

**MIT 2.852**  
**Manufacturing Systems Analysis**  
**Lectures 22–?**

*Quality and Quantity*

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Spring, 2007

# Quality and Quantity

- *Quantity* is about how much is produced, when it is produced, and what resources are required to produce it.
- *Quality* is about how well it is made, and how much of it is made well.
  - ★ *Design quality* is about giving customers what they would like.
  - ★ *Production quality* is about not giving customers what they would not like.

# Quality and Quantity

- Most literature is all quantity or all quality.
- *Quantity measures* include production rate, lead time, inventory, utilization.
- *Quality measures* include yield and output defect rate.

# Quality and Quantity

- *Quantity strategies* include optimizing local inventories, optimizing global inventory, release/dispatch policies, make-to-order vs. make-to-stock, etc.
- *Quality strategies* include inspection, statistical process control, etc.

# Quality and Quantity

The problem is that, conventionally, ...

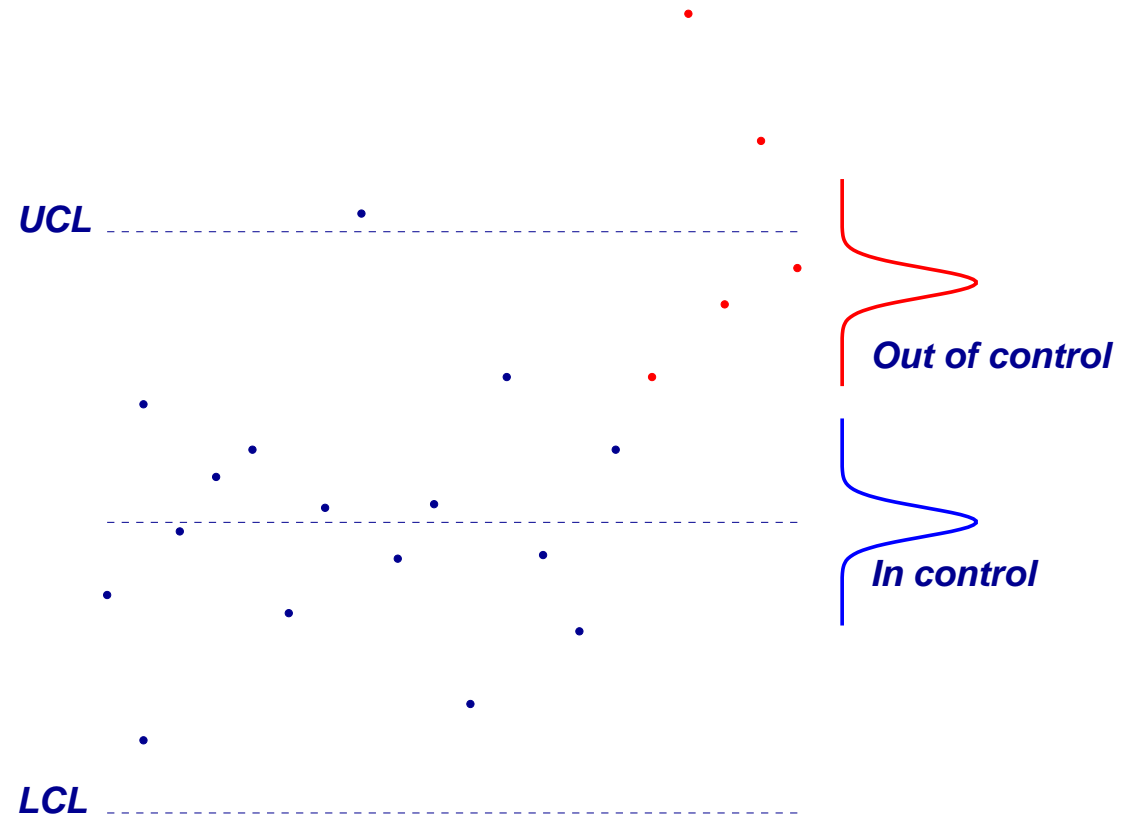
- Quantity strategies are selected according to how they affect quantity measures, and
- Quality strategies are selected according to how they affect quality measures, but ...
- in reality, *both affect both* .

# Quality and Quantity

## Quality

### *Example:* Statistical Process Control

- Goal is to determine when a process has gone *out of control* in order to maintain the machine.
- Upper and lower control limits (UCL, LCL) usually chosen to be  $6\sigma$  apart.
- Basic idea: which is the most likely distribution that sample comes from?



# Quality and Quantity

## Quantity

*Example:*

Everything we have been discussing so far.

# Taxonomy of Issues

- Failure dynamics
- Inspection
  - ★ Binary (good/bad) vs. measurement
  - ★ Accuracy (false positives and negatives)
  - ★ Spatial and temporal frequency
- Actions on parts and machines
- Topology of system
- Performance measures



## Taxonomy of Issues

- *Definition:* How the quality of a machine changes over time.
- The quality literature distinguishes between *common causes* and *special causes* . (Other terms are also used.)
  - ★ Common cause: successive failures are equally likely, regardless of past history.  
GGGGGBGGGBGGGGGGGBGGGBGGGGBBGGGGGGGG . . . . .
  - ★ Special cause: something happens to the machine, and failures become much more likely.  
GGGGBGGGGGBGGGGGGGGBBBBBBBGBBBBBBB . . . . .
- We use this concept to extend quantity models.

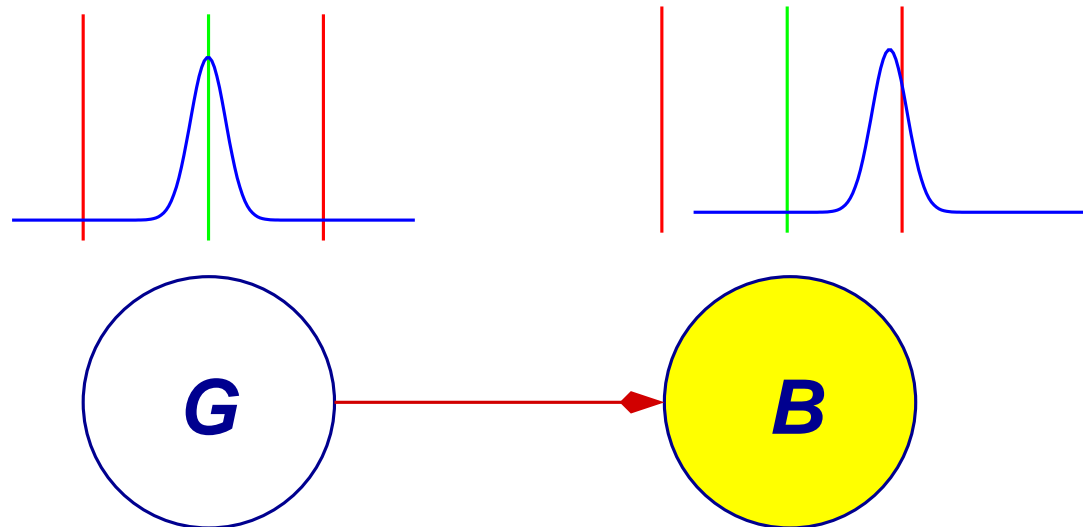
# Taxonomy of Issues

- *Bernoulli or common cause*: independent.
- *Persistent or special cause*: all parts after the first bad part are bad, until the repair.
- *Multi-Yield*: generalization of persistent.

# Taxonomy of Issues

## Failure Dynamics

The relationship between quality dynamics and statistical process control:



*Note:* The operator **does not know** when the machine is in the bad state until it has been detected.

