://www.livevideo.com/video/16A18C6512B945C29547A8658E890AF1/crane-drops-steamroller-on-car.aspx

An unpleasant (I hope) example

Common actuators for mechanical systems

Biological

Pneumatic/Hydraulic

Electromagnetic

Electrostatic

Piezo

Thermal

Biological

People powered machines

Energy

Power

Load

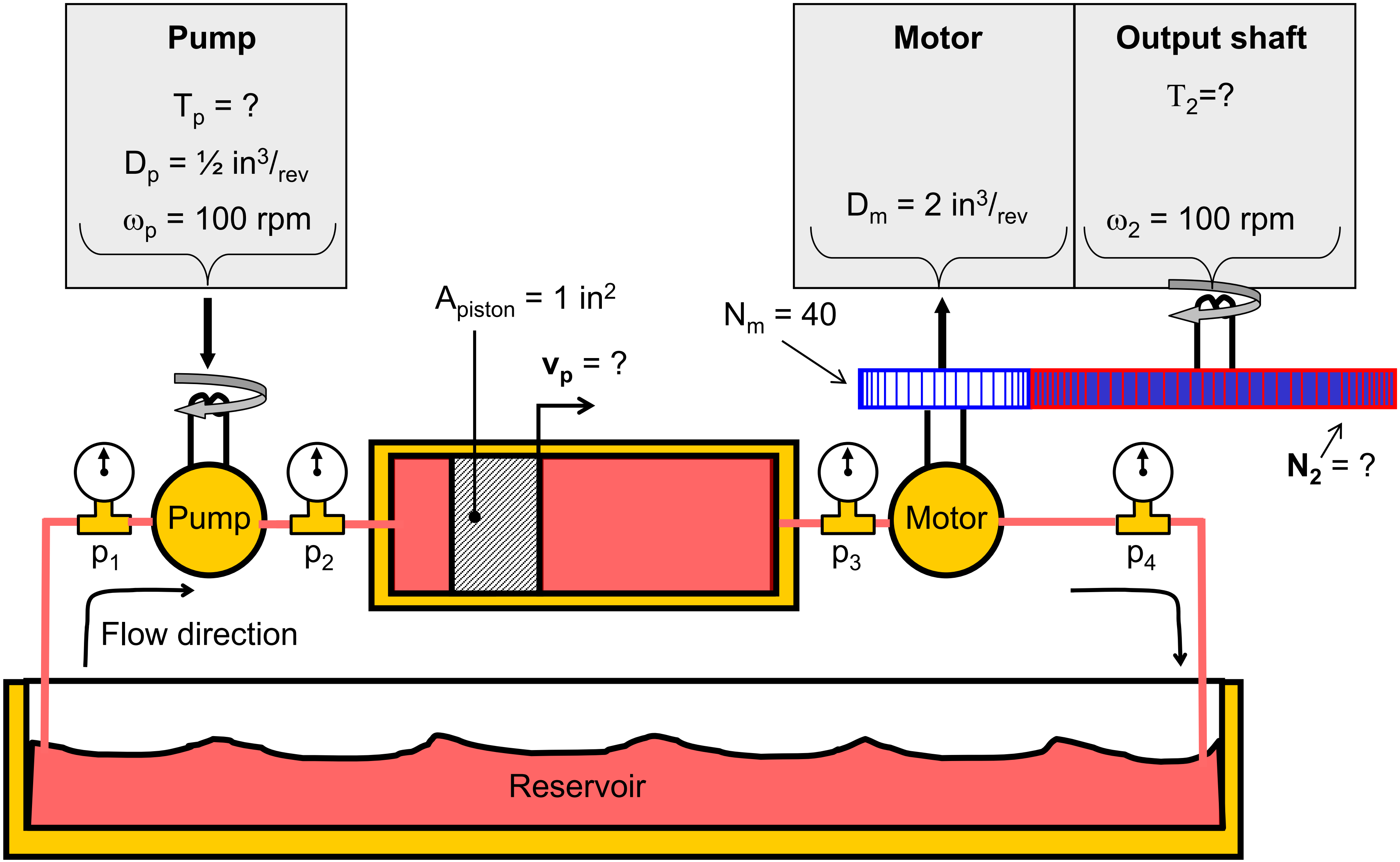
Speed

Bandwidth

Why is it important to understand what humans can do?

Hydraulics
Basic principles

Sub-system design



Examples: Real but practical ;) ?

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http://darkdiamond.net/wp-content/uploads/2006/08/115638952415_hugegundam1.JPG

http://gizmodo.com/

Other less than practical examples

Please see HydraulicGuitar. “Hydraulic Guitar.” September 10, 2006. YouTube. Accessed November 25, 2009. http://www.youtube.com/watch?v=Elt1XriaQXU

Other less than practical examples

Please see any video of a hydraulic low rider assembly.

Other less than practical examples

Please see arefadib. “The Flying Steamroller.” October 17, 2006. YouTube. Accessed November 25, 2009. http://www.youtube.com/watch?v=sKGRRIiR5xA

Hydraulic systems in machines

Advantage:

- High force/torque and routing of power

Disadvantage:

- Leaking and wear due to contaminants

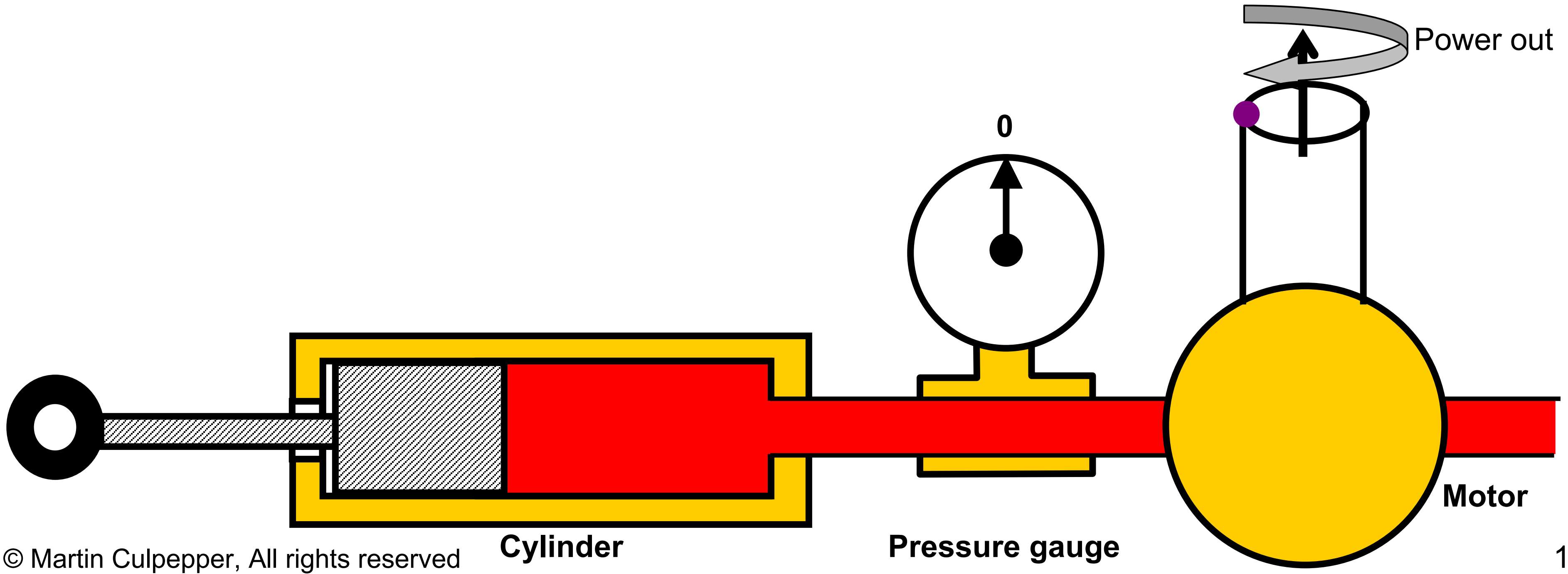
Liquids & gases in fluid-based machinery

- Hydraulics: Fluid is a liquid
- Pneumatics: Fluid is a gas

Example: Piston pump doing work

Hydraulic machines can be used to do work

- Load on the system extracts energy from the liquid
- Pressure increases between the input and output components
- Pressure is used to do work on internal parts of hydraulic devices
- Power is input/extracted via shaft (motor) or rod (cylinder)



Volume flow rate and displacement

Displacement (D)

- Displacement = volume of fluid moved / cycle
- Cycle = rotation (drill pump) or stroke (cylinder)

Q = volume moved per unit time

- D * f = Q

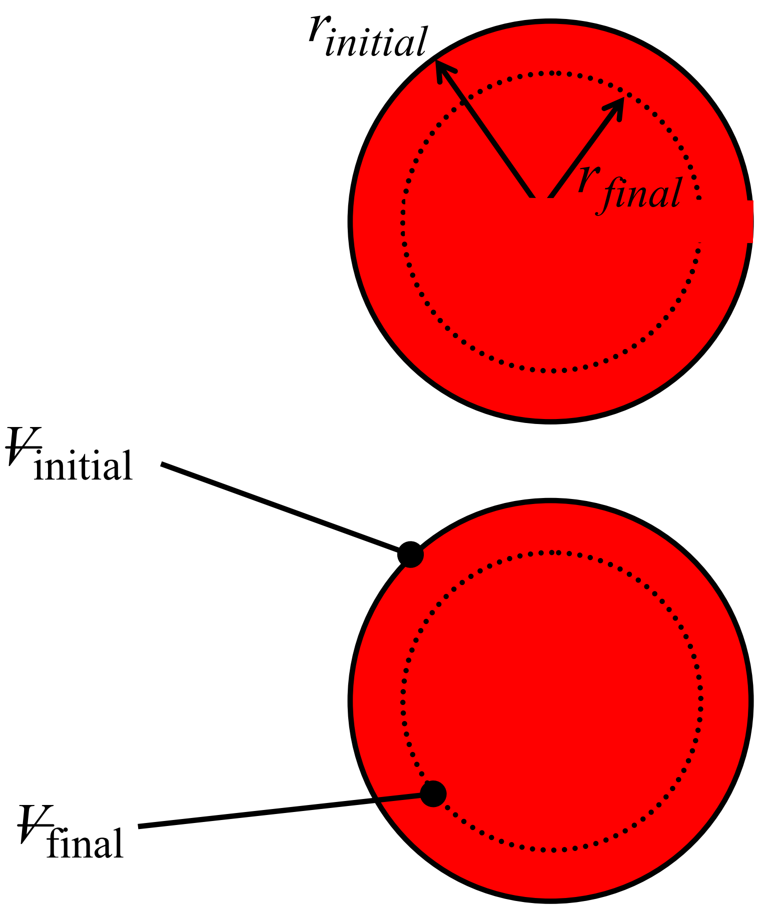
$$\frac{\text{Volume}}{\text{Cycle(s)}} * \frac{\text{Cycles}}{\text{second}} = \frac{\text{Volume}}{\text{second}}$$

- F is the frequency of a machine's cycle
  - For hydraulic pumps, f = speed of the shaft = $\omega/(2\pi)$ ω [rad/s]
  - For hydraulic motors, f = speed of the shaft = $\omega/(2\pi)$ ω [rad/s]
  - For cylinders, f = strokes/second   f [Hertz]

Displacement: Physical example

D = volume pumped per cycle

1 Cycle = expansion + contraction



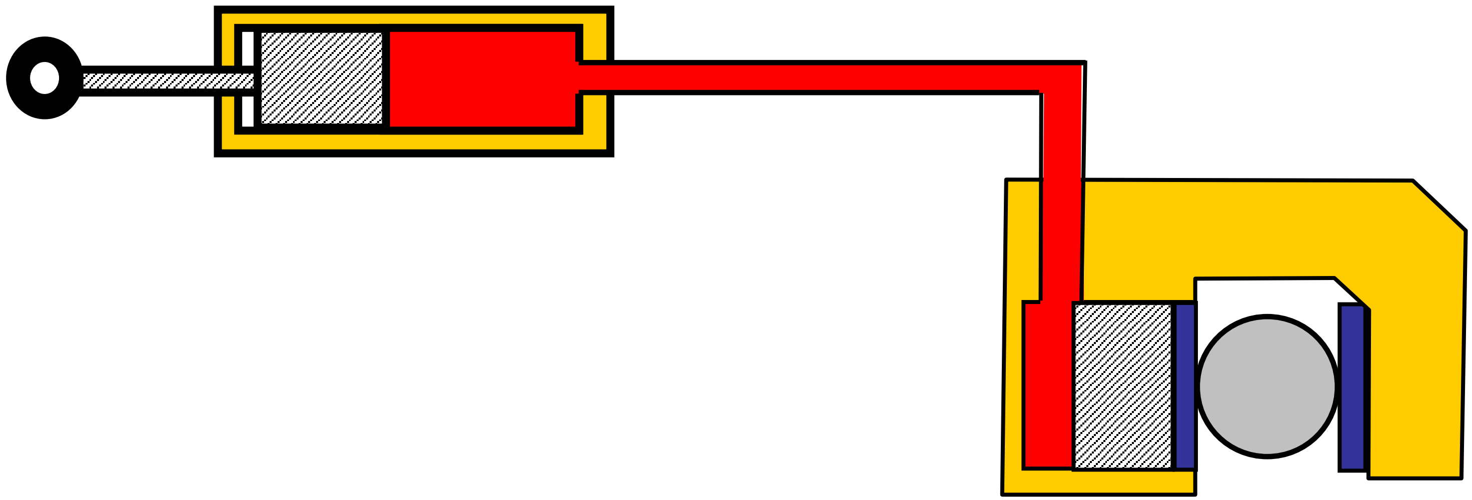
$$V_{\text{initial}} = \frac{4}{3} \cdot \pi \cdot \left(r_{initial}\right)^3$$

$$V_{\text{final}} = \frac{4}{3} \cdot \pi \cdot \left(r_{final}\right)^3$$

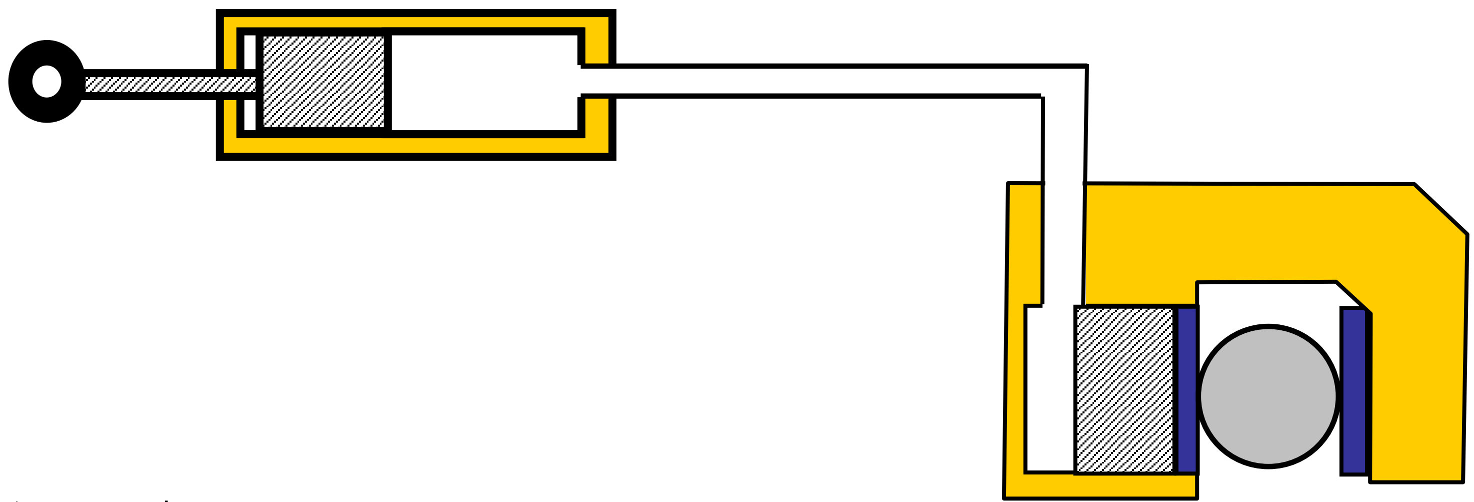
$$\text{Displacement} = D = V_{\text{initial}} - V_{\text{final}} = \frac{4}{3} \cdot \pi \cdot \left[\left(r_{initial}\right)^3 - \left(r_{final}\right)^3\right]$$

Incompressibility

Incompressible fluid:



Compressible fluid:



Why is incompressibility important?

