Topic two: Production line profit maximization subject to both time window constraint and production rate constraint

Motivation

A time-window constraint between operations means that the time for a part waiting for the next operation after the previous operation should be kept less than a fixed value, to guarantee the quality of the part. Such a constraint is common in semiconductor industry (Kitamura, Mori, and Ono 2006). As examples,

- Robinson and Giglio (1999) mentioned that a baking operation must be started within two hours of a prior clean operation. If more than two hours elapse, the lot must be sent back to be cleaned again.
- Lu, Ramaswamy, and Kumar (1994) studied the efficient scheduling policies to reduce mean and variance of cycle-time, and pointed out that the shorter the period that wafers are exposed to aerial contaminants while waiting for processing, the smaller is the yield loss.
- Yang and Chern (1995) indicated the consideration of such a time-window constraint in food production, chemical production, and steel production.
- For surveys, see Neacy, Brown, and McKiddie (1994) and Uzsoy, Lee, and Martin-Vega (1992).

Mathematical expression of the constraint

We transform this constraint (for a specific buffer $B_{\hat{i}}$) into our mathematical model through Little's law (Little 1961), as

$$\bar{n}_{\hat{i}} = w_{\hat{i}} P(N_1, \cdots, N_{k-1})$$

where $w_{\hat{i}}$ is the average waiting time for a part in buffer $B_{\hat{i}}$, $\bar{n}_{\hat{i}}$ is the average inventory of buffer $B_{\hat{i}}$. If we further let $\hat{w}_{\hat{i}}$ be the time constraint, then we require $w_{\hat{i}} \leq \hat{w}_{\hat{i}}$, or

$$\bar{n}_{\hat{i}} \leq \hat{w}_{\hat{i}} P(N_1, \cdots, N_{k-1})$$

Note that constraint above guarantees the average part waiting time, **NOT** the maximal part waiting time, to be upper bounded by $\hat{w}_{\hat{i}}$. To resolve this concern, we may consider to reduce $\hat{w}_{\hat{i}}$, by a certain multiplier $0 < \delta < 1$, to $\delta \hat{w}_{\hat{i}}$ in the constraint, such that the probability that the waiting time for a part is less than or equal to $\hat{w}_{\hat{i}}$ will satisfy a specific confidence level.

The optimization problem

$$
\begin{aligned}
\max_{\mathbf{N}} \quad J(\mathbf{N}) &= AP(\mathbf{N}) - \sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i(\mathbf{N}) \\
\text{s.t.} \quad P(\mathbf{N}) &\geq \hat{P} \\
\bar{n}_{\hat{i}} &\leq \hat{w}_{\hat{i}} P(\mathbf{N}) \\
N_i &\geq N_{\min}, \forall i = 1, \cdots, k-1
\end{aligned}
$$

The optimization problem has 5 cases in general. They are

- The production rate constraint conflicts with the time-window constraint. Therefore, there is no feasible solution to the problem.
- The optimal solution exists. Both the production rate constraint and the time window constraint are active.
- The optimal solution exists. The production rate constraint is active, while the time window constraint is inactive.
- The optimal solution exists. The production rate constraint is inactive, while the time window constraint is active.
- The optimal solution exists. Both the production rate constraint and the time window constraint are inactive.

Consider a three machine two buffer line with machine parameters $r_1 = .15, p_1 = .01, r_2 = .15, p_2 = .01, r_3 = .09$ and $p_3 = .01$. In addition, consider these 5 cases below:

- Case 1: $\hat{P} = .89$ and $\hat{w}_1 = 2$.
- Case 2: $\hat{P} = .88$ and $\hat{w}_1 = 7$.
- Case 3: $\hat{P} = .88$ and $\hat{w}_1 = 15$.
- Case 4: $\hat{P} = .86$ and $\hat{w}_1 = 6.5$.
- Case 5: $\hat{P} = .86$ and $\hat{w}_1 = 15$.