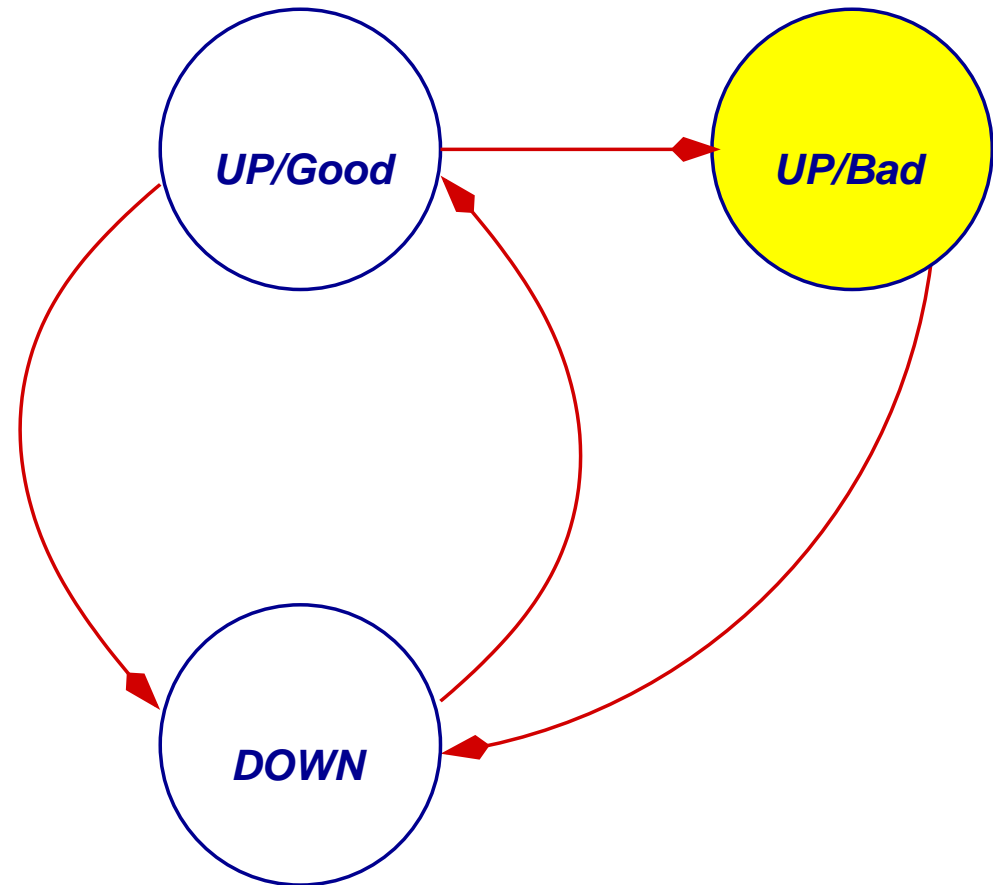
# Taxonomy of Issues

## Failure Dynamics

### Simplest model

Versions:

- The *Good* state has 100% yield and the *Bad* state has 0% yield.
- The *Good* state has high yield and the *Bad* state has low yield.



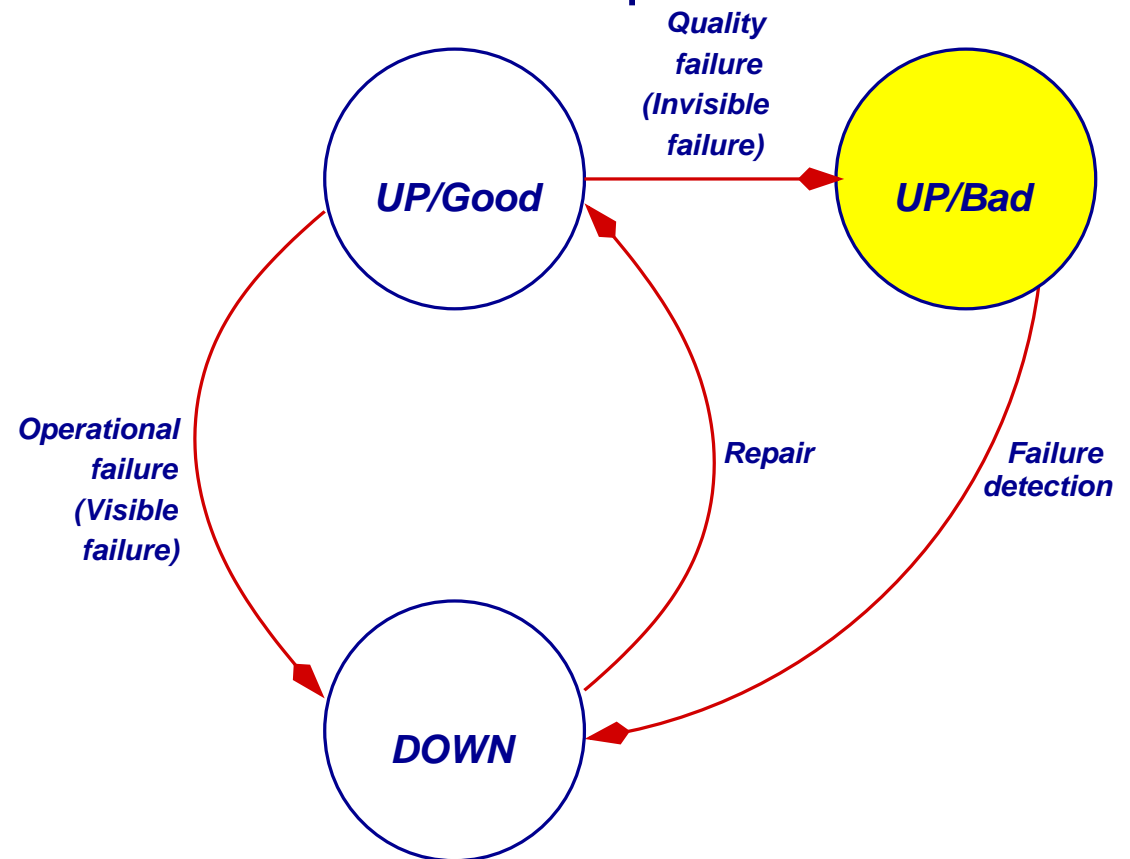
# Taxonomy of Issues

## Failure Dynamics

### Simplest model

The three-state machine model is much too simple.

- No matter how the machine arrived at the *DOWN* state, it gets the same repair. Since the next state is always the *UP/Good* state, there must have been a quality repair.
- Quality repairs are expensive, and not necessary for operational failures.