Mass balances

- The mass density (ρ_m) of fluids changes with pressure (Δp)
- Compressible fluids: exhibit large ($\Delta\rho_m$) for small (Δp)
- If ($\Delta\rho_m$) is large, it is possible to store significant mass in a machine
- This complicates our analysis

$$\Sigma \dot{m}_{in} = \Sigma \dot{m}_{out} + \frac{d}{dt} m_{stored}$$

Why is incompressibility important?

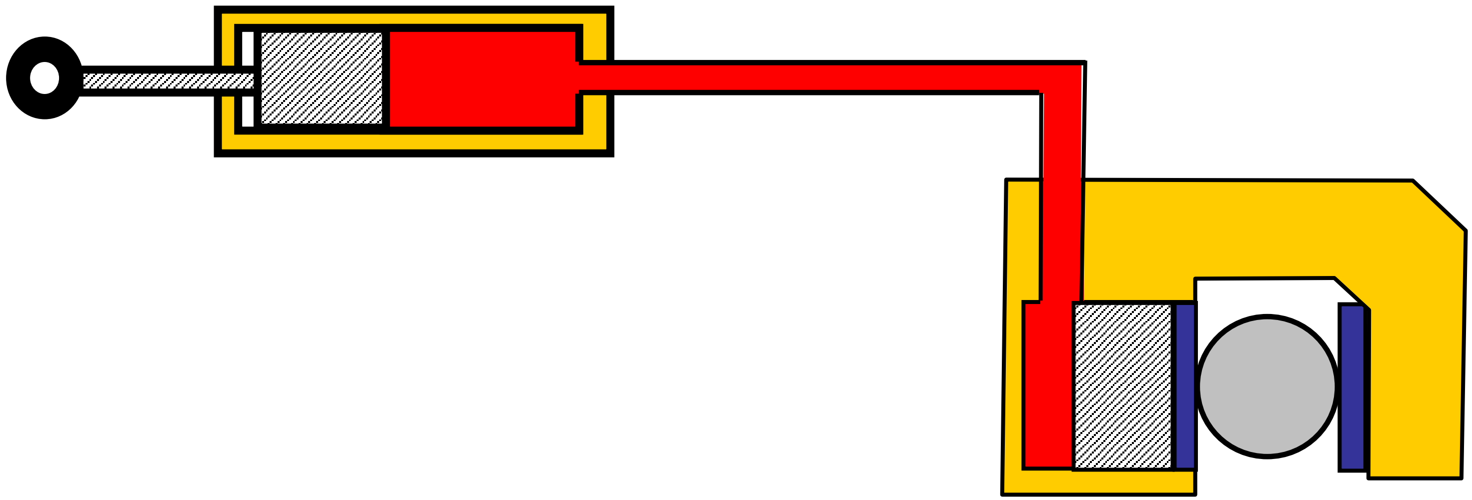
Energy balances

- All fluids store energy when compressed (similar to a spring)
- Compressible fluids store A LOT of energy (think balloons!!)
- Stored energy complicates analysis (calculating can be difficult)

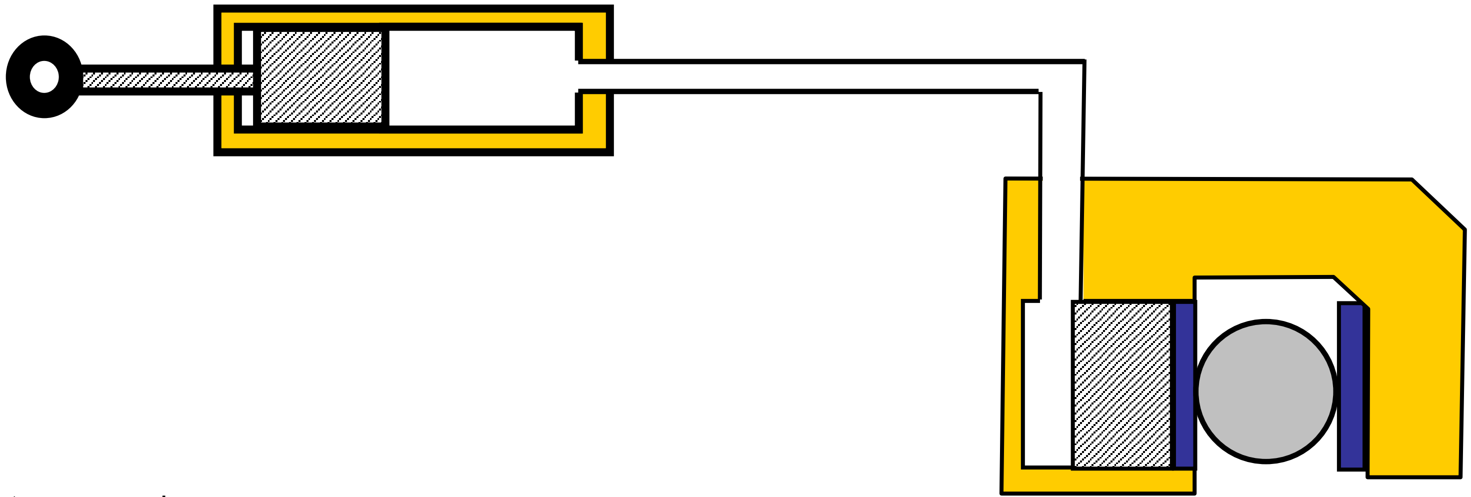
$$\Sigma E_{in} = \left[\Sigma E_{out}\right] + \Sigma E_{stored}$$

Example: "Locked" piston positions

Incompressible fluid:



Compressible fluid:



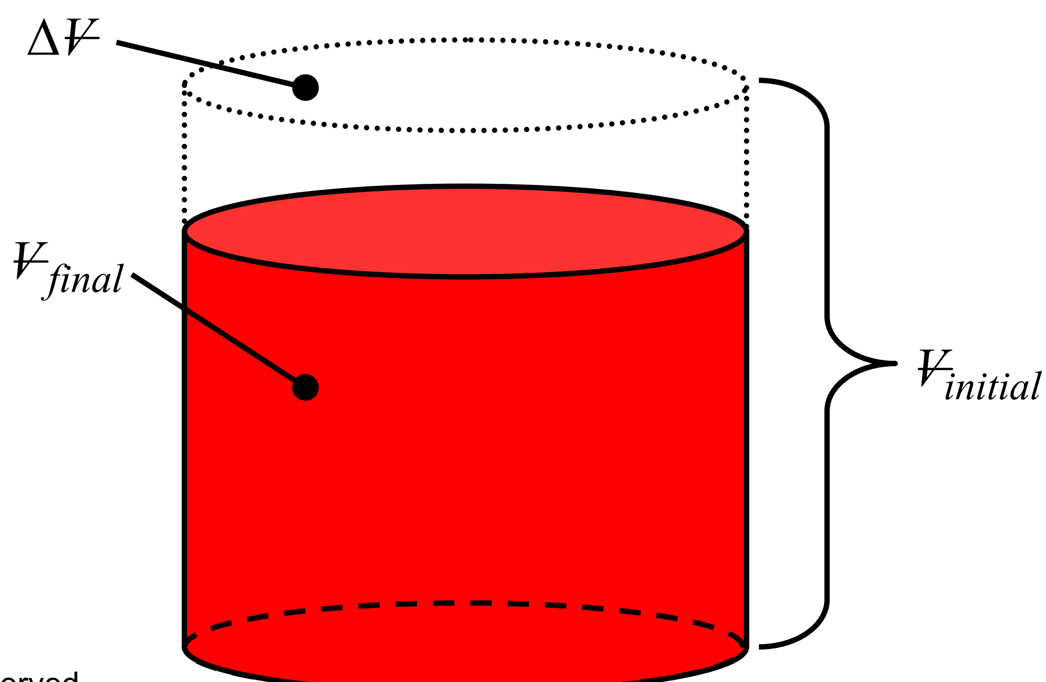
Incompressibility

Bulk modulus: Measures of resistance to Δvolume

$$\beta = -\frac{dp}{\left(\frac{dV}{V_{initial}} \right)}$$

For small (incremental) changes in volume: $\beta \approx -\frac{\Delta p}{\left(\frac{\Delta V}{V_{initial}} \right)}$

Example: Fluid in tube exposed to pressure increase



Incompressibility

Hydraulics, pneumatics and incompressibility

- Pneumatics = gas: Low β, usually compressible
- Hydraulics = liquid: High β, usually incompressible

What makes a good assumption?

- Depends on the error you are willing to live with

Example: Incompressibility of water (e.g. H_2O)

$$\beta_{H_2O} = 2.2 \times 10^9 \frac{\text{N}}{\text{m}^2} = 3.2 \times 10^5 \frac{\text{lbf}}{\text{in}^2} \longrightarrow \left(\Delta V / V \right) \approx -\frac{\Delta p}{\beta}$$

Example for H_2O where $\Delta p = 2500$ psi, $\Delta V / V = -0.006 = -0.6\%$

Is this OK?

Volume flow rate, Q

Link between mass flow rate & volume flow rate:

$$Q = \text{time rate of volume flow through a hydraulic system}$$

From mass conservation

From 8.01

$$Q_i = \frac{\dot{m}_i}{\rho_{mi}} = \frac{\rho_{mi} \cdot A_i \cdot \mathrm{v}_i}{\rho_{mi}}$$

$$\frac{d}{dt}\left(\forall_{stored}\right) \sim 0$$

$$\frac{\Sigma \dot{m}_{in}}{\rho_{min}} = \frac{\Sigma \dot{m}_{out}}{\rho_{mout}} + \frac{\frac{d}{dt} m_{stored}}{\rho_{mstored}} \rightarrow Q_{in} = Q_{out} + \frac{d}{dt}\left(\forall_{stored}\right)$$

Mass densities are equal and cancel out of equation if fluid is incompressible

For incompressible flow in a pipe: $A_{in} \cdot \mathrm{v}_{in} = Q_{in} = Q_{out} = A_{out} \cdot \mathrm{v}_{out}$

Vane pumps

Series of vanes extending radially from rotating core

- Vanes can slide in/out or deform depending upon design

How a sliding vane pump works:

- Step 1: Fluid enters when volume between vanes is increasing

- Step 2: Fluid travels when volume between vanes does not change

- Step 3: Fluid exits at when volume between vanes is decreasing

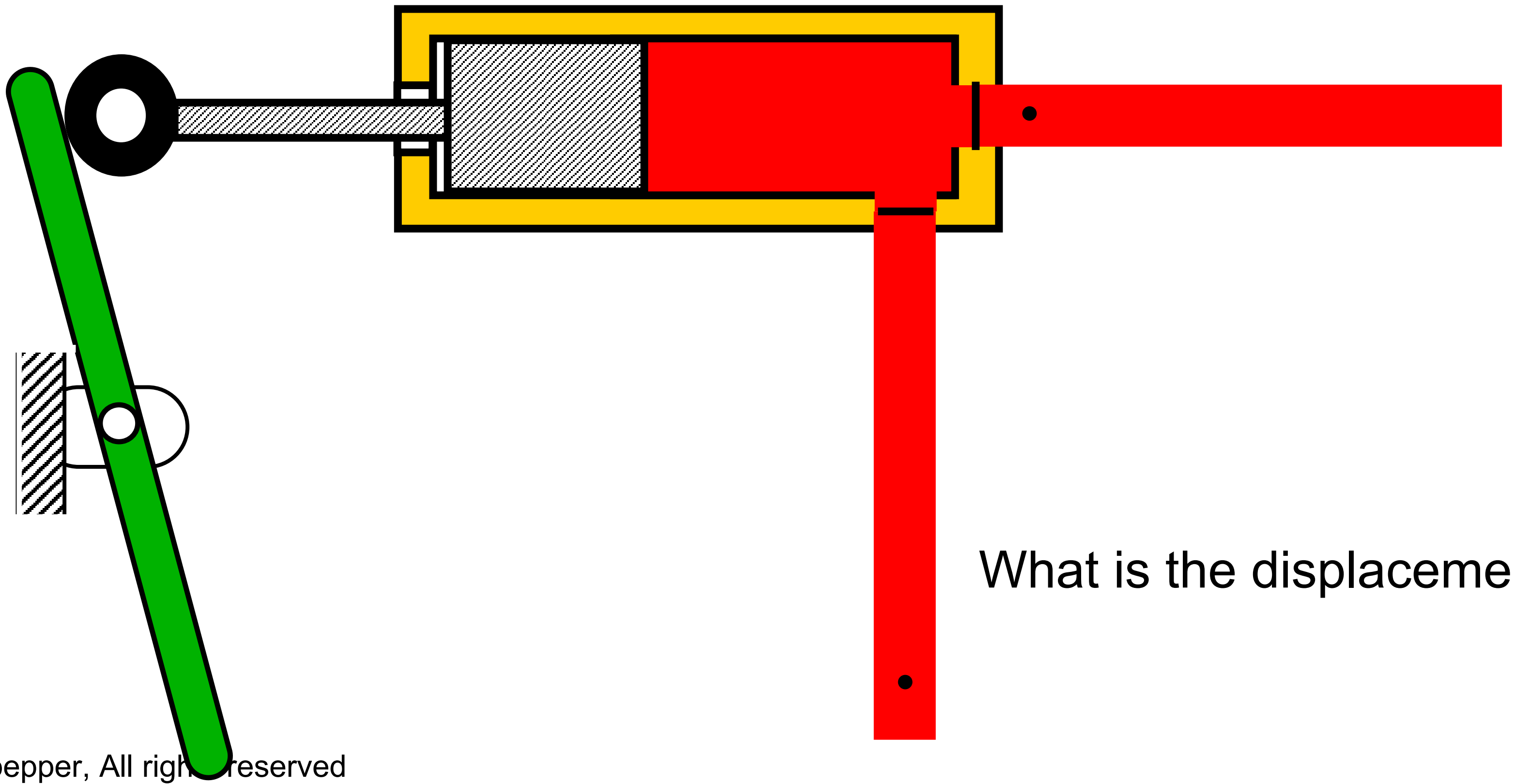
Image removed due to copyright restrictions. Please see
http://pumpschool.com/images/vnsteps.gif

credit: pumpschool.com

Pump types: Piston

How it works:

- Step 1: Piston forces fluid out during initial stroke
- Step 2: Valves change fluid path (only allows flow into pump)
- Step 3: Piston recharged with fluid, cycle starts again



What is the displacement?