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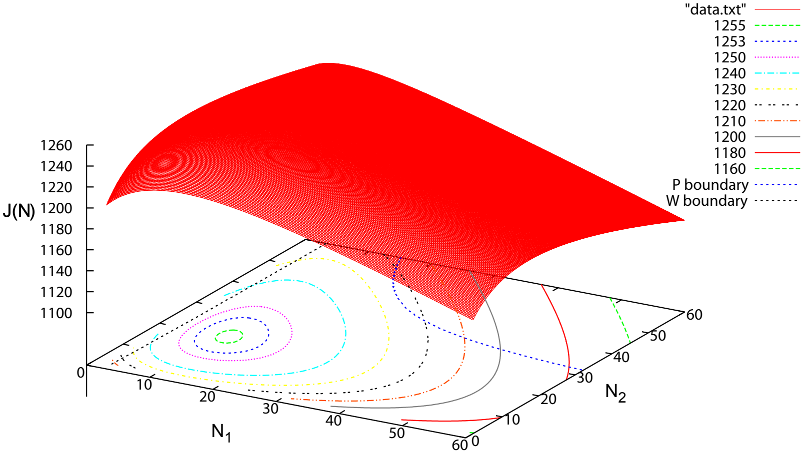
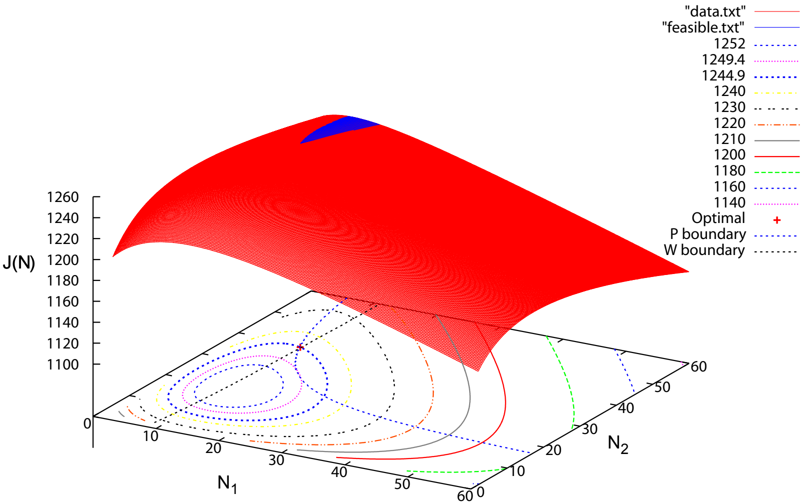
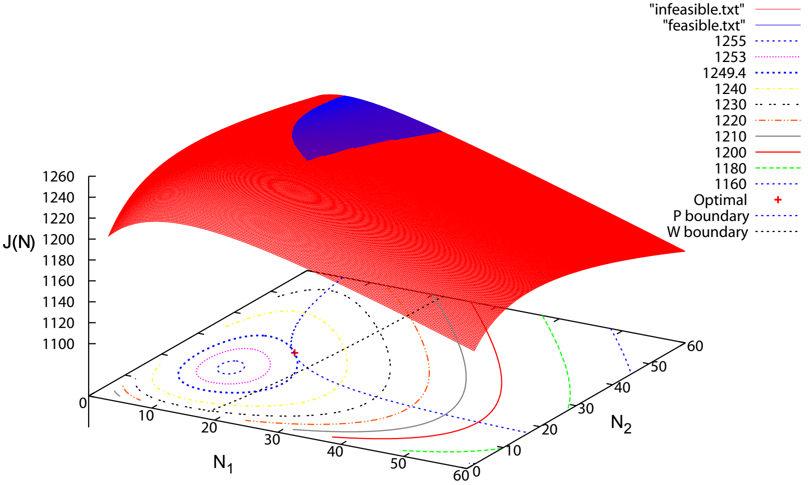


The optimal solution exists. Both the production rate constraint and the time window constraint are active.



The optimal solution exists. The production rate constraint is active, while the time window constraint is inactive.



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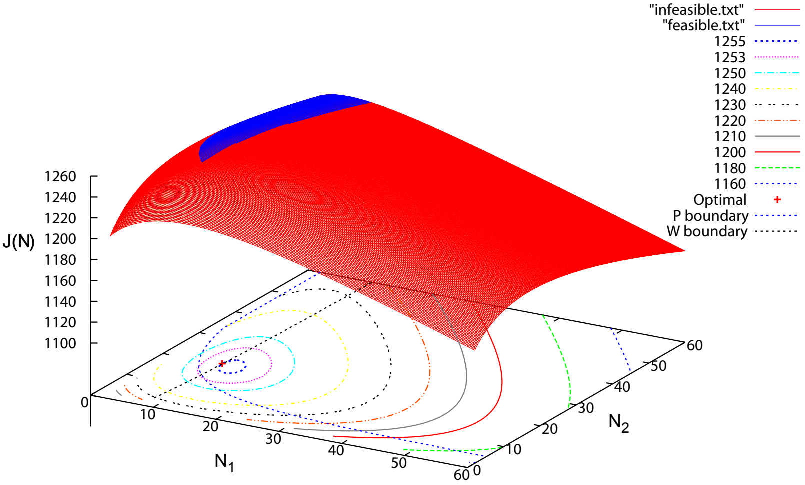