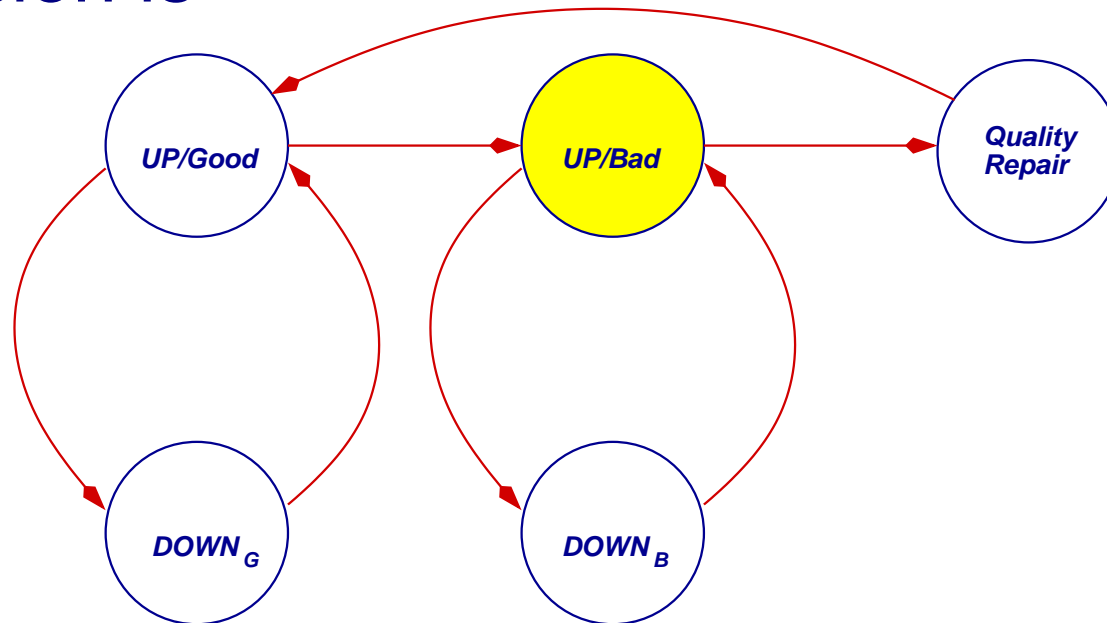


# Taxonomy of Issues

## Failure Dynamics

Simplest model

- One extension is



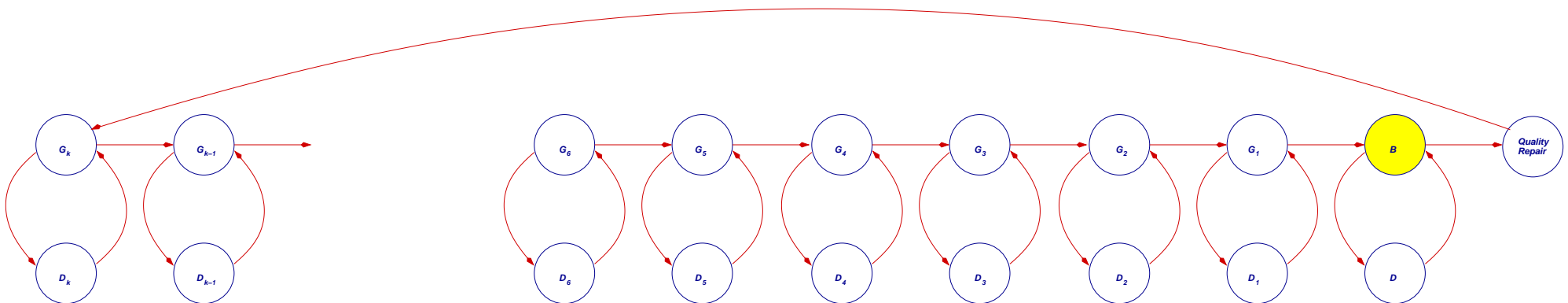
- ... but even this leaves out important features.

# Taxonomy of Issues

## Failure Dynamics

### Simplest model

- Another extension is



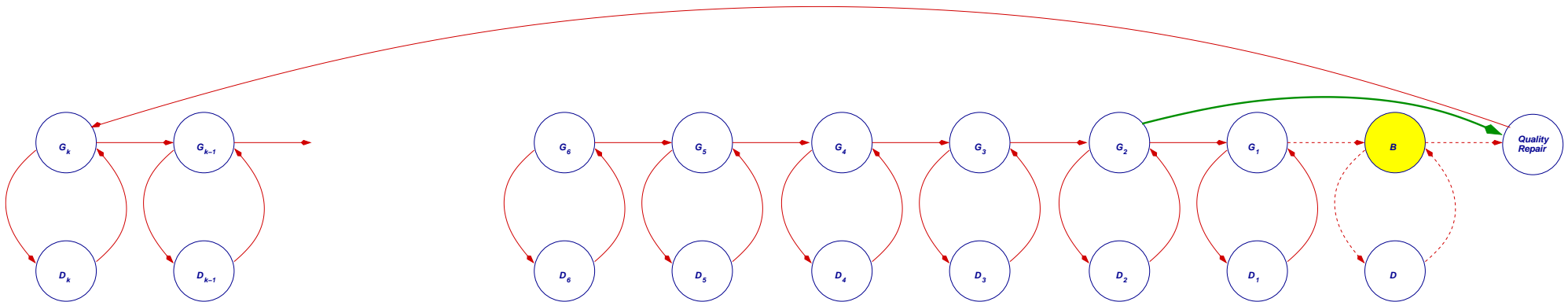
- This allows very general wear or aging models.

# Taxonomy of Issues

## Failure Dynamics

### Simplest model

- A maintenance strategy could be modeled as



*if* we have perfect knowledge of the machine state.

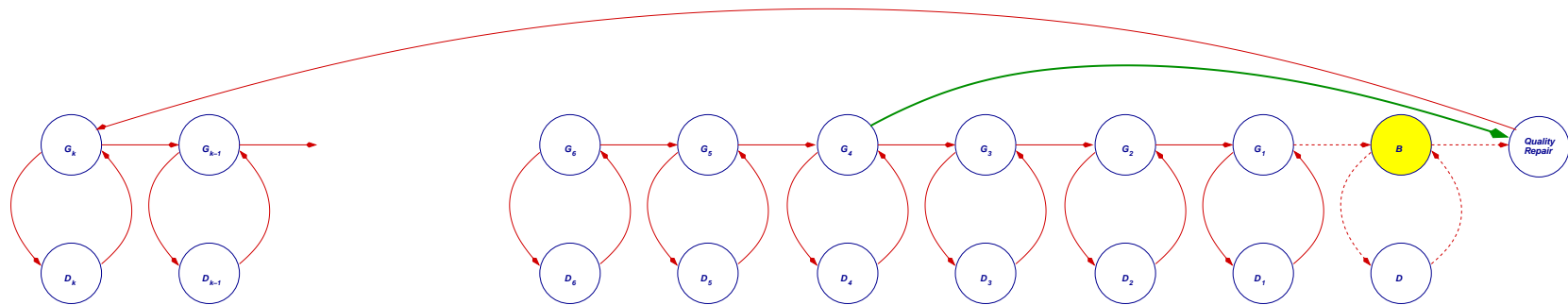


# Taxonomy of Issues

## Failure Dynamics

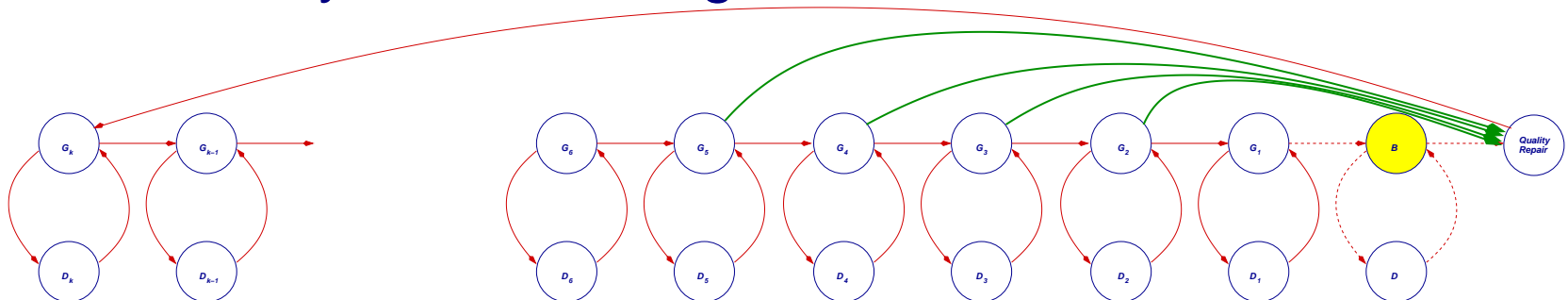
### Simplest model

- A maintenance strategy could be *implemented* as



if we do not have perfect knowledge of the machine state.

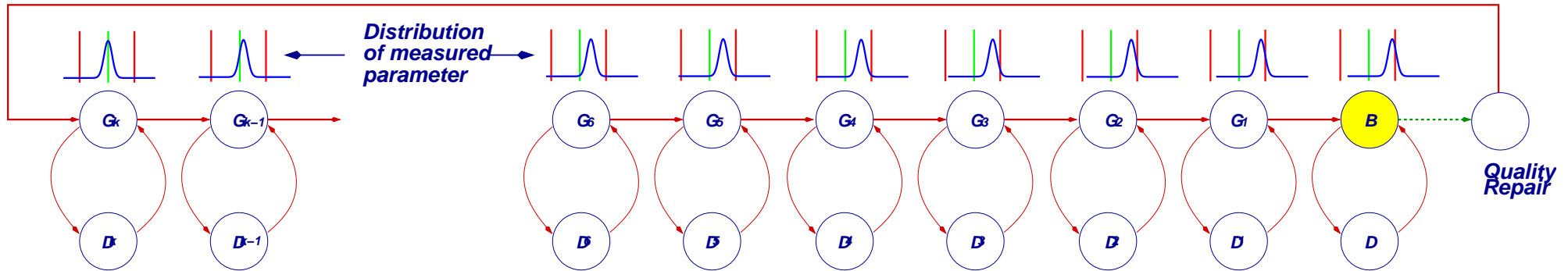
- It would be analyzed according to



# Taxonomy of Issues

## Failure Dynamics

### Simplest model



# Taxonomy of Issues

- Motivation — why inspect?
  - ★ To take action on parts and machines.
- Objectives of inspection:
  - ★ Bad parts rejected or reworked.
  - ★ Machine maintained when necessary.
- Effects of inspection errors:
  - ★ Some good parts rejected or reworked; some bad parts accepted.
  - ★ Unnecessary downtime *and/or* more bad parts.