Pump types: Piston

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http://www.animatedsoftware.com/pics/pumps/wobble.gif
http://www.flexicad.com/bilder/Rhino_Galerie/Kolpenpumpe.jpg

http://gallery.mcneel.com/fullsize/11155.jpg

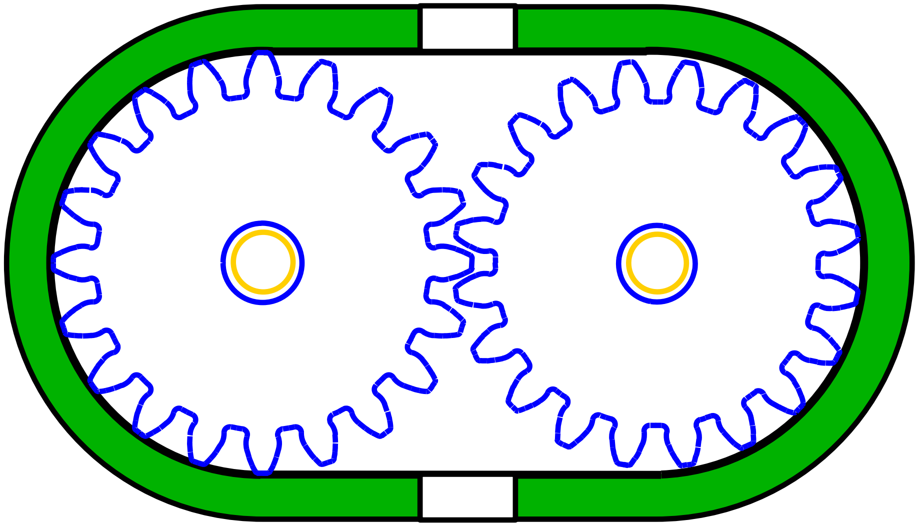
www.animatedsoftware.com/pumpglos/wobble.htm

Pump types: External gear pump

Only one gear is driven, the other spins free

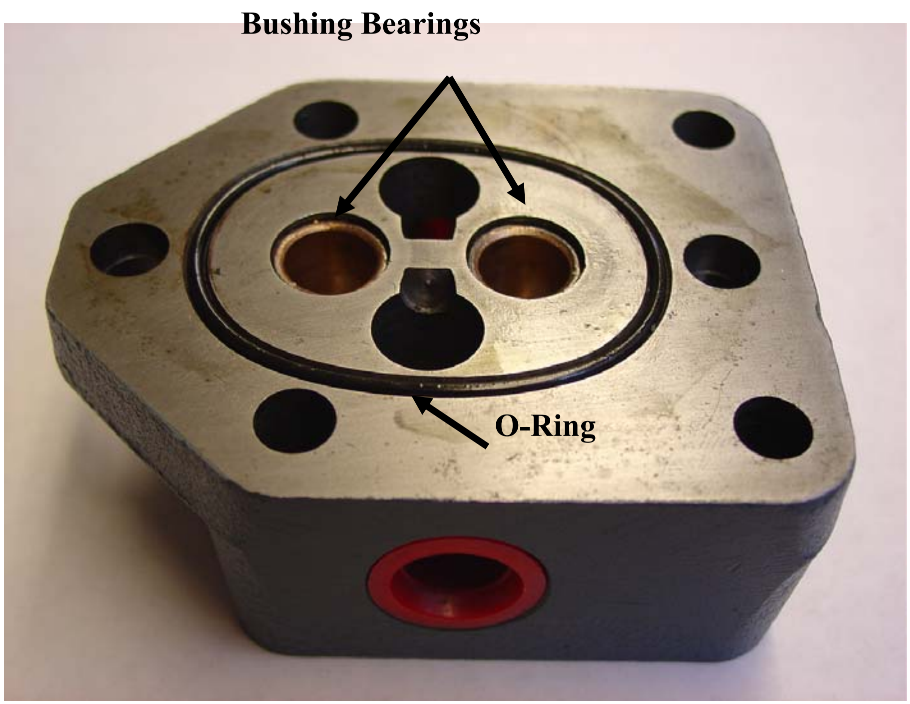
Which way does the flow go?

- Step 1: Fluid comes in at ?
- Step 2: Fluid travels through ?
- Step 3: Fluid exits at ?



What is the displacement?

Pump types: External gear pump



a) Pump Body

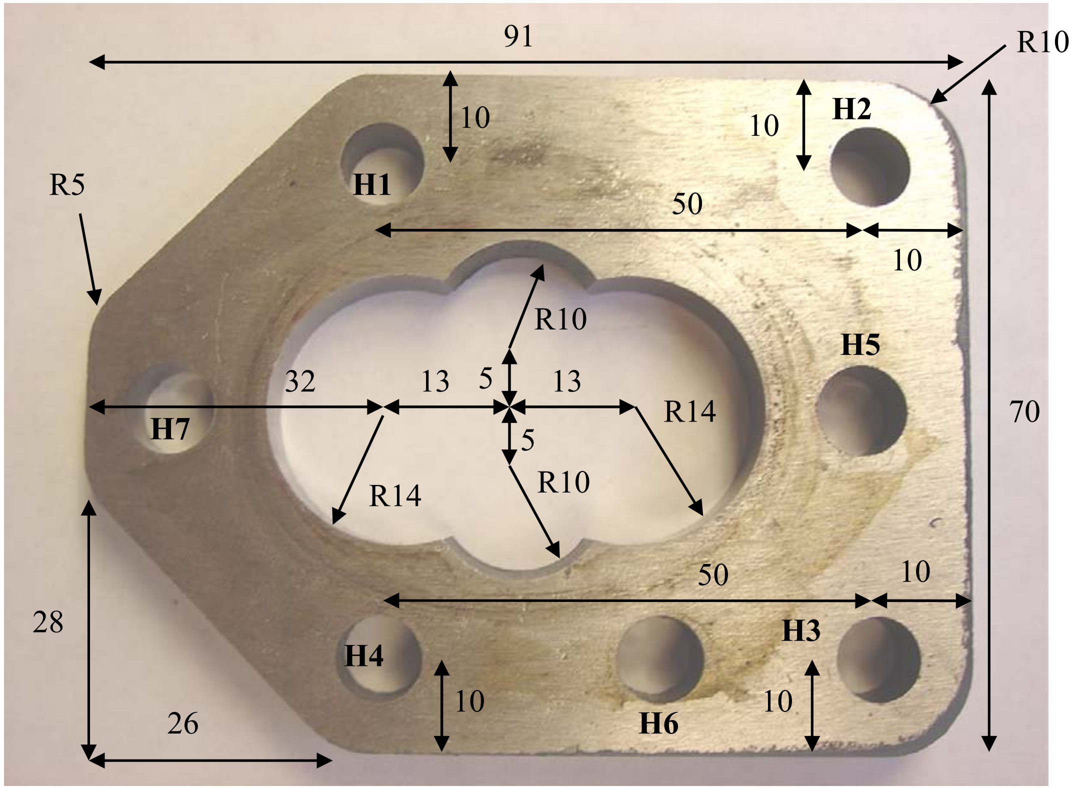
a) Gear and Shaft

Hydraulics Exercise

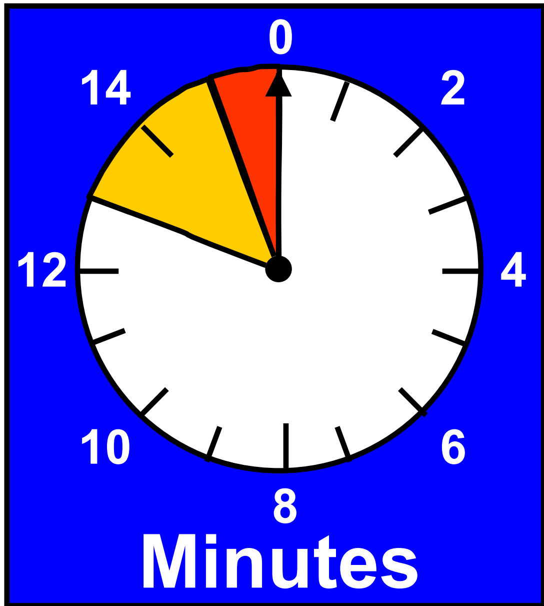
Competition: Pump

Form group

- In 10 minutes, make best estimate of gear pump displacement
- Hand in answer/analysis at end of exercise (with all names)
- Sketches, calculations, etc… must be handed in before bell sounds

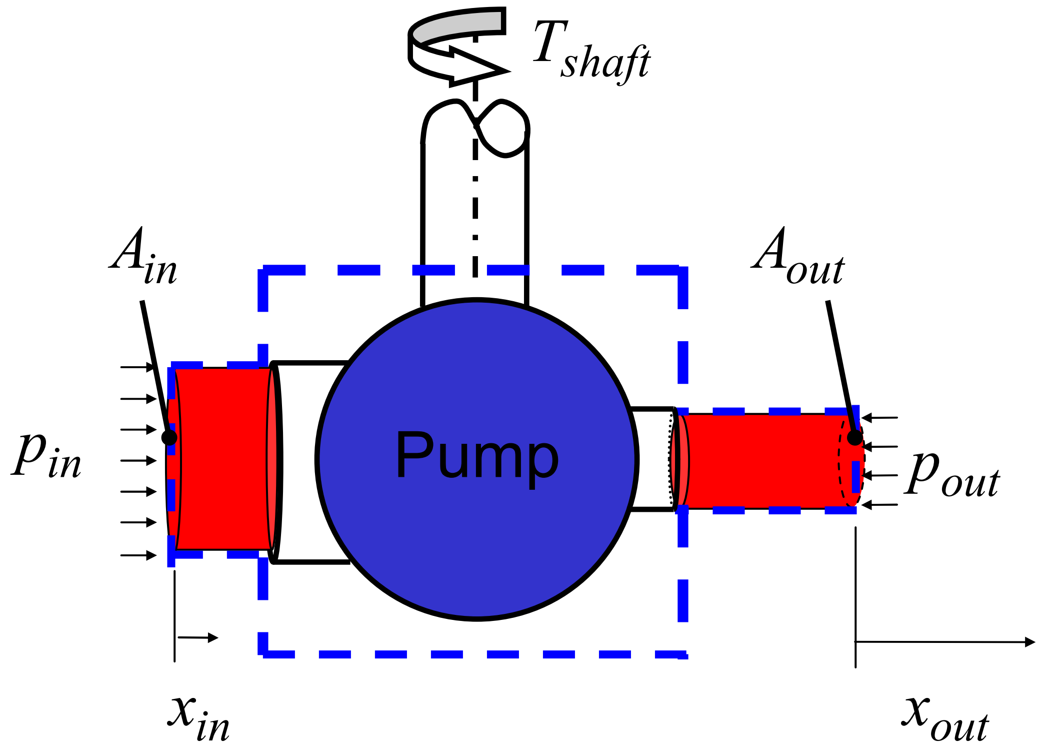


Gear Pump Cavity Plate with Dimensions
All Dimensions in mm



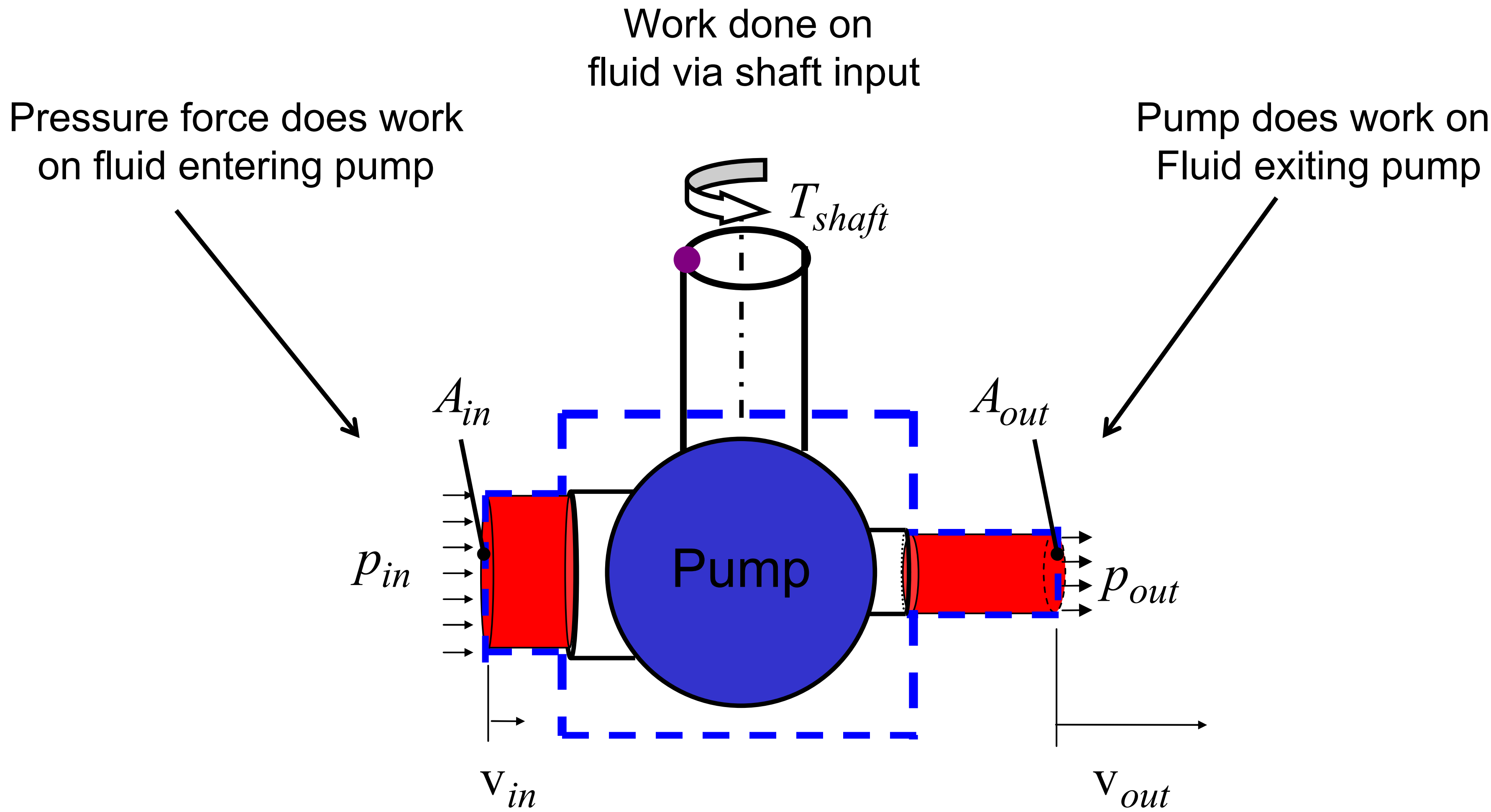
Hydraulics power

Power example: Pump at steady state



$$A_{in} \cdot \mathrm{v}_{in} = Q_{in} = Q_{out} = A_{out} \cdot \mathrm{v}_{out}$$