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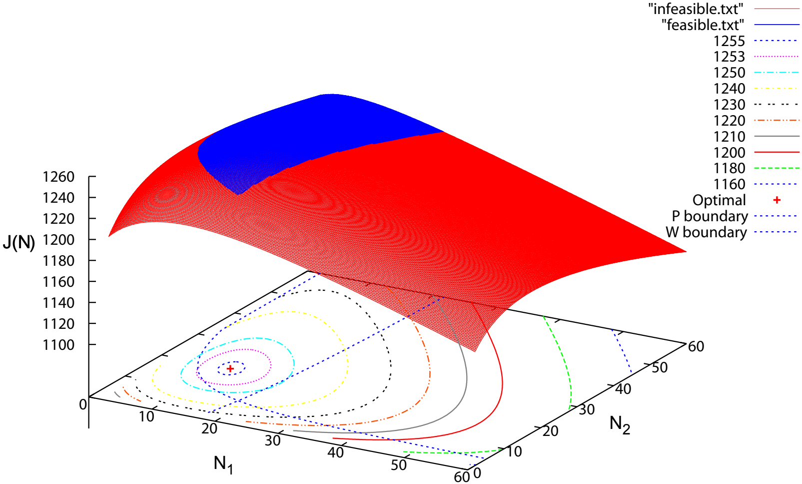


We extend the algorithm in Topic 1 to solve the new optimization problem with both time-window constraint and production rate constraint. For the case where both of the two constraints are active, one constraint qualification we can use to guarantee the existence of Lagrange multipliers is that $\nabla(\hat{n}_{\hat{i}}(\mathbf{N}^\star) - \hat{w}_{\hat{i}} P(\mathbf{N}^\star))$ and $\nabla(\hat{P} - P(\mathbf{N}^\star))$ are linearly independent.

This is equivalent to require that $\nabla \hat{n}_{\hat{i}}(\mathbf{N}^\star)$ and $\nabla P(\mathbf{N}^\star)$ are linearly independent. Since all components of $\nabla P(\mathbf{N}^\star)$ are positive due to the monotonicity of $P(\mathbf{N})$, but $\nabla \hat{n}_{\hat{i}}(\mathbf{N}^\star)$ has both positive and negative components, they are linearly independent[2].

[2]We will provide formal proof for this in the future.

Applying the KKT conditions, we have

$$-\begin{pmatrix} \frac{\partial J(\mathbf{N}^\star)}{\partial N_1} \\ \vdots \\ \frac{\partial J(\mathbf{N}^\star)}{\partial N_{\hat{i}}} \\ \vdots \\ \frac{\partial J(\mathbf{N}^\star)}{\partial N_{k-1}} \end{pmatrix} + \mu_0^\star \begin{pmatrix} \frac{\partial \bar{n}_{\hat{i}}(\mathbf{N}^\star)}{\partial N_1} - \hat{w}_{\hat{i}} \frac{\partial P(\mathbf{N}^\star)}{\partial N_1} \\ \vdots \\ \frac{\partial \bar{n}_{\hat{i}}(\mathbf{N}^\star)}{\partial N_{\hat{i}}} - \hat{w}_{\hat{i}} \frac{\partial P(\mathbf{N}^\star)}{\partial N_{\hat{i}}} \\ \vdots \\ \frac{\partial \bar{n}_{\hat{i}}(\mathbf{N}^\star)}{\partial N_{k-1}} - \hat{w}_{\hat{i}} \frac{\partial P(\mathbf{N}^\star)}{\partial N_{k-1}} \end{pmatrix} - \mu_1^\star \begin{pmatrix} \frac{\partial P(\mathbf{N}^\star)}{\partial N_1} \\ \vdots \\ \frac{\partial P(\mathbf{N}^\star)}{\partial N_{\hat{i}}} \\ \vdots \\ \frac{\partial P(\mathbf{N}^\star)}{\partial N_{k-1}} \end{pmatrix} = \begin{pmatrix} 0 \\ \vdots \\ 0 \\ \vdots \\ 0 \end{pmatrix} \tag{14}$$

$$\mu_0^\star(\hat{n}_{\hat{i}}(\mathbf{N}^\star) - \hat{w}_{\hat{i}} P(\mathbf{N}^\star)) = 0 \tag{15}$$

and

$$\mu_1^\star(\hat{P} - P(\mathbf{N}^\star)) = 0 \tag{16}$$

Solving those three conditions is equivalent to searching μ_0 and μ_1 and solving the following optimization problem

$$\max_{\mathbf{N}} \quad \bar{J}(\mathbf{N}) = (A + \mu_0 \hat{w}_{\hat{i}} + \mu_1) P(\mathbf{N}) - \sum_{i=1}^{k-1} b_i N_i - \sum_{i=1}^{k-1} c_i \bar{n}_i(\mathbf{N}) - \mu_0 \bar{n}_{\hat{i}}(\mathbf{N})$$

$$\text{s.t.} \quad N_{\min} - N_i \leq 0, \forall i = 1, \cdots, k-1 \tag{17}$$

until its solution $\mathbf{N}^c$ satisfies $P(\mathbf{N}^c) = \hat{P}$ and $\hat{n}_{\hat{i}}(\mathbf{N}^c) = \hat{w}_{\hat{i}} P(\mathbf{N}^c)$. Then, $\mathbf{N}^\star = \mathbf{N}^c$.