# Taxonomy of Issues

- Destructive vs. non-destructive
- Domain
- Sampling
- Inspection time
- Accuracy (and goal of inspection)

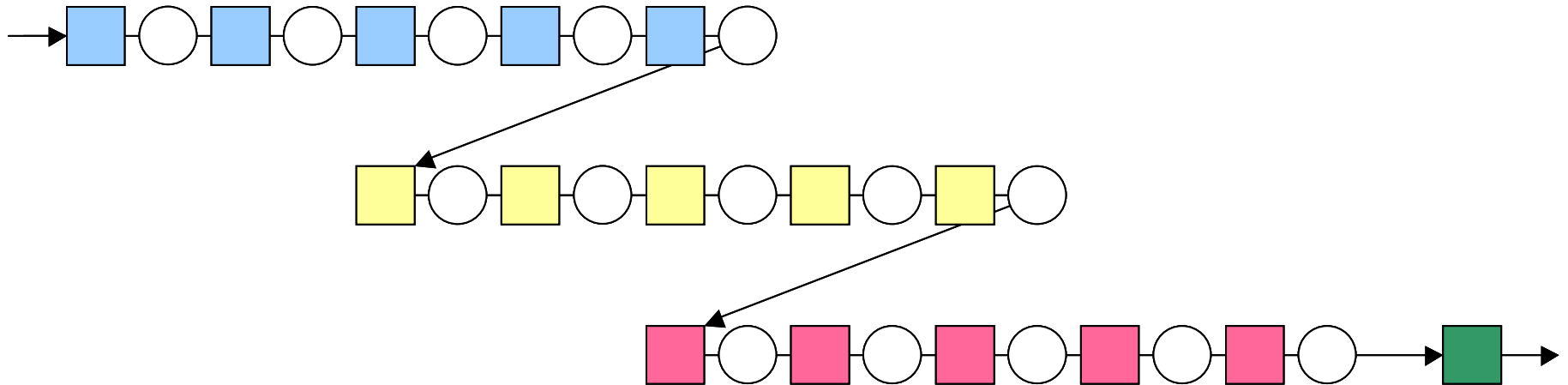
# Taxonomy of Issues

## Actions on parts and machines

- Actions on parts: accept, rework, or scrap.
- Actions on machines: leave alone or repair.

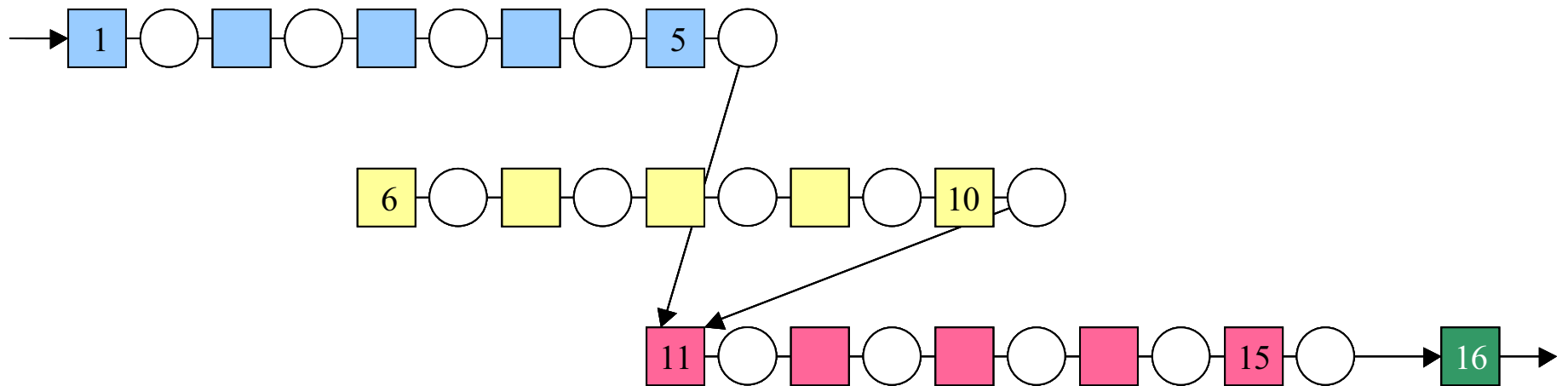
# Taxonomy of Issues

## Topology of system



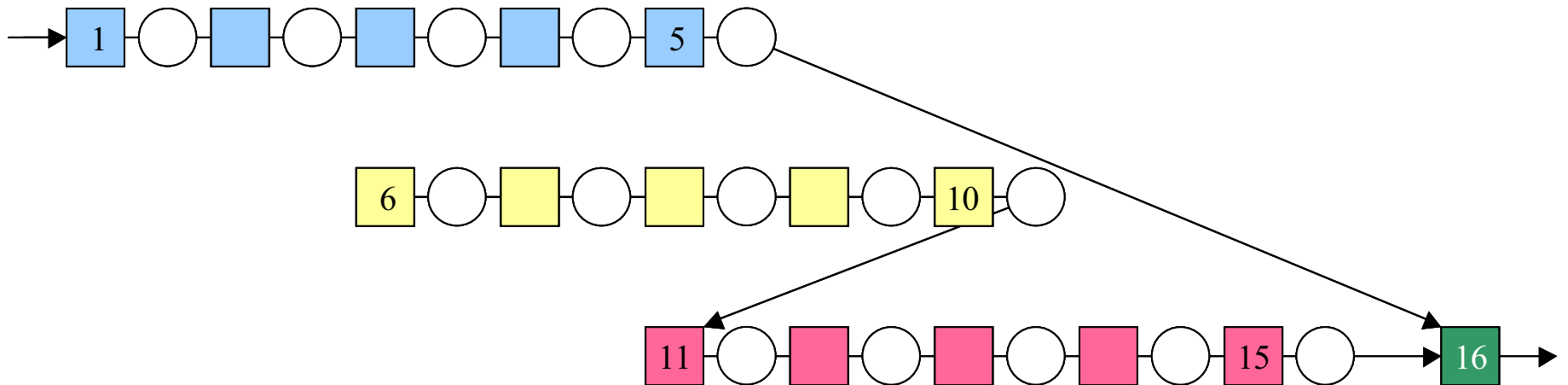
# Taxonomy of Issues

## Topology of system



# Taxonomy of Issues

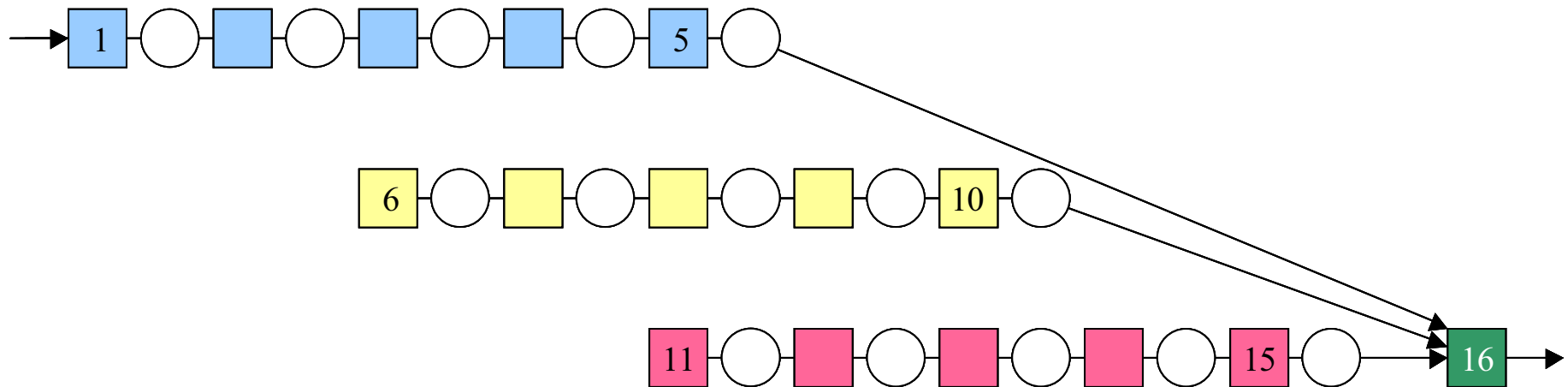
## Topology of system





# Taxonomy of Issues

## Topology of system



# Taxonomy of Issues

- Expected total production rate
- Expected good production rate
- Yield
- Expected inventory.
- Miss and waste
- Production lead time

They are easy to calculate in a single-machine model.