Example: Pump at steady state



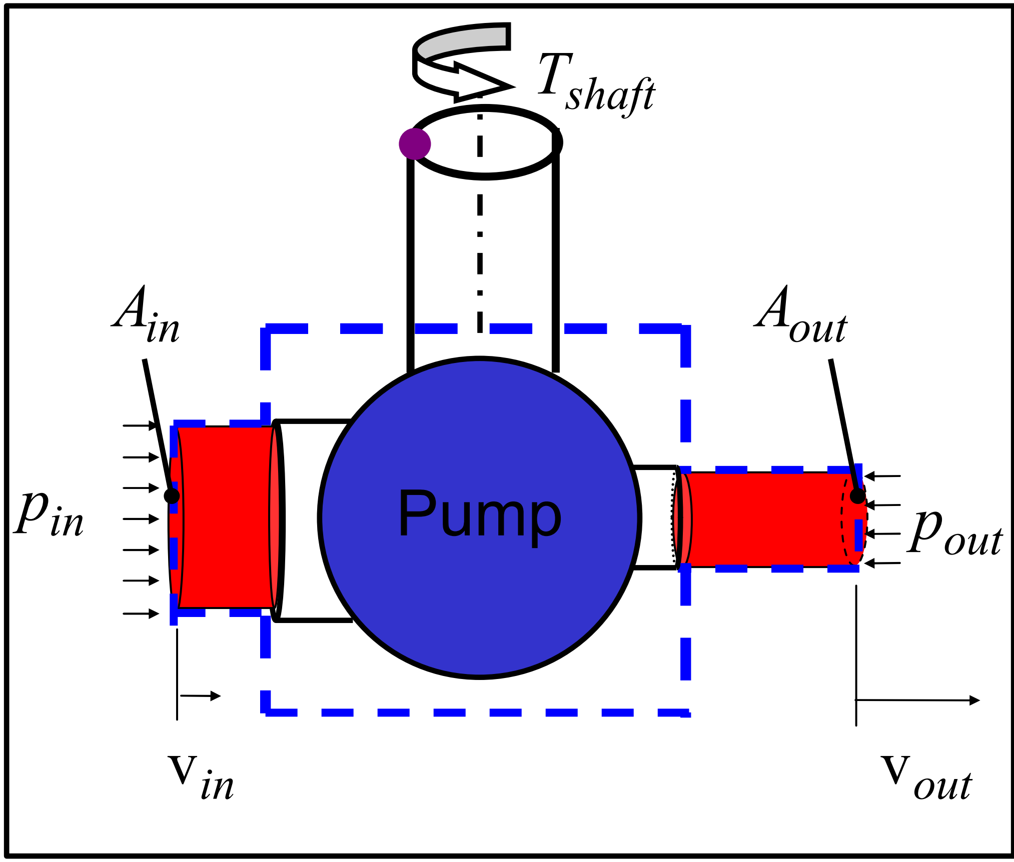
Example: Pump at steady state

$$\Sigma P_{in} = [\Sigma P_{out}] + \frac{d}{dt}(E_{stored}) \rightarrow P_{inlet} + P_{shaft} = [P_{outlet} + P_{loss}] + \frac{d}{dt}(E_{stored})$$

$$P_{inlet} = [F_{in}] \cdot \text{v}_{in} = [p_{in} \cdot A_{in}] \cdot \text{v}_{in}$$

$$P_{shaft} = T_{shaft} \cdot \omega_{shaft}$$

$$P_{outlet} = [F_{out}] \cdot \text{v}_{out} = [p_{out} \cdot A_{out}] \cdot \text{v}_{out}$$



If can assume A&B, P_{loss} & d(E)/dt can be neglected

A. $\frac{d}{dt}(E_{stored}) <<< P_{in} - P_{out}$

B. $P_{loss} <<< P_{in} - P_{out}$

Substituting into energy balance (top equation on sheet)

$$[p_{in} \cdot A_{in}] \cdot \text{v}_{in} + T_{shaft} \cdot \omega_{shaft} = [[p_{out} \cdot A_{out}] \cdot \text{v}_{out} + \sim 0] + \sim 0$$

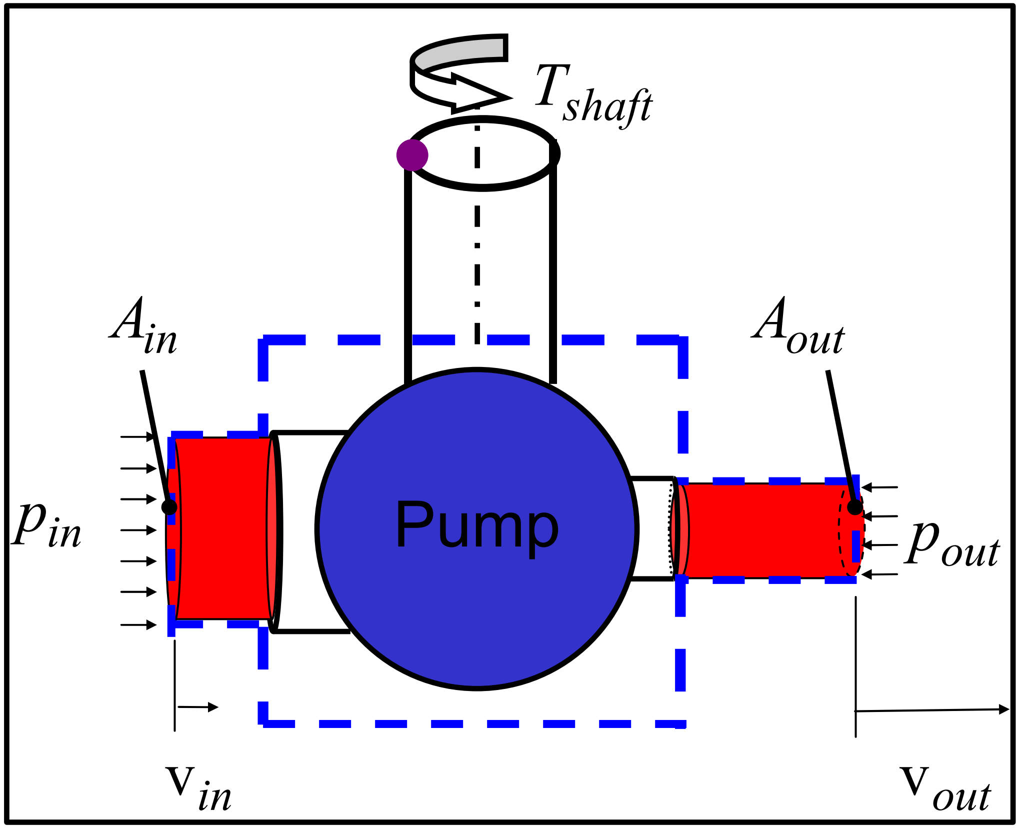
Power example: Pump at steady state

$$[p_{in}\cdot A_{in}]\cdot \mathrm{v}_{in} + T_{shaft}\cdot\omega_{shaft} = [[p_{out}\cdot A_{out}]\cdot \mathrm{v}_{out} + \sim 0] + \sim 0$$

$$A_{in}\cdot \mathrm{v}_{in} = Q_{in} = Q_{out} = A_{out}\cdot \mathrm{v}_{out}$$