Algorithm summary

1. Check the feasibility of the problem.
2. Solve the problem with the production rate constraint, and let $\mathbf{N}^P$ denote the solution. Check if $\hat{n}_{\hat{i}}(\mathbf{N}^P) \leq \hat{w}_{\hat{i}} P(\mathbf{N}^P)$. If yes, then we are done and $\mathbf{N}^\star = \mathbf{N}^P$. If not, go to step 3.
3. Solve the problem with the time-window constraint, and let $\mathbf{N}^T$ denote the solution. Check if $P(\mathbf{N}^T) \geq \hat{P}$. If yes, then we are done and $\mathbf{N}^\star = \mathbf{N}^T$. If not, go to step 4.
4. Solve the problem with both constraints.

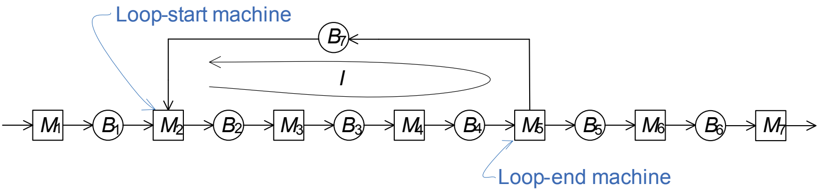
Numerical results

Consider a 6-machine 5-buffer line with constraints $\hat{P} = .83$ and $\hat{w}_3 = 7$. In addition, $A = 3000$ and all b_i and c_i are 1.

machine	M_1	M_2	M_3	M_4	M_5	M_6
r	.15	.15	.09	.10	.11	.10
p	.01	.01	.01	.01	.01	.01

	$\hat{P}$ Surface Search	The algorithm	Error
$P(\mathbf{N}^\star)$	.8300	.8300	
$\bar{n}_3(\mathbf{N}^\star)/P(\mathbf{N}^\star)$	6.9999	7.0884	1.26%
$N_1^\star$	8.4800	8.5010	0.25%
$N_2^\star$	21.2800	21.1960	0.39%
$N_3^\star$	12.5200	12.6625	1.14%
$N_4^\star$	40.5200	39.8855	1.57%
$N_5^\star$	24.5200	24.6071	0.36%
$\bar{n}_1$	5.6656	5.6818	0.29%
$\bar{n}_2$	13.5400	13.4803	0.44%
$\bar{n}_3$	5.8099	5.8834	1.27%
$\bar{n}_4$	13.3901	13.2503	1.04%
$\bar{n}_5$	8.0201	8.0387	0.23%
Profit ($)	2336.2582	2336.8139	0.02%

Research in process and Research extensions



- Processes that utilize pallets or fixtures can be viewed as loops since the number of pallets/fixtures that are in the system remains constant.

- Control policies such as Constant Work-in-process (CONWIP) and Kanban create conceptual loops by imposing a limit on the number of parts that can be in the system at any given time.

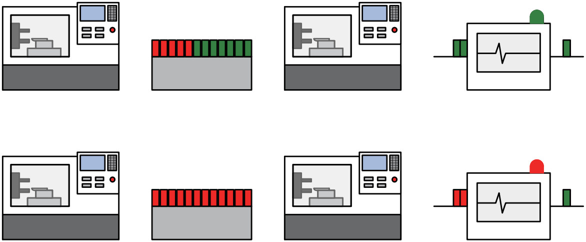
Prior research and assumption

- Single loop evaluation by Gershwin and Werner (2005), multiple loop evaluation by Zhang (2006).
- Concavity of $P(\mathbf{N}, I)$.

Work

- Develop analytical solutions for two-machine-line evaluation with **no delay** machines based on Tolio, Matta, and Gershwin (2002).
- Extend and improve loop evaluation algorithm for single arbitrary loops.
- Present the optimization algorithm, which is an extension of the algorithm for line optimization.
- Prove this algorithm theoretically by the KKT conditions of nonlinear programming, and verify this algorithm numerically.

By taking account of quality control, we assume that machines generate both good parts and bad parts. Unfortunately, buffers delay the inspection of bad parts.



Kim and Gershwin (2005) pointed out that in the case of our example above, an increase of buffer size could either increase or decrease the production rate for different lines.

This means whenever we decide to establish a buffer between two machines (or machine sets), we introduce a fixed buffer set-up cost. After the buffer is established, the buffer space cost will be proportional to its size. So, in this case, buffer space cost will be 0 if $N_i < N_{\min}$ or $a_i + b_i N_i$ if $N_i \geq N_{\min}$ for buffer B_i.

Thank you!

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