# One- and Two-Machine Systems

## Note:

All the material up to Slide 43 is taken from

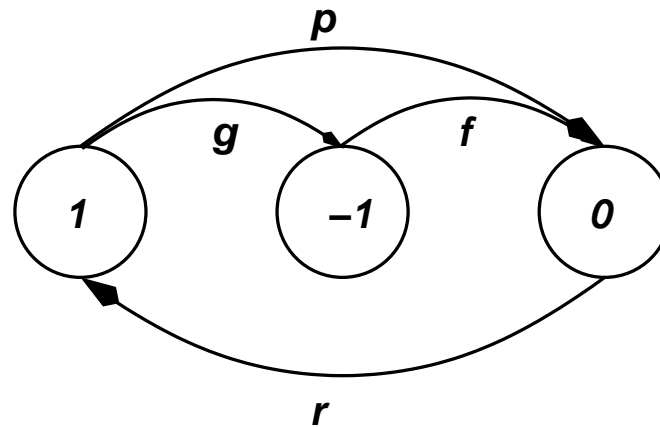
Kim and Gershwin, “Integrated Quality and Quantity Modeling of a Production Line,” *OR Spectrum*, Volume 27, Numbers 2-3, pp. 287–314, June, 2005.

and

Jongyoon Kim, “Integrated Quality and Quantity Modeling of a Production Line,” Ph. D thesis, MIT Mechanical Engineering, November, 2004.

# One- and Two-Machine Systems

## Single Machine



$$(g + p)P(1) = rP(0)$$

$$fP(-1) = gP(1)$$

# One- and Two-Machine Systems

## Single Machine

$$rP(0) = pP(1) + fP(-1)$$

$$P(0) + P(1) + P(-1) = 1$$

$$P(1) = \frac{1}{1 + (p + g)/r + g/f}$$

$$P(0) = \frac{(p + g)/r}{1 + (p + g)/r + g/f}$$

$$P(-1) = \frac{g/f}{1 + (p + g)/r + g/f}$$

# One- and Two-Machine Systems

## Single Machine

The *total production rate*, including good and bad parts, is

$$P_T = \mu(P(1) + P(-1)) = \mu \frac{1 + g/f}{1 + (p + g)/r + g/f}$$

The *effective production rate*, the production rate of good parts only, is

$$P_E = \mu P(1) = \mu \frac{1}{1 + (p + g)/r + g/f}$$

(This quantity is also called the *good production rate*.) Since there is no scrapping, the rate at which parts enter the system is equal to the rate at which parts leave the system, so that the *yield* is

$$Y = \frac{P_E}{P_T} = \frac{P(1)}{P(1) + P(-1)} = \frac{f}{f + g}$$

# One- and Two-Machine Systems

## Lines with Infinite Buffers

*Two-Machine, Infinite-Buffer Line:*

$$P_T^\infty = \min \left[ \frac{\mu_1(1 + g_1/f_1)}{1 + (p_1 + g_1)/r_1 + g_1/f_1}, \frac{\mu_2(1 + g_2/f_2)}{1 + (p_2 + g_2)/r_2 + g_2/f_2} \right]$$

$$P_E^\infty = \frac{f_1 f_2}{(f_1 + g_1)(f_2 + g_2)} P_T^\infty$$

# One- and Two-Machine Systems

## Lines with Zero Buffers

*Two-Machine, Zero-Buffer Line:*

$$P_T^0 = \frac{\min[\mu_1, \mu_2]}{1 + \frac{f_1^b(p_1^b + g_1^b)}{r_1(f_1^b + g_1^b)} + \frac{f_2^b(p_2^b + g_2^b)}{r_2(f_2^b + g_2^b)}}$$

$$P_E^0 = \frac{f_1^b f_2^b}{(f_1^b + g_1^b)(f_2^b + g_2^b)} P_T^0$$



# One- and Two-Machine Systems

## Two-Machine-One-Buffer Lines

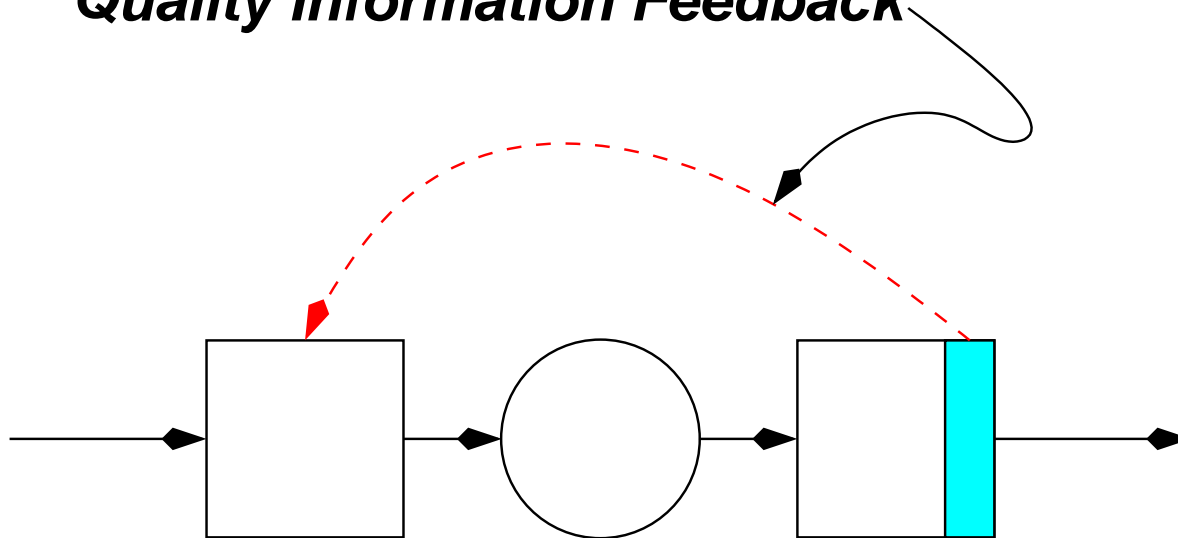
- Continuous material
- Three-state machine
- *Quality information feedback*
  - ★ Defects produced by the first machine are detected, after a delay, by the second machine.
  - ★ The length of the delay depends on the number of parts in the buffer.
- As buffer size increases, total production rate increases and yield decreases. But good production rate behavior is harder to predict.

# One- and Two-Machine Systems

## Two-Machine-One-Buffer Lines

### Quality Information Feedback

*Quality Information Feedback*



# One- and Two-Machine Systems

## Two-Machine-One-Buffer Lines

### Solution Technique

The two-machine, one-buffer line with known parameters can be solved using standard methods.

All parameters of the two-machine, one-buffer line are known except  $h_{12}$ , the transition rate from the bad quality state of  $M_1$  to the down state due to the inspection at  $M_2$ . This depends on the number of parts in the buffer  $\bar{x}$ .

# One- and Two-Machine Systems

## Two-Machine-One-Buffer Lines

### Solution Technique

#### *Procedure:*

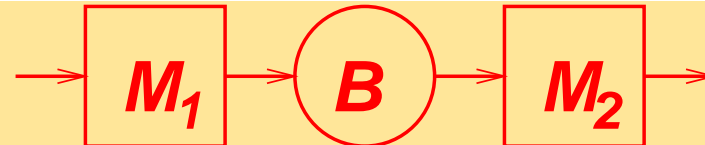
- Guess  $\bar{x}$ .
- Calculate  $h_{12}$ .
- Solve the two-machine line. Recalculate  $\bar{x}$  and iterate.

# One- and Two-Machine Systems

## Intuition

- Quantity-oriented people tend to assume that increasing a buffer *increases* the production rate.
- Quality-oriented people tend to assume that increasing a buffer *decreases* the production rate of good items.
- However, we have found that the picture is not so simple.