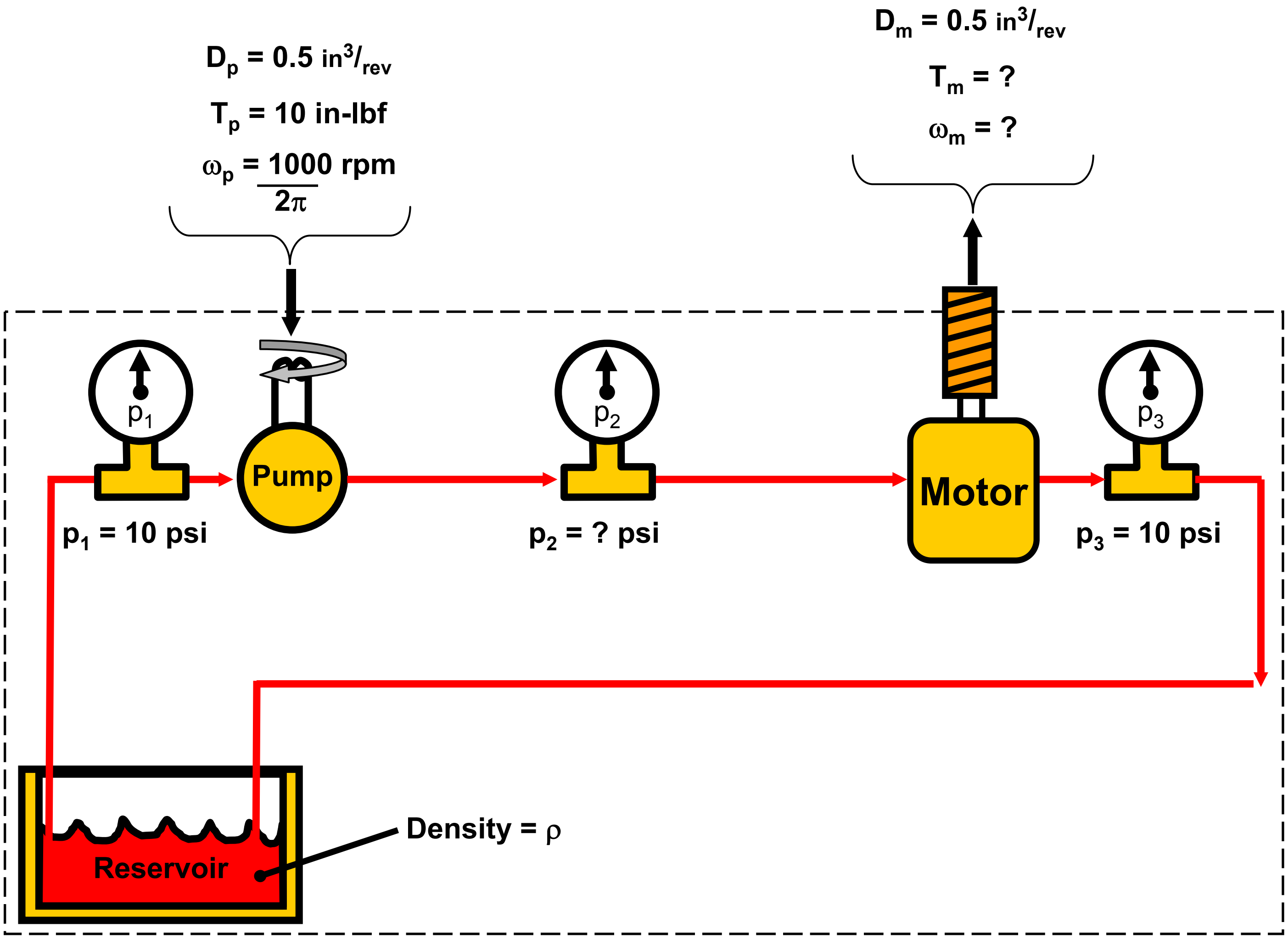
$$Q_{in} = Q_{out} = Q$$

$$[p_{out} - p_{in}]\cdot Q = T_{shaft}\cdot\omega_{shaft}$$



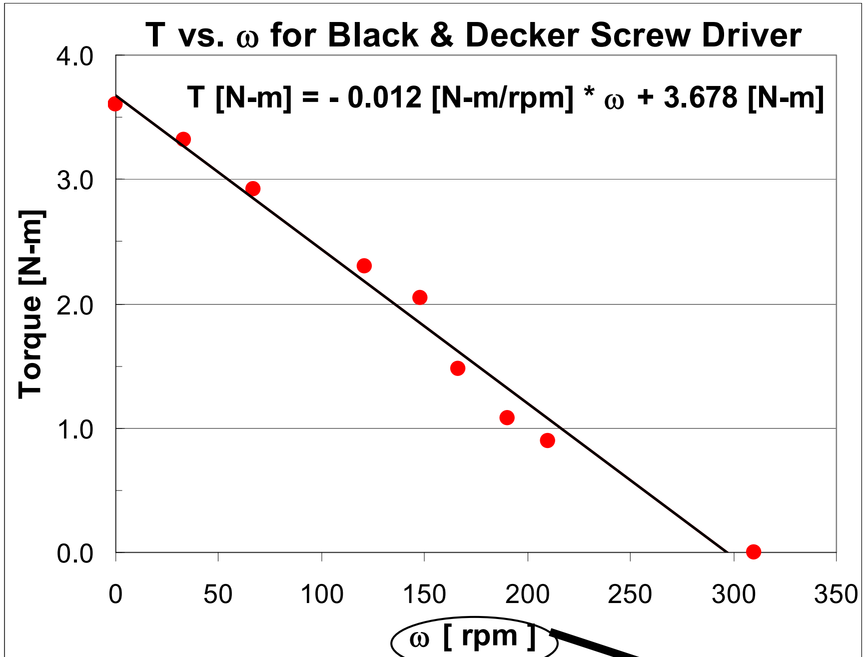
Hydraulics System example

Power example: Pump at steady state

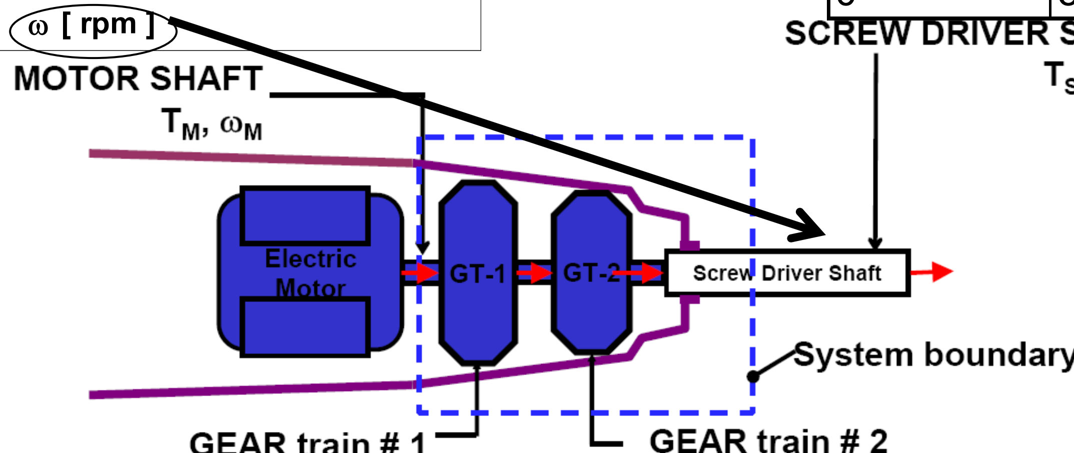


DC permanent magnet motors

DC Permanent magnet motor

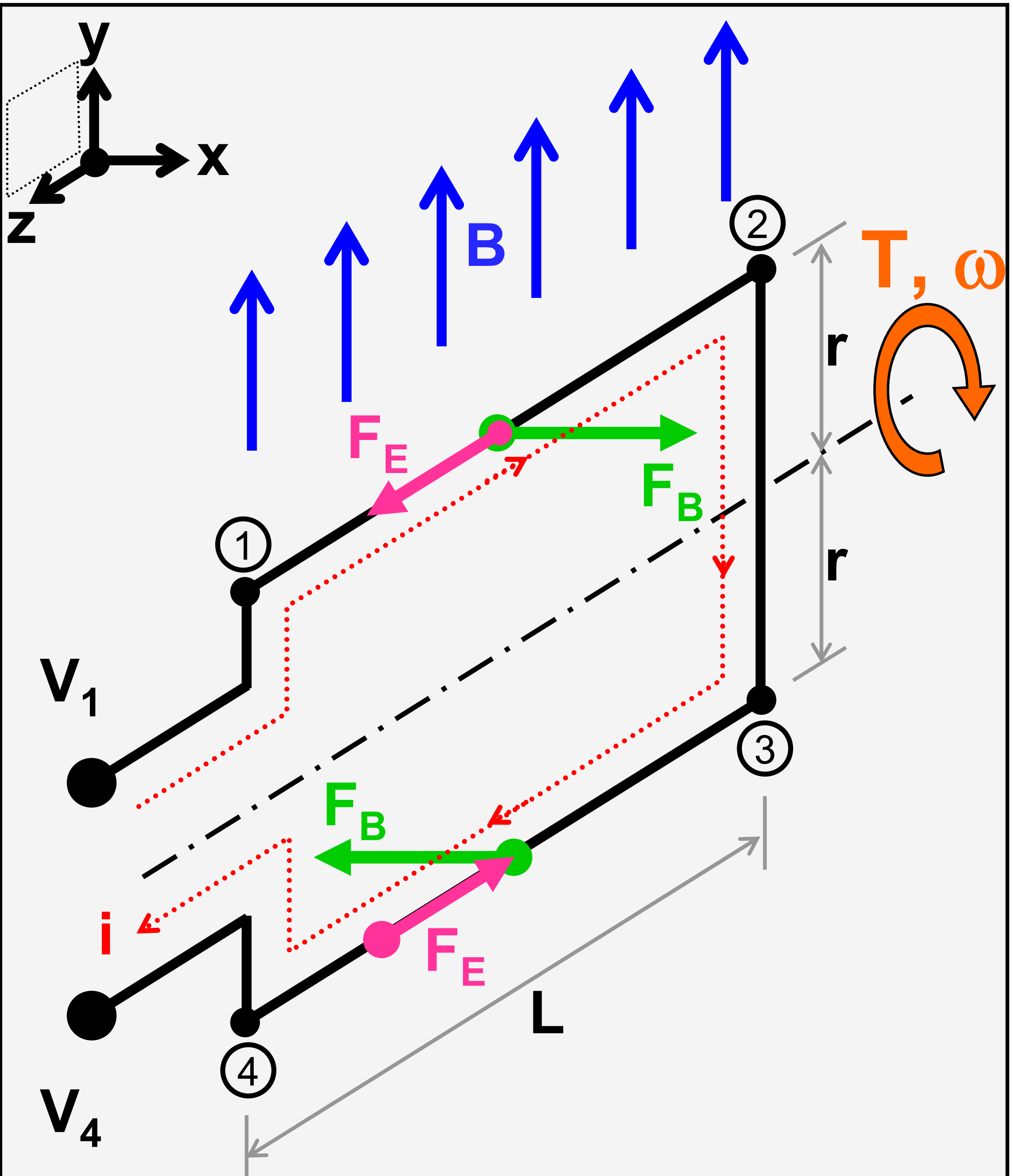


ω rpm Data	T N m Data	T N m Fitted	% Error
310	0.00	-0.09	N/A
210	0.90	1.11	23
190	1.08	1.34	24
167	1.47	1.63	10
148	2.04	1.85	-9
121	2.30	2.17	-6
67	2.92	2.83	-3
33	3.32	3.23	-3
0	3.60	3.63	1



$$T(\omega) = T_{Stall}\left[1 - \frac{\omega}{\omega_{NL}}\right]$$

Understanding the model



Simple 1 loop model

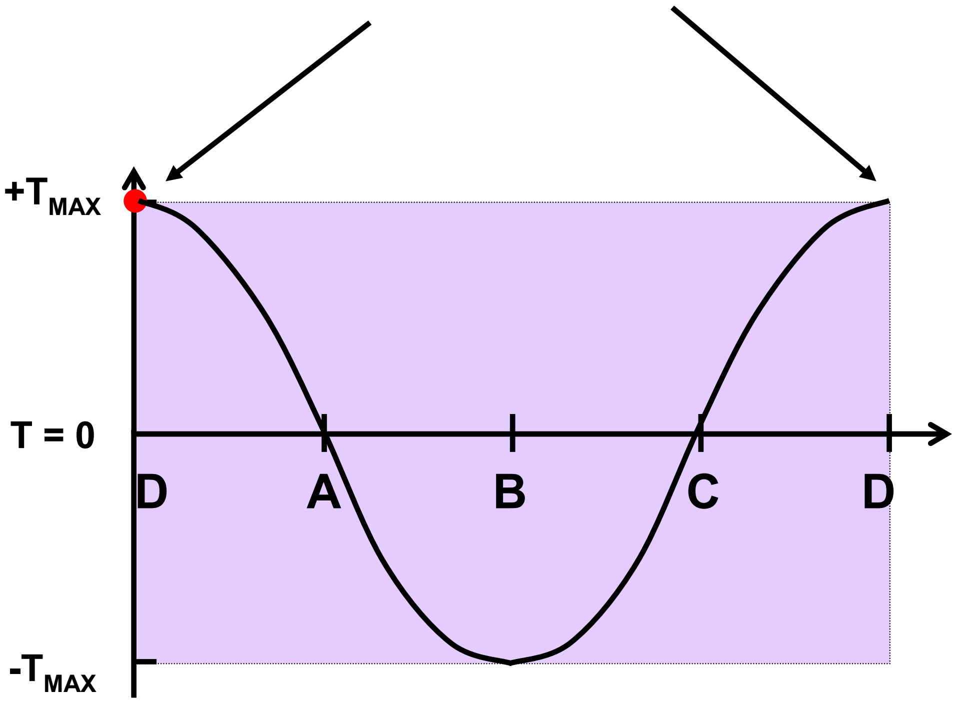
- Goal: understand trends

Assumptions

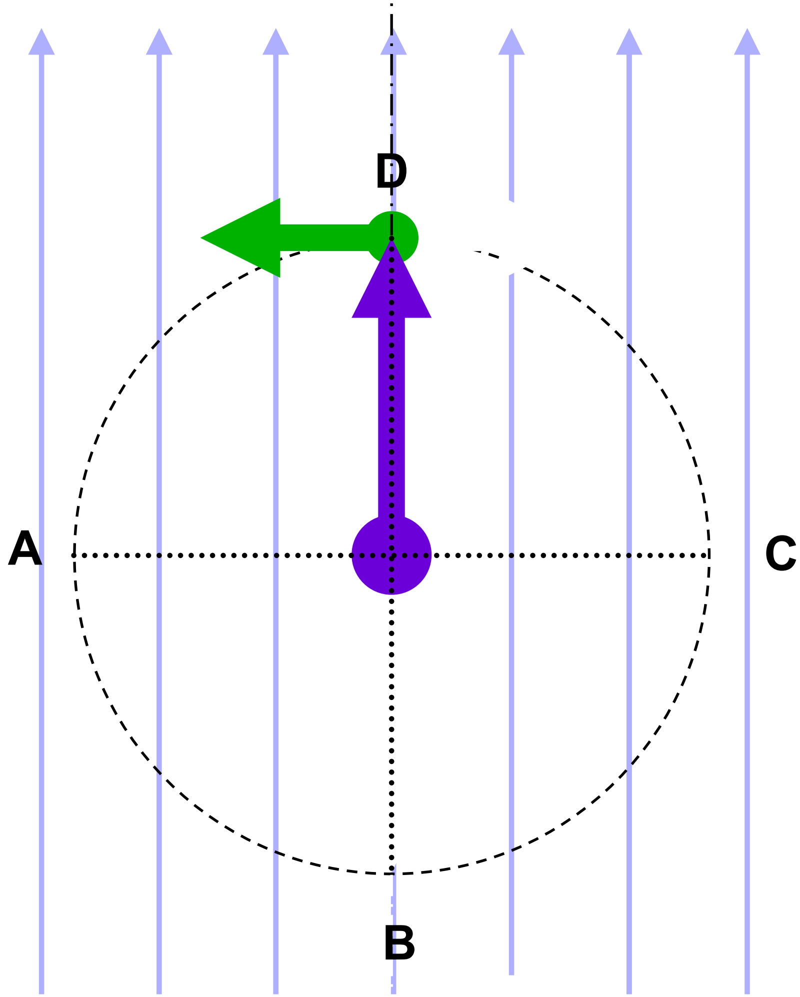
- Low loss in wires
- Steady state
- Single loop
- No ferrous cores
- Snap shot with loop plan in the y-z plane