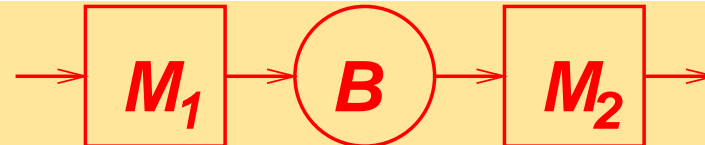
# One- and Two-Machine Systems



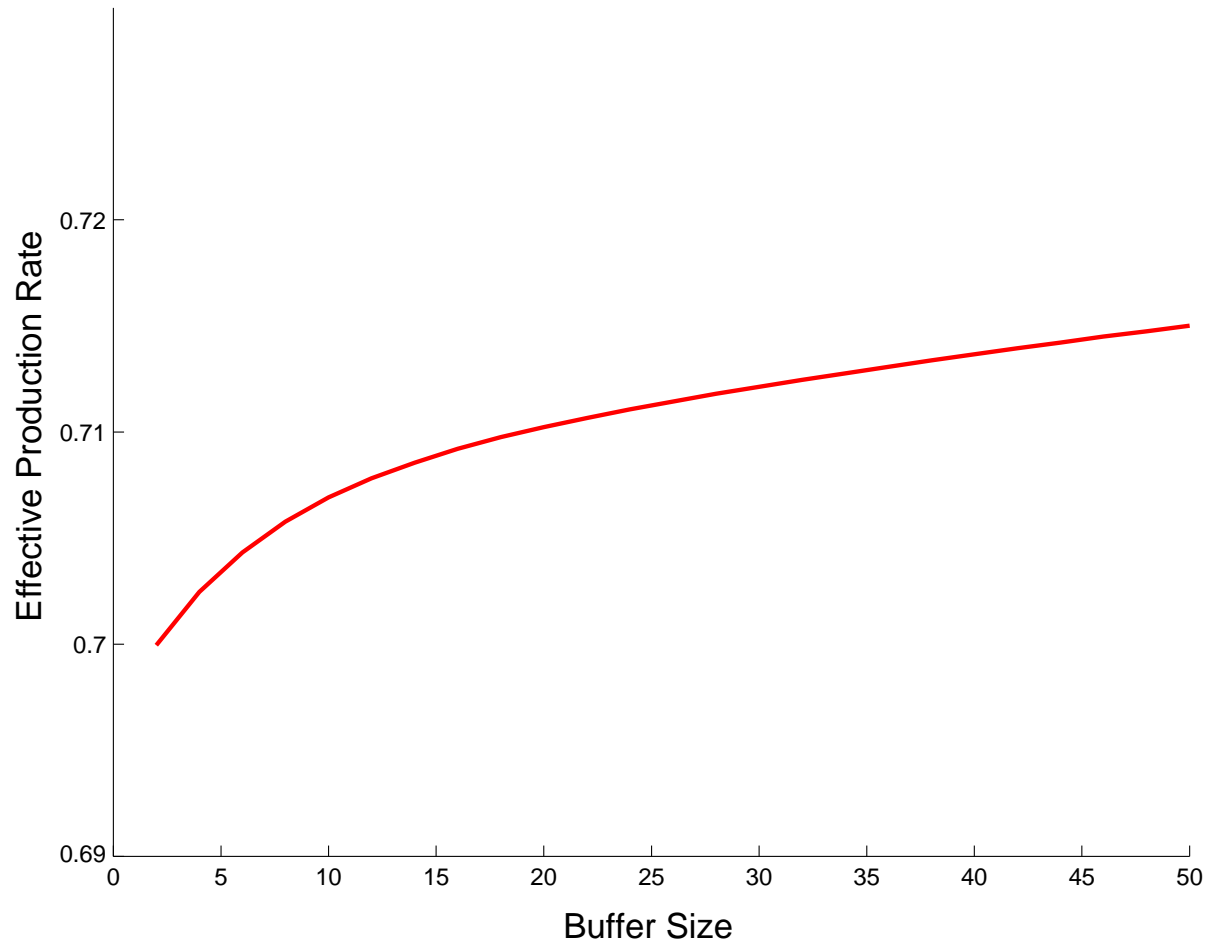
## Assumptions

- $M_1$  has variable quality; the inspection occurs at  $M_2$ .
- $M_1$  makes only good parts in the  $G$  state and only bad parts in the  $B$  state.
- Stoppages occur at both machines at random times for random durations.
- The buffer is operated according to FIFO.
- Detection of the  $M_1$  state change cannot take place until a bad part reaches  $M_2$ .

# One- and Two-Machine Systems

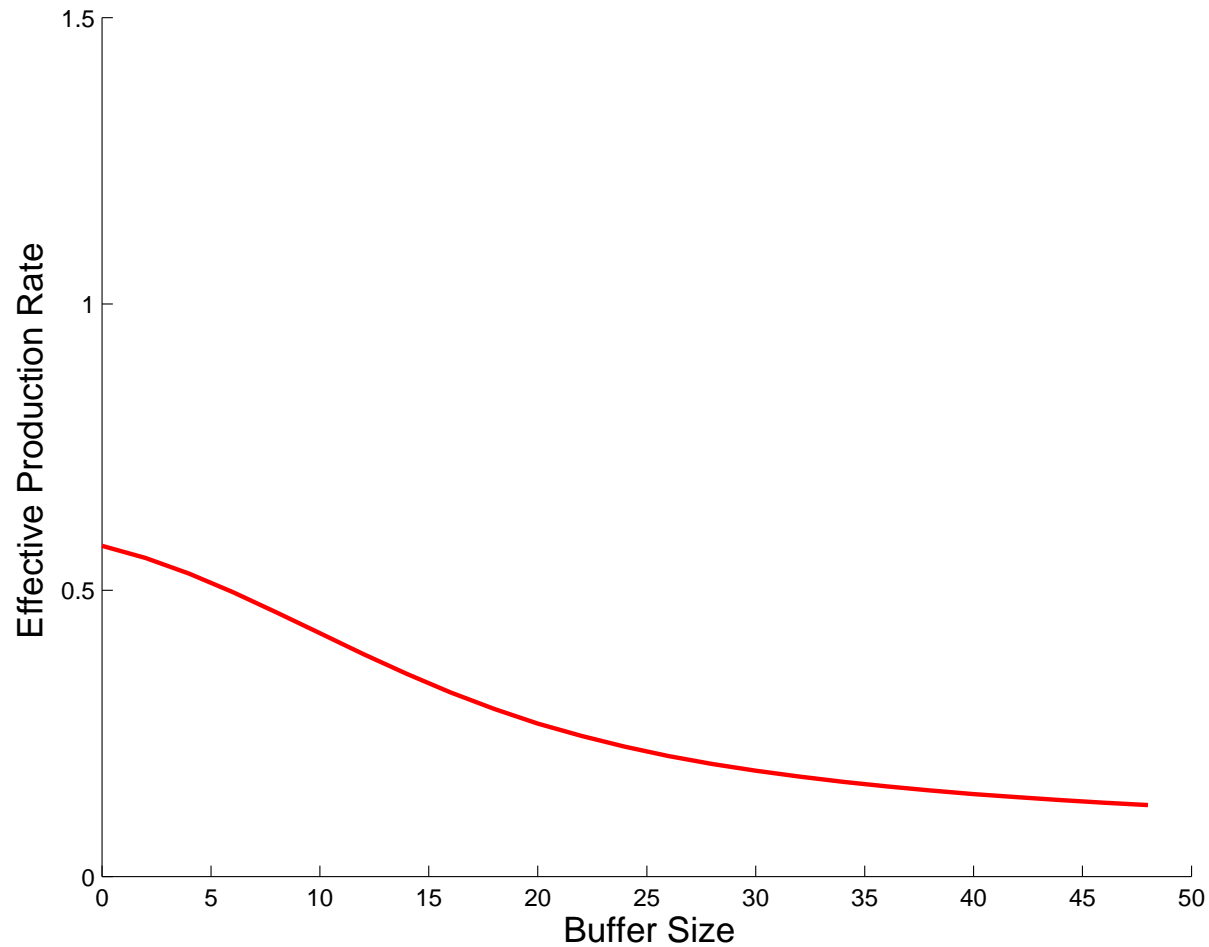
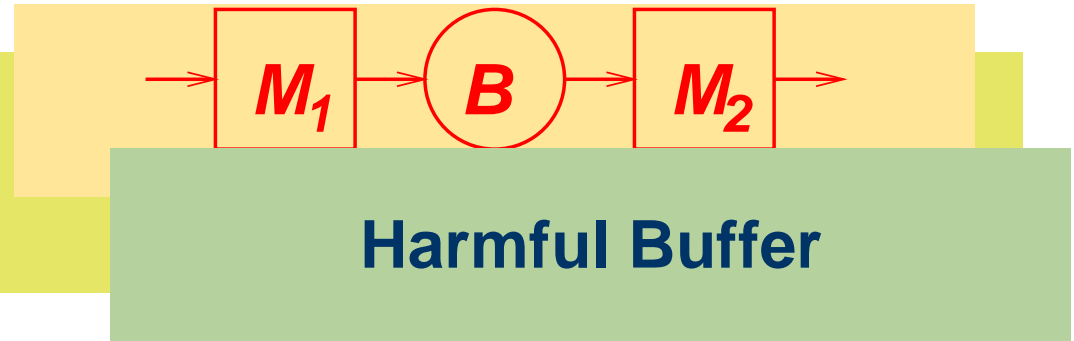


**Beneficial Buffer**



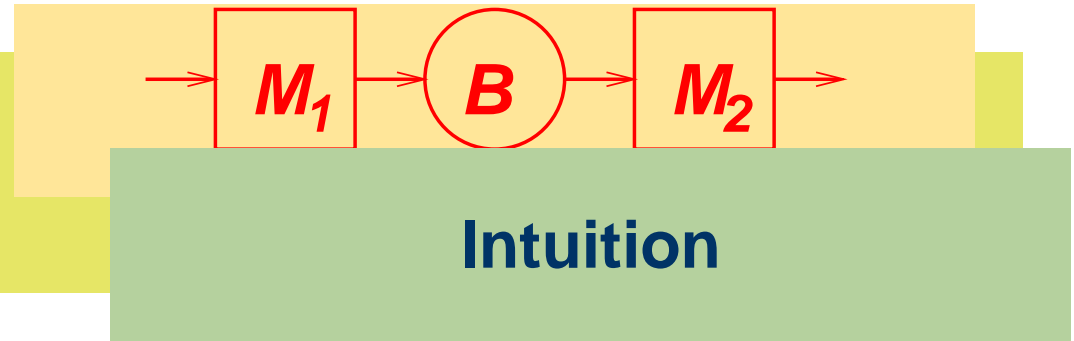
*Effective production rate* = production rate of good parts.

# One- and Two-Machine Systems



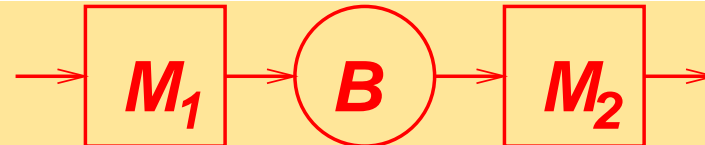


# One- and Two-Machine Systems



- When the inspection detects the first bad part after a good part, the buffer contains *only* bad parts.
- In the harmful buffer case, the first machine has a higher isolated total production rate than the second. Therefore, the buffer is usually close to full, *no matter how large the buffer is* .
- Increasing the size of the buffer increases the number of bad parts in the system when the  $M_1$  state change is detected.
- It also increases the total production rate, but not as much as it increases the production rate of bad parts.

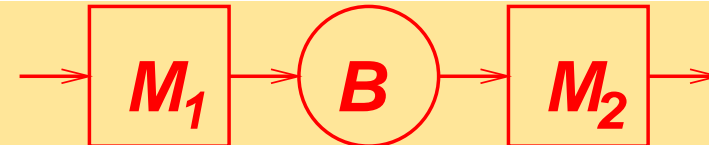
# One- and Two-Machine Systems



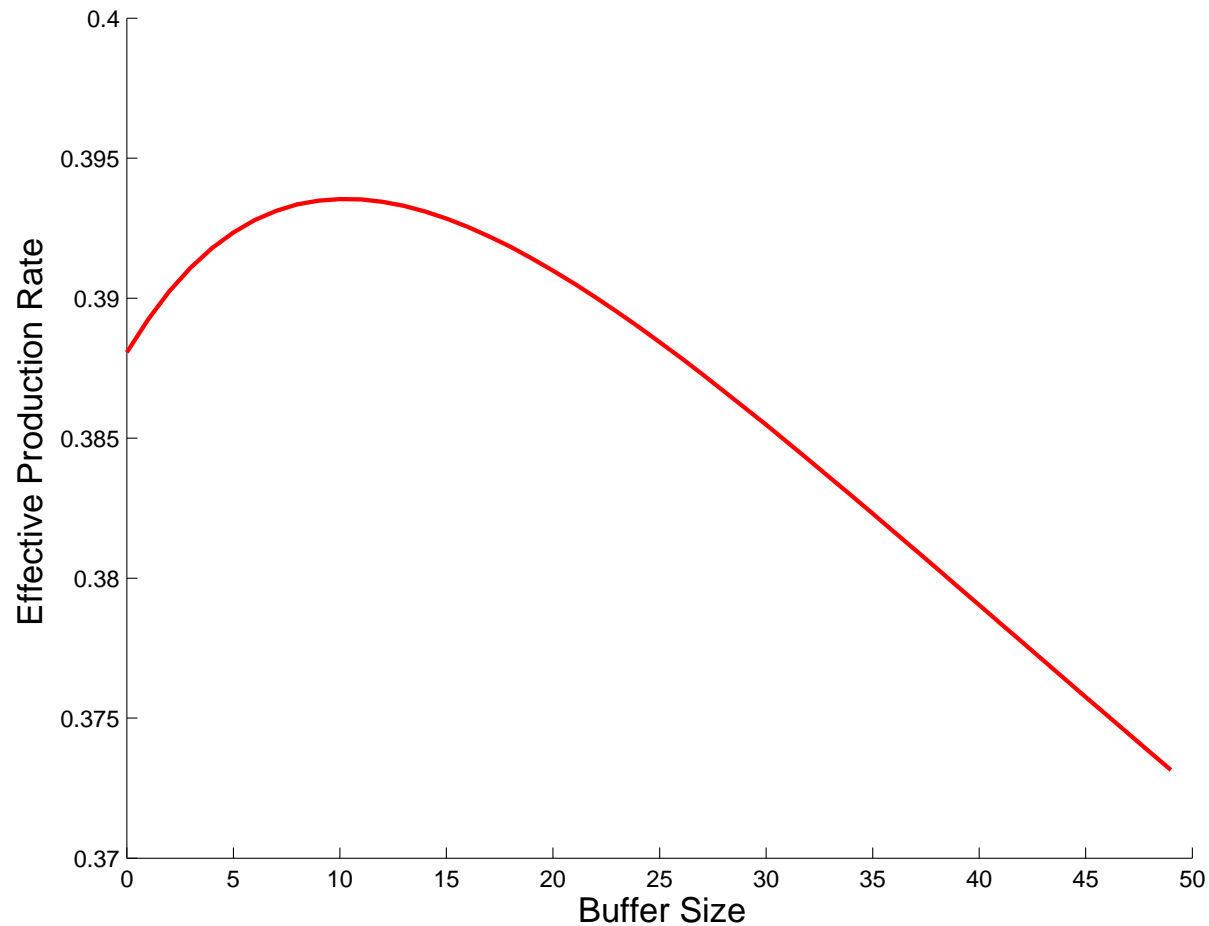
## Intuition

- In the beneficial buffer case, the first machine has a smaller isolated production rate than the second.
- Therefore, even if the buffer size increases, the number of parts in the system is almost always small.
- Therefore it is rare for there to be many bad parts in the buffer when the first bad part is inspected.
- Consequently, the production rate of bad parts remains limited even as the buffer size increases.