Point we will study

Points we are studying

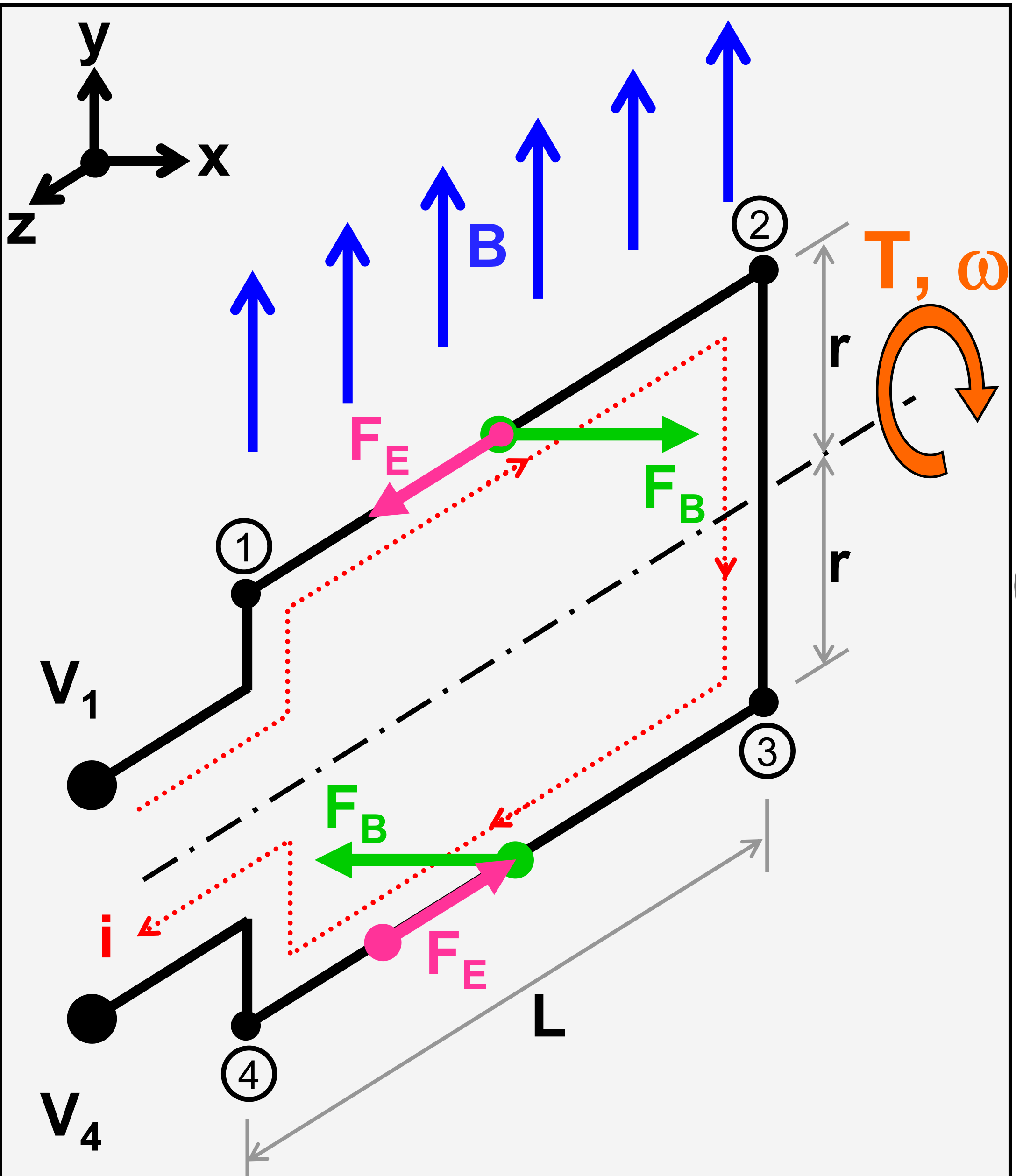


Torque curve of simple loop



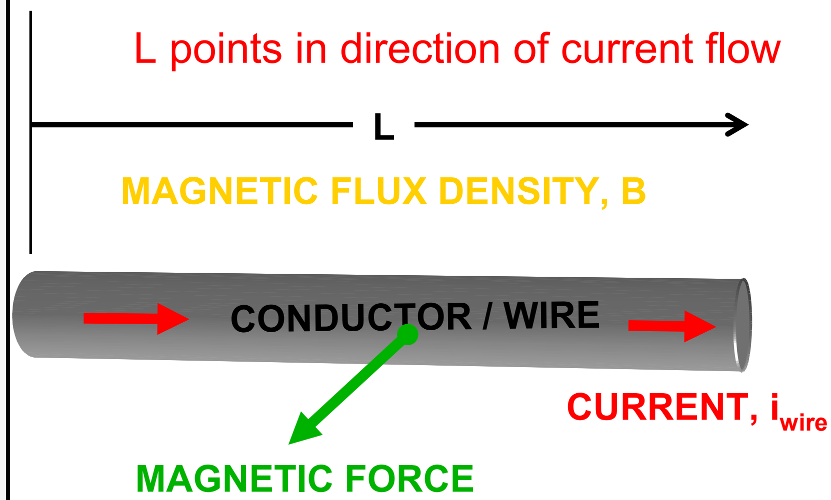
Side view of simple loop

Forces



Force on wire

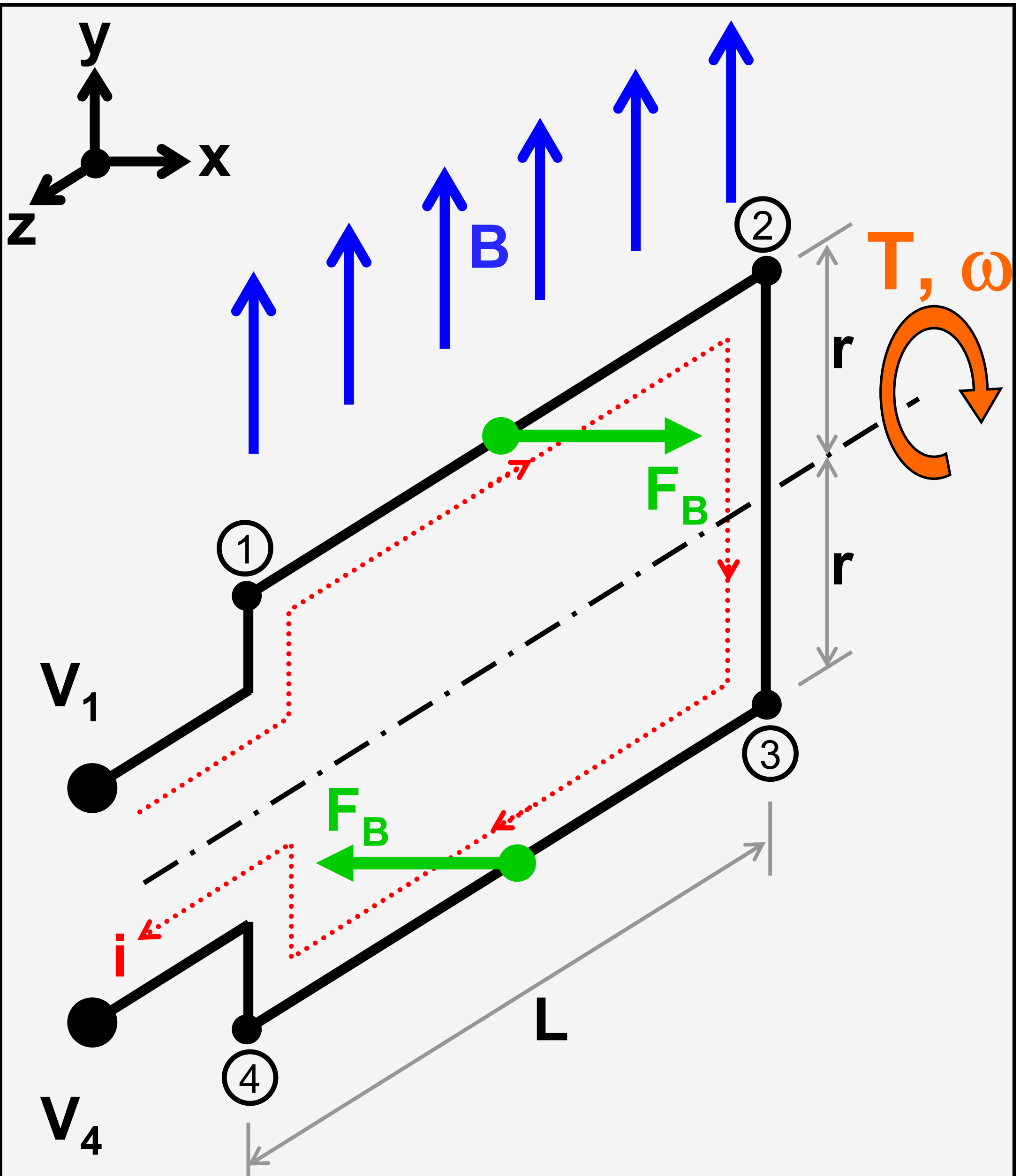
$$\vec{F}_B = i \cdot \left(\vec{L} \times \vec{B} \right)$$



Lorentz Force

$$\vec{F}_E = q \cdot \vec{E} + q \cdot \vec{v} \times \vec{B}$$

Torque inducing forces on wire



Force on wire

$$\vec{F}_B = i \cdot \left(\vec{L} \times \vec{B} \right)$$

Torque at ω = 0

$$\vec{T} = 2 \cdot \left(\vec{r} \times \vec{F}_B \right)$$

90° in y-z plane

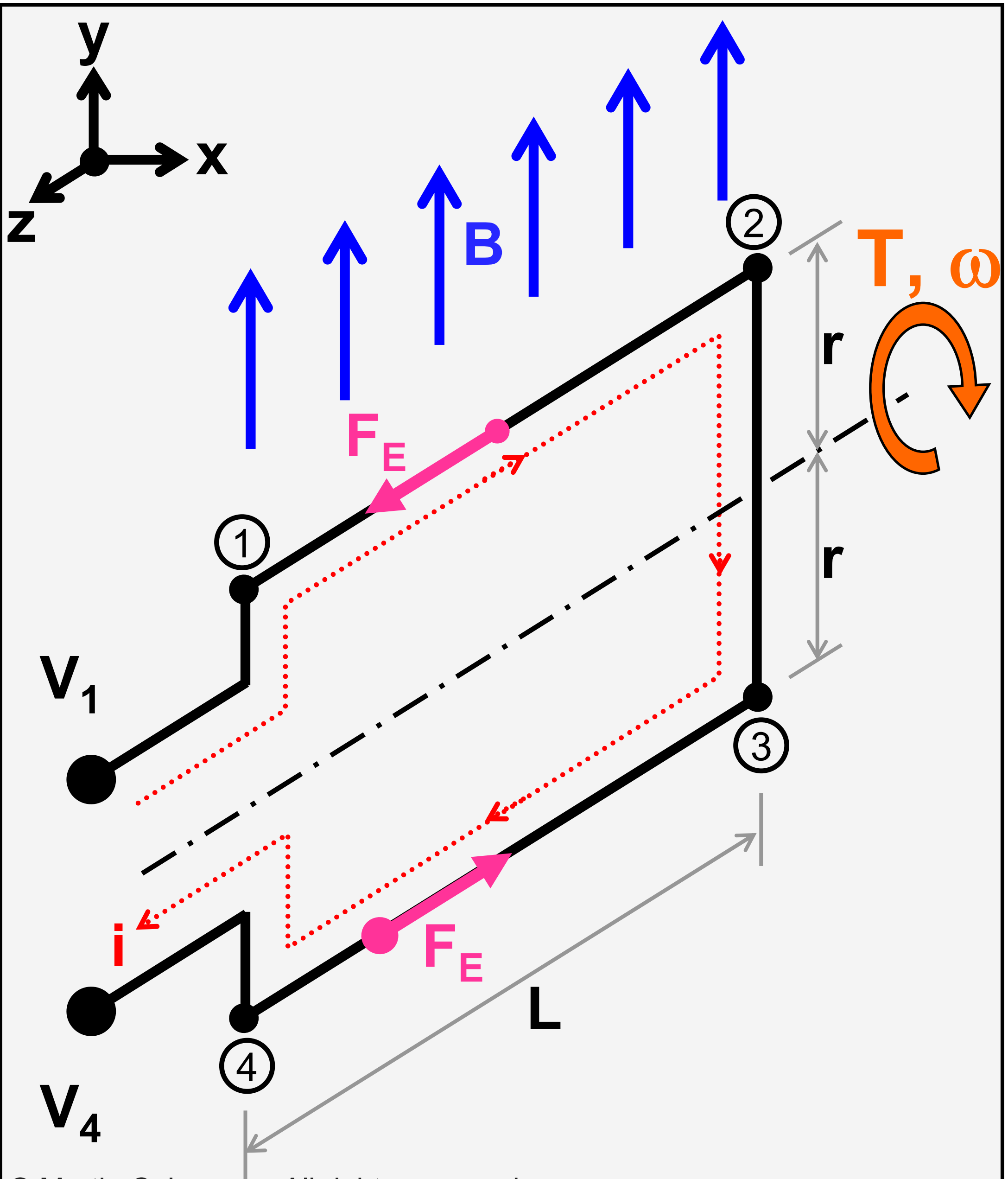
$$\left| T \right| = 2 \cdot r \cdot \left(i \cdot L \cdot B \cdot \right) \cdot \sin\left(\theta_{r-F} \right)$$

$$T = 2 \cdot r \cdot \frac{\left(V_1 - V_4 \right)\big|_{Battery}}{R} \cdot L \cdot B$$

$$\left(V_1 - V_4 \right)\big|_{Battery} = \frac{T \cdot R}{2 \cdot r \cdot L \cdot B}$$

Lorentz force

$$E = -\frac{dV}{dx}$$



Force due to E & B

$$\vec{F}_E = q \cdot \vec{E} + q \cdot \left(\vec{v} \times \vec{B}\right)$$

$$|E| = |v| \cdot |B| \cdot \sin\left(\theta_{v-B}\right)$$

Wire 1-2

$$\frac{\left(V_2 - V_1\right)\big|_\omega}{L} = \left(r \cdot \omega\right) \cdot B$$

Wire 2-3

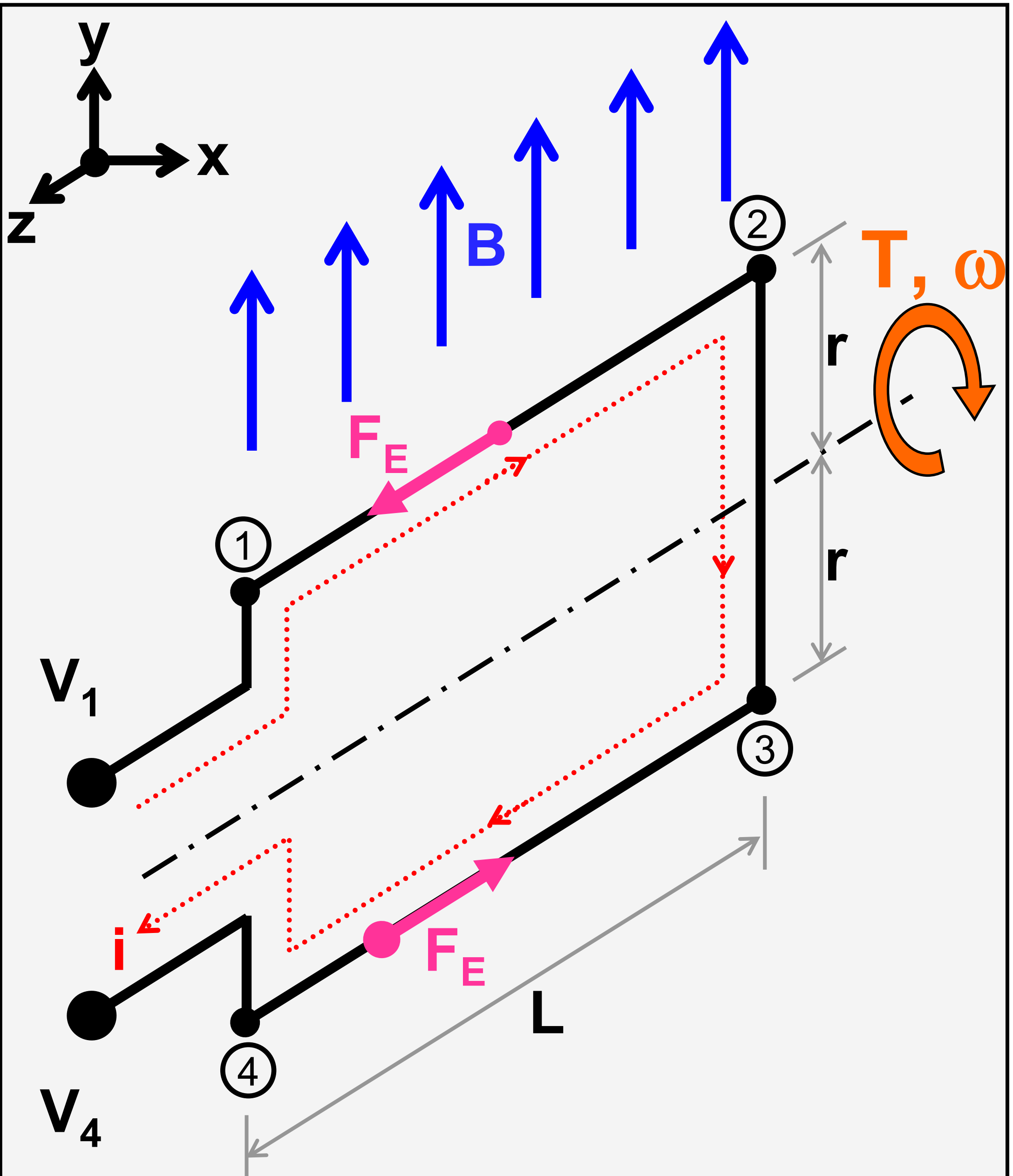
$\vec{v} \times \vec{B}$ not along r

$$V_2\big|_\omega = V_3\big|_\omega$$

Wire 3-4

$$\frac{\left(V_4 - V_3\right)\big|_\omega}{L} = \left(r \cdot \omega\right) \cdot B$$

Induced voltage due to rotation



ΔV due to rotation, ω

$$\frac{\left(V_2 - V_1\right)\big|_{\omega}}{L} = (r \cdot \omega) \cdot B$$

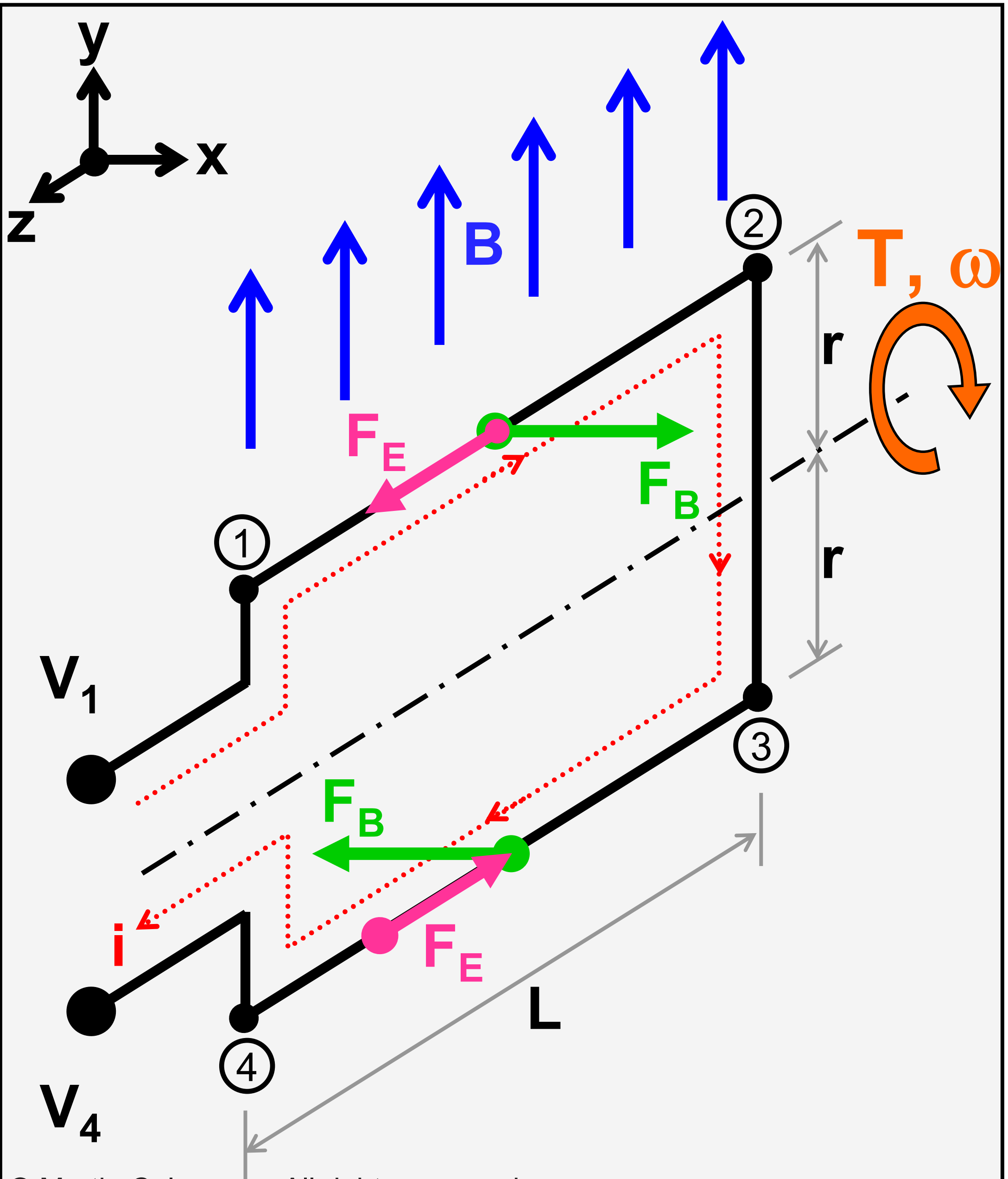
$$V_2\big|_{\omega} = V_3\big|_{\omega}$$

$$\frac{\left(V_4 - V_3\right)\big|_{\omega}}{L} = (r \cdot \omega) \cdot B$$

$$\frac{\left(V_4 - V_1\right)\big|_{\omega}}{L} = 2 \cdot (r \cdot \omega) \cdot B$$

$$\left(V_1 - V_4\right)\big|_{\omega} = -2 \cdot (r \cdot \omega) \cdot B \cdot L$$

Total voltage



ΔV due to rotation, ω

$$\left(V_1 - V_4\right)\Big|_{\omega} = -2\cdot\left(r\cdot\omega\right)\cdot B\cdot L$$

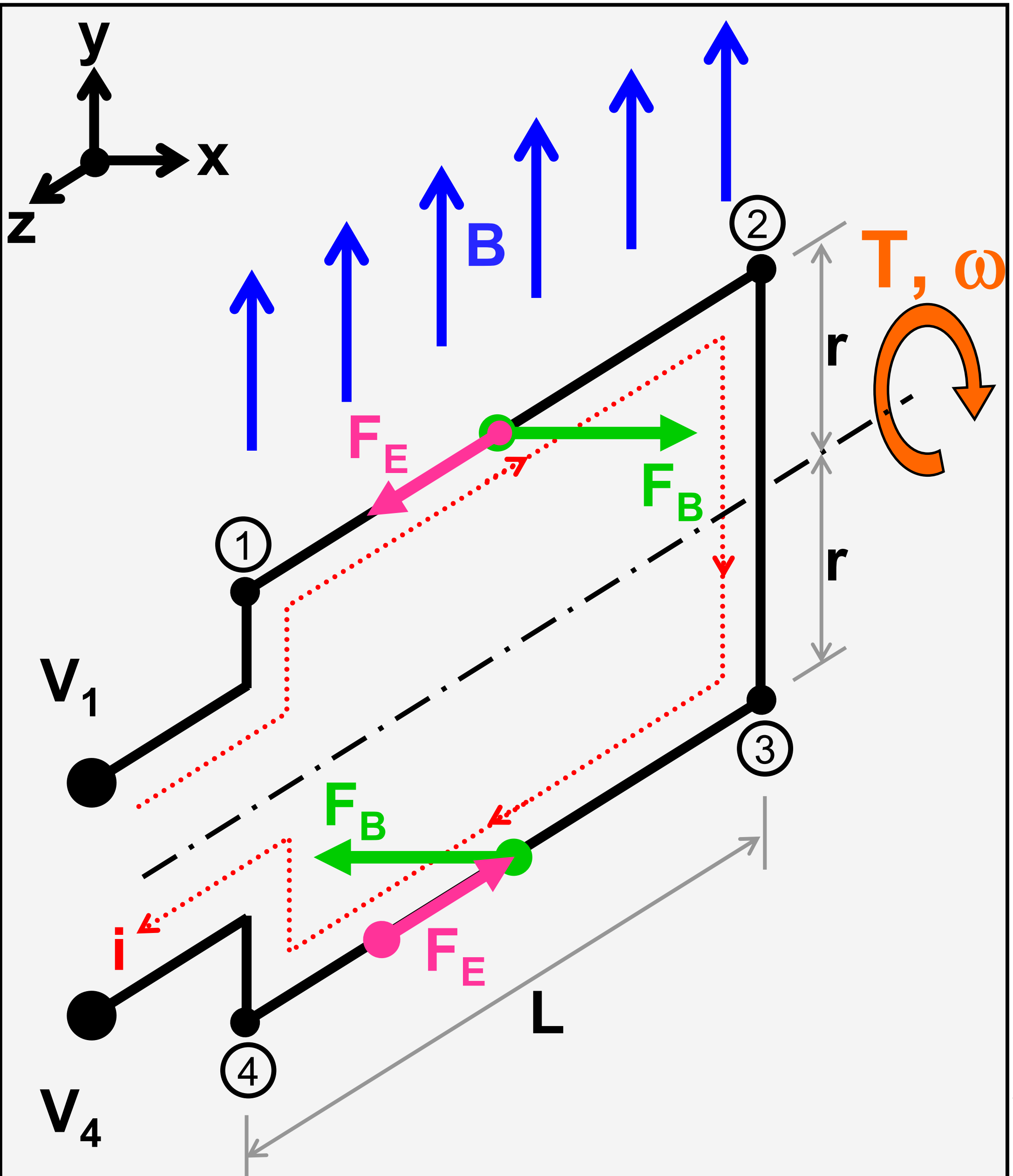
ΔV due to battery

Total potential diff.

$$\left(V_1 - V_4\right)\Big|_{Battery} = \frac{T\cdot R}{2\cdot r\cdot L\cdot B}$$

$$\Delta V = \left(V_1 - V_4\right)\Big|_{Battery} + \left(V_1 - V_4\right)\Big|_{\omega}$$

$$\Delta V = \frac{T\cdot R}{2\cdot r\cdot L\cdot B} - 2\cdot\left(r\cdot\omega\right)\cdot B\cdot L$$



Total potential diff.

$$\Delta V = \frac{T \cdot R}{2 \cdot r \cdot L \cdot B} - 2 \cdot (r \cdot \omega) \cdot B \cdot L$$

Ohm's law

$$\Delta V = i \cdot R$$

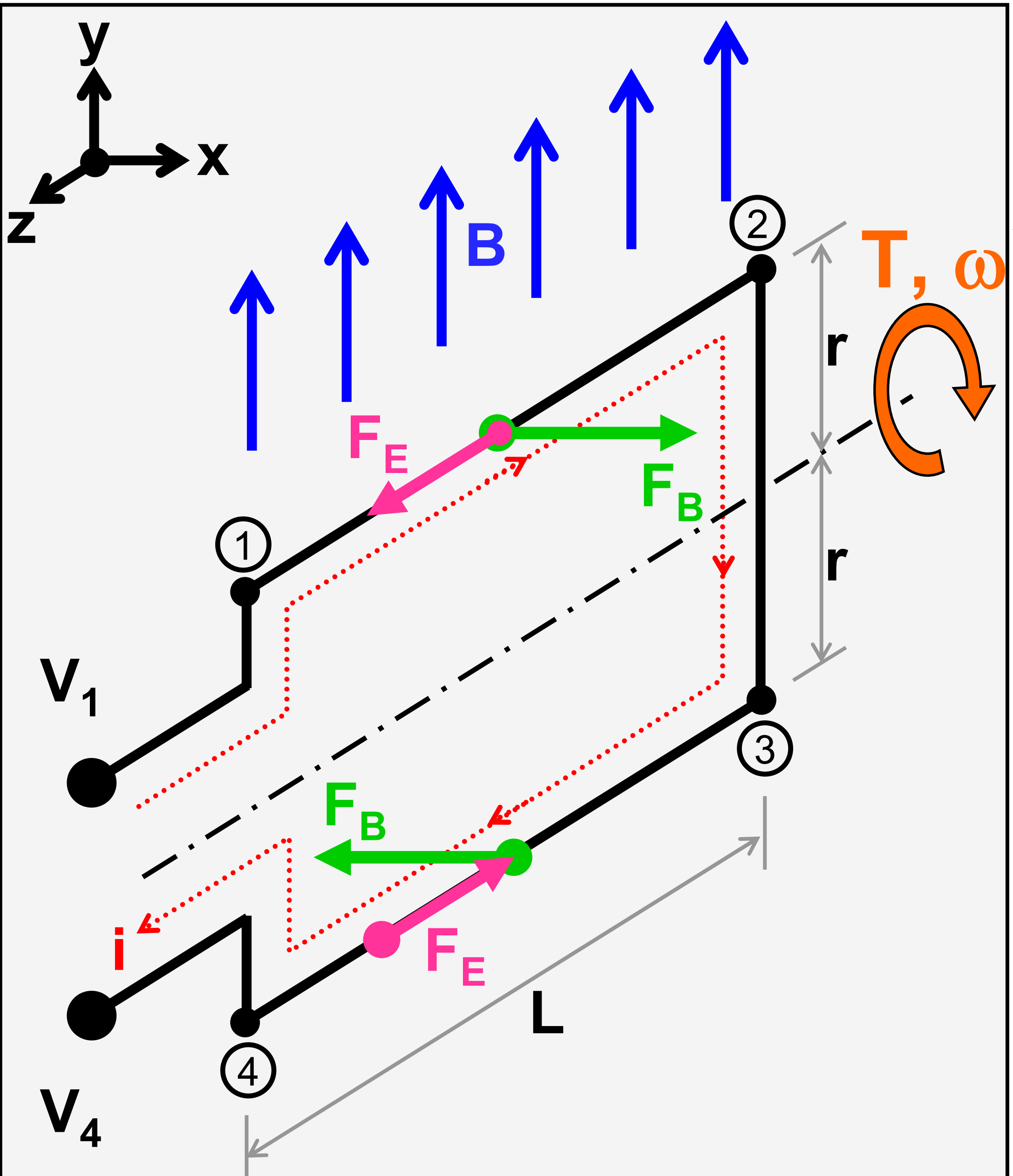
Total potential diff.

$$i \cdot R = \frac{T \cdot R}{2 \cdot r \cdot L \cdot B} - 2 \cdot (r \cdot \omega) \cdot B \cdot L$$

T- ω relationship

$$T = 2 \cdot i \cdot L \cdot r \cdot B - \frac{4 \cdot r^2 \cdot L^2 \cdot B^2}{R} \omega$$

Torque – ω relationship cont.



T- ω relationship

$$T = 2 \cdot i \cdot L \cdot r \cdot B - \frac{4 \cdot r^2 \cdot L^2 \cdot B^2}{R}\omega$$

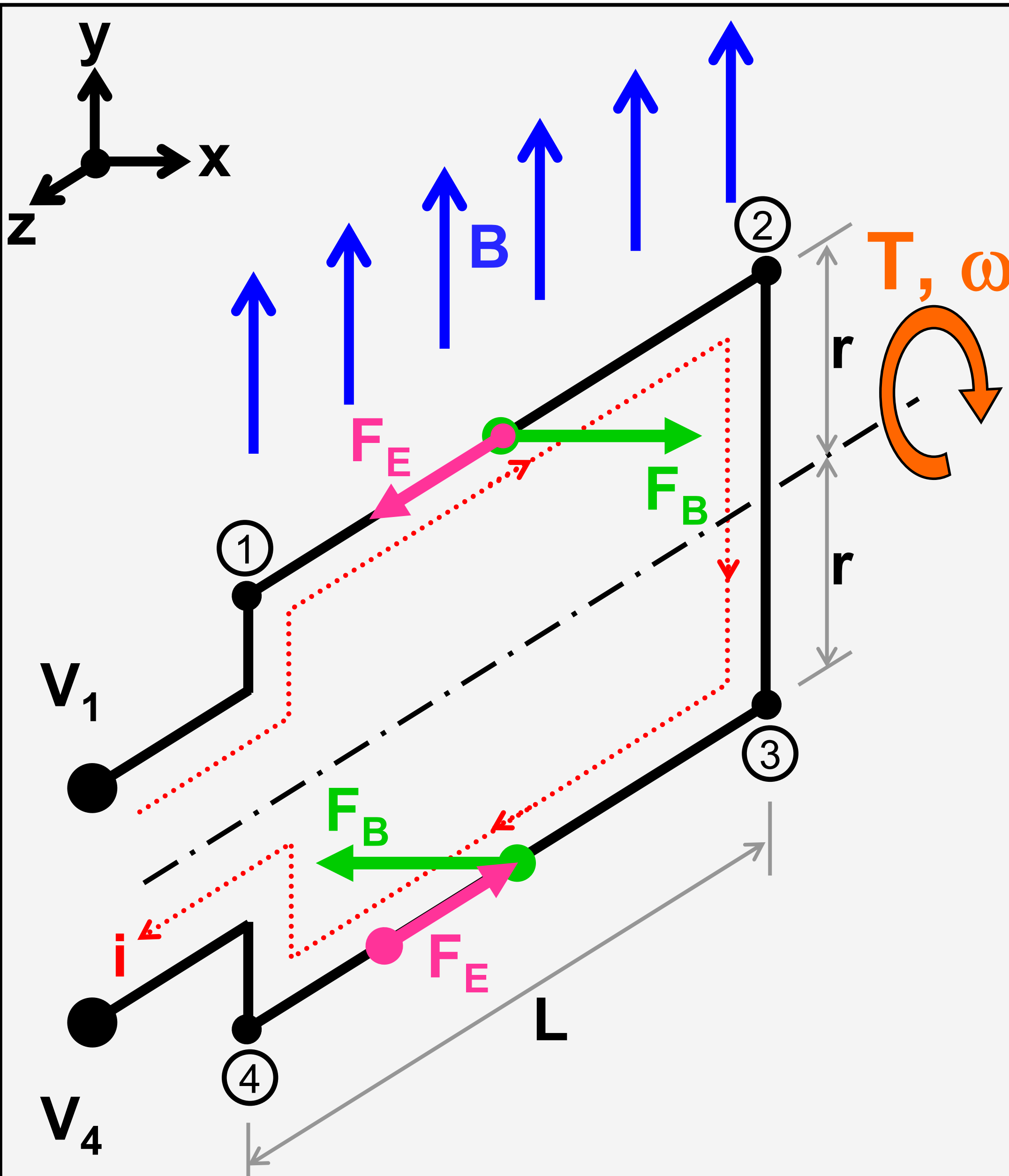
Stall torque

$$T_{Stall} = 2 \cdot i \cdot L \cdot r \cdot B$$

T- ω relationship

$$T = T_{Stall} - \frac{4 \cdot r^2 \cdot L^2 \cdot B^2}{R}\omega$$

Torque – ω relationship cont.