# One- and Two-Machine Systems



Mixed-Benefit Buffer



# Long Lines with Finite Buffers

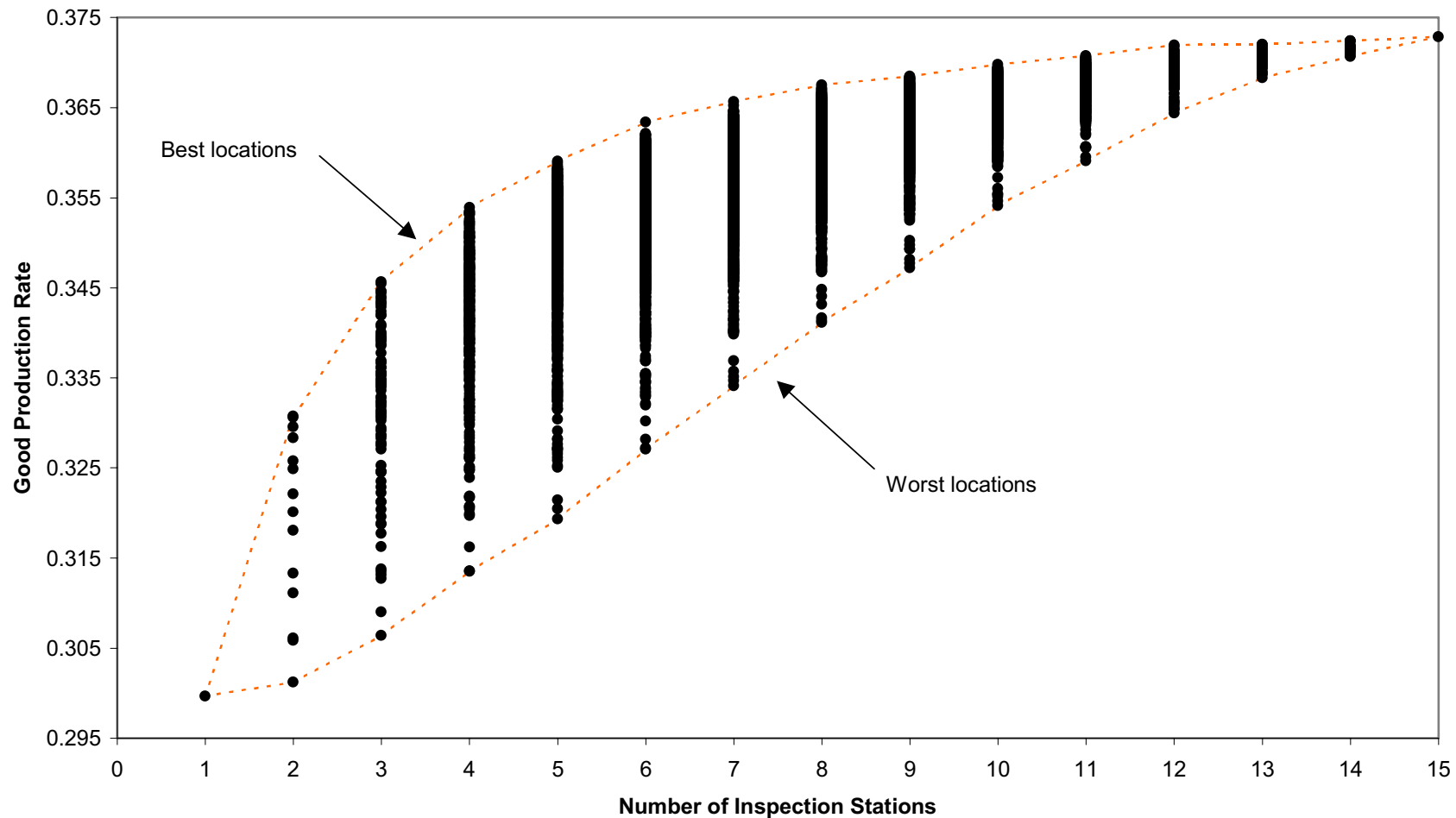
- Intuition: more inspection improves quality.
- Reality: increasing inspection can actually reduce quality, if it is not done intelligently.

# Long Lines with Finite Buffers

- We simulated a 15-machine, 14-buffer line.
- All machines and buffers were identical.
- We looked at all possible combinations of inspection stations in which all operations were inspected.
  - ★ *Example:* Inspection stations just after Machines 6, 9, 13, and 15.
  - ★ The first inspection looks at the results from Machines 1 – 6; the second looks at results from Machines 7, 8, and 9; the third from 10 – 13; and the last from 14 and 15.
  - ★ There is always one inspection after Machine 15.
- A total of  $2^{14}=16,384$  cases were simulated.

# Long Lines with Finite Buffers

## Simulation



# Long Lines with Finite Buffers

## Observations

*A few inspection stations deployed well can do as well or better than many stations deployed poorly.*

- The best distribution of **3** stations has a higher effective production rate than the worst distribution of **7** stations.
- The best distribution of **8** stations performs almost as well as **15** inspection stations.

# Long Lines with Finite Buffers

## Decomposition

Three structures analyzed

Details are in

Jongyoon Kim, “Integrated Quality and Quantity Modeling of a Production Line,” Ph. D thesis, MIT Mechanical Engineering, November, 2004.

and

Kim and Gershwin, “ Modeling and analysis of long flow lines with quality and operational failures,” *IIE Transactions*, to appear.

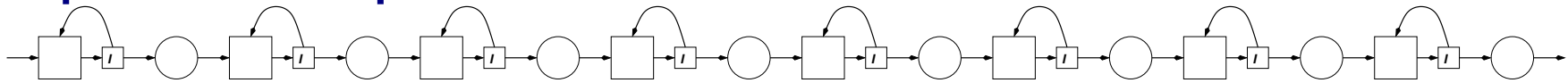


# Long Lines with Finite Buffers

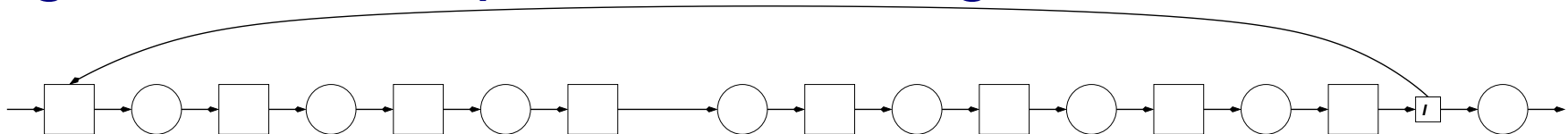
## Decomposition

Three structures analyzed

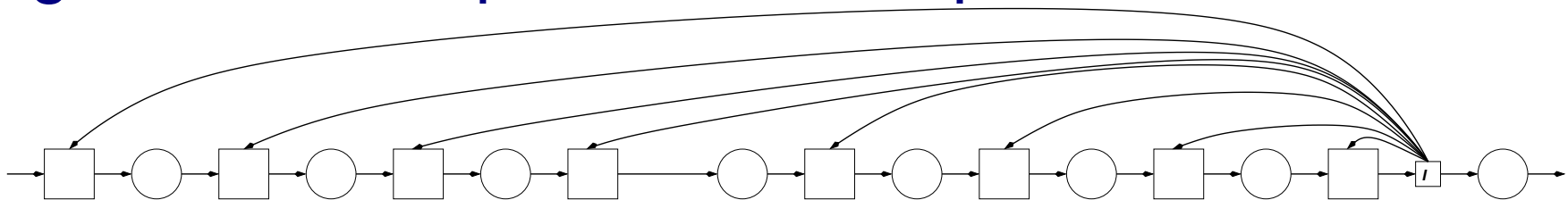
Ubiquitous inspection:



Single remote inspection of a single machine:



Single remote inspection of multiple machines:



# Long Lines with Finite Buffers

- Procedure:
  - ★ Guess  $\bar{x}_i$ .
  - ★ Calculate required  $h_{ij}$  parameters.
  - ★ Transform the 3-state machines into approximate 2-state machines.
  - ★ Solve the resulting line by a standard decomposition technique.
  - ★ Recalculate  $\bar{x}_i$  and iterate.
- Comparison with simulation is reasonable.

# Long Lines with Finite Buffers

The system yield is the product of individual machine yields *using the final  $h_{ij}$  values* .

The effective production rate is the total production rate times the system yield.

# Conclusions

- *Yield* is a system attribute. It is not a simple function of machine yields. It depends on the operation policy, the buffer sizes, etc.
- The Q/Q area is important but has not been studied systematically with engineering rigor as much as other areas have. Much work remains to be done.
- Factory designers and operators must use intuition and simulation.

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