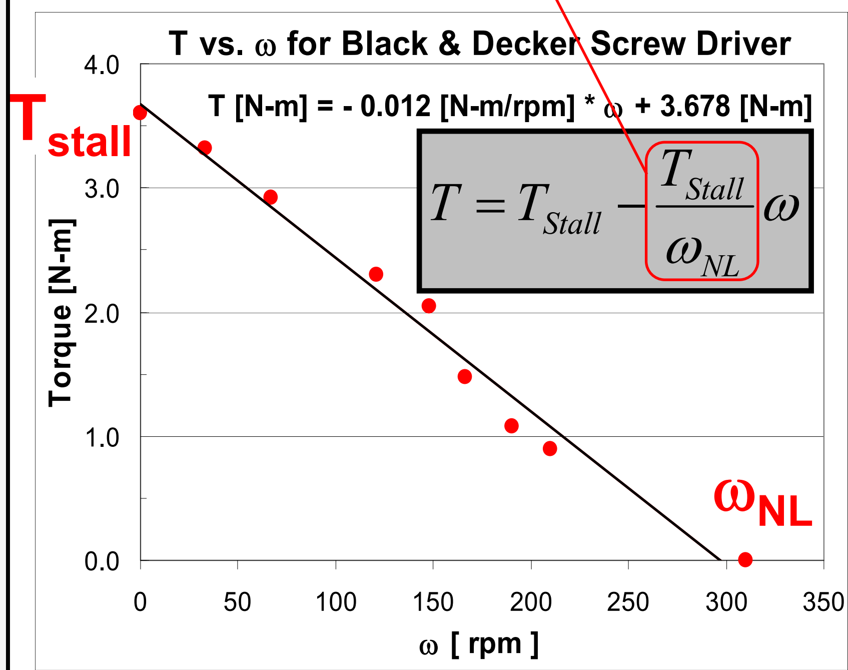
T- ω relationship

$$T = T_{Stall} - \frac{4 \cdot r^2 \cdot L^2 \cdot B^2}{R}\omega$$

Loop constant

T- ω curve

Motor constant



Scaling

Follow up on micro-actuator lecture

Electrostatics

How does electrostatic physics scale?

$$U_E = \frac{\varepsilon_o \cdot L \cdot L \cdot V^2}{2 \cdot z}$$

How does ratio of $F_{Electric}$ scale to F_{Body}?

$$\left| \frac{F_{Electric}}{F_{Body}} \right| \sim \frac{1}{L}$$

What does this mean for MuSS interaction?

- What happens if you downsize each by factor of 10?

Electrostatics

$$U_{Electric-z} = \frac{\varepsilon_o \cdot L \cdot L \cdot V^2}{2 \cdot z} \longrightarrow F_{Electric-z} = -\frac{dU}{dz} \longrightarrow F_{Electric-z} = \frac{\varepsilon_o \cdot L^2 \cdot V^2}{2 \cdot z^2}$$

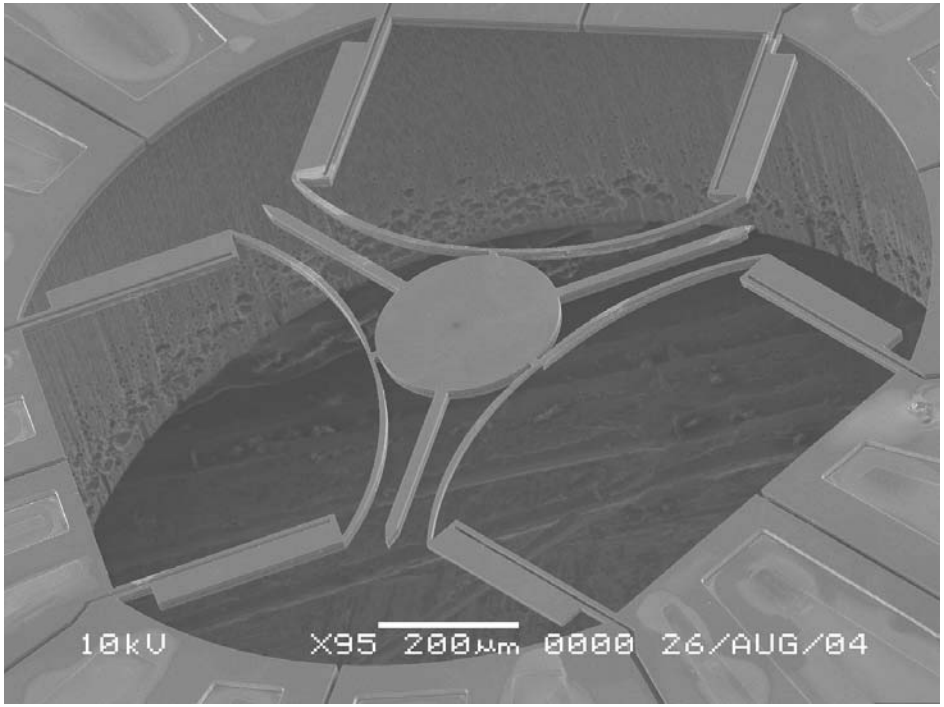
$$F_{body} = \rho \cdot V^3 \longrightarrow \left| \frac{F_{Electric}}{F_{Body}} \right| \sim \frac{1}{L}$$

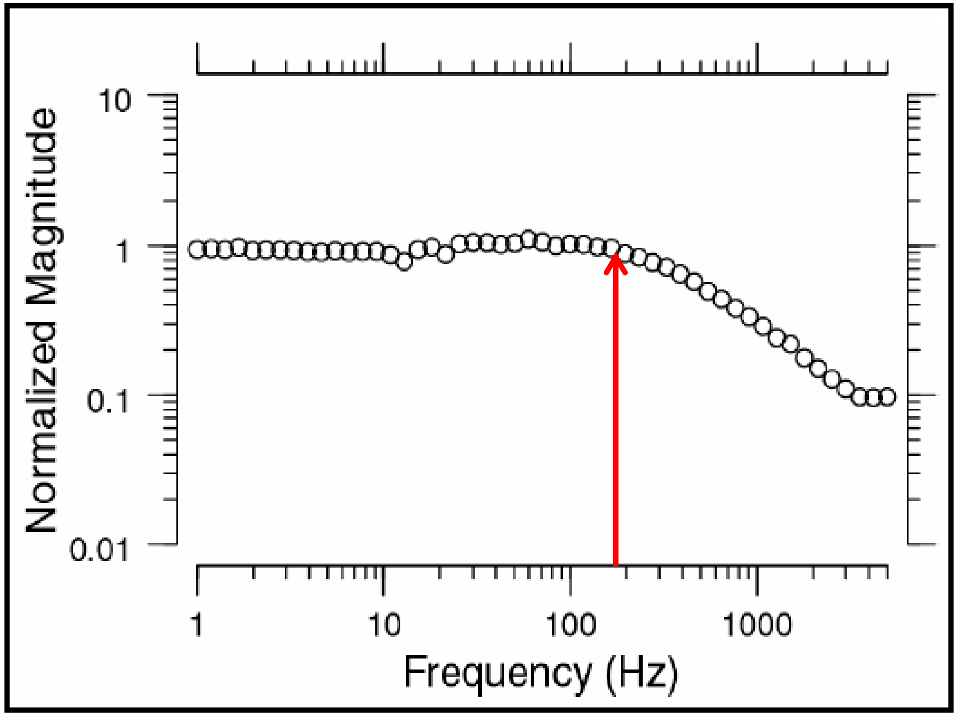
Table removed due to copyright restrictions. Please see http://www.sizes.com/built/clean_rooms.htm

Semi-intuitive example

Cooling…

Heating…



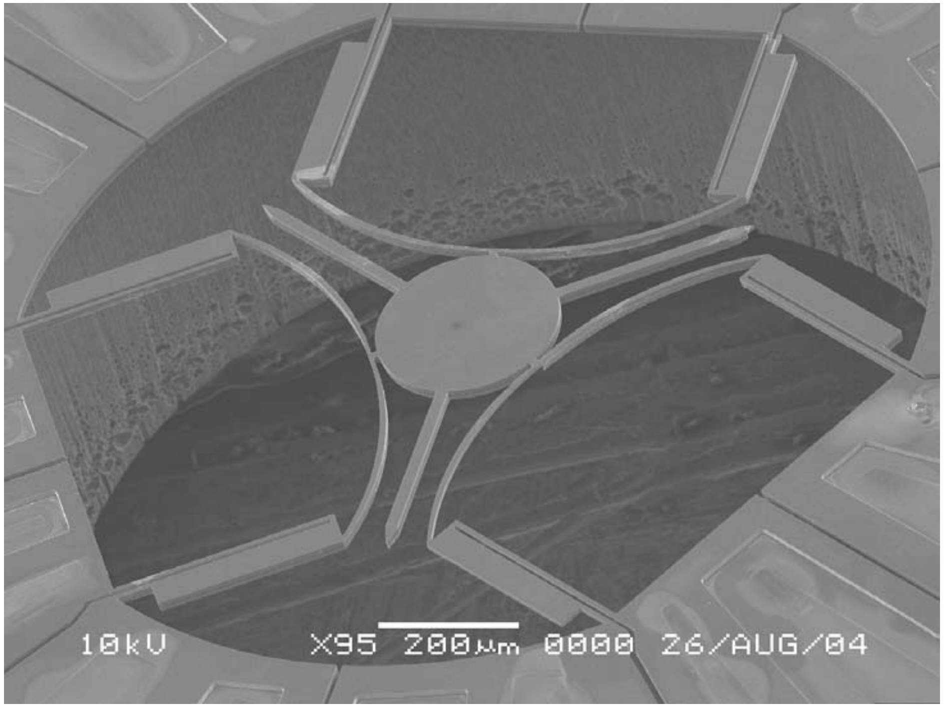


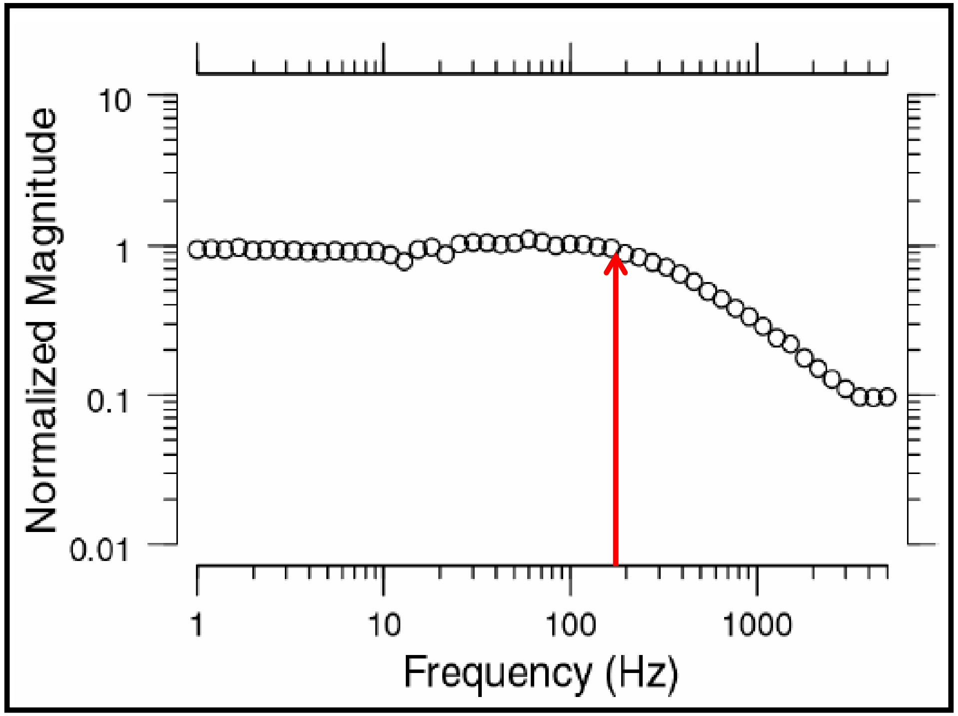
Thermal behavior

How does thermal physics scale (small Bi #)?

$$e^{\left[-\left(\frac{h\cdot A}{\rho\cdot V\cdot c}\right)\cdot t\right]} = \frac{\theta}{\theta_{\inf}} = \frac{T - T_{\inf}}{T_{initial} - T_{\inf}}$$

$$Bi = \frac{h\cdot L}{k} \sim \frac{Convection}{Conduction}$$





Thermal behavior

How does thermal physics scale?

$$-h \cdot A \cdot (T - T_{\inf}) = \rho \cdot c \cdot \not{V} \cdot \frac{dT}{dt}$$

$$Bi = \frac{h \cdot L}{k}$$

$$e^{\left[-\left(\frac{h \cdot A}{\rho \cdot \not{V} \cdot c}\right) \cdot t\right]} = \frac{\theta}{\theta_{\inf}} = \frac{T - T_{\inf}}{T_{initial} - T_{\inf}}$$

$$\tau \sim \frac{\rho \cdot \not{V} \cdot c}{h \cdot A} \to L$$

Is this a good or a bad thing for MEMS actuators?

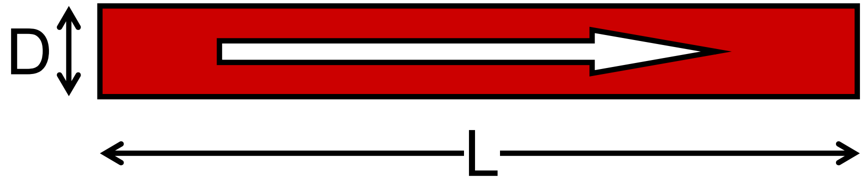
For the STM?

Fluidics

How do fluid-based physical phenomena scale?

$$Q = \frac{\pi \cdot r^4 \cdot \Delta p}{8 \cdot \mu \cdot L}$$

$$Q = U \cdot \pi \cdot r^2$$

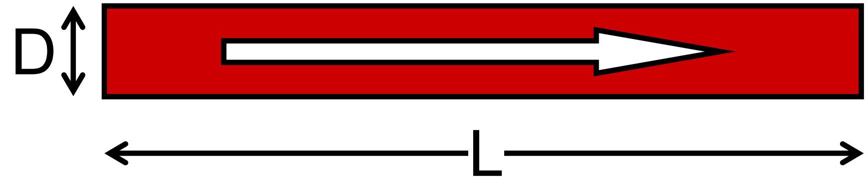


$$\Delta p = -\frac{8 \cdot \mu \cdot U}{r^2} \cdot L$$

High pressure change over narrow flow paths…

Fluidics

Reynolds number



$$Re = \frac{\rho \cdot U \cdot D}{\mu}$$ → Ratio of inertial forces to viscous forces

D = 50 μm   U = 500 μm/s   L = 1000 μm

Re_{Air} and Re_{H2O} << 1

What does this mean:

- Heavily damped
- Limits response time (ms vs. μs)