Chapter 2 (and 3)

Cross-Sections

- TA
- Lewis 2.1, 2.2 and 2.3

Learning Objectives

- Understand different types of nuclear reactions
- Understand cross section behavior for different reactions
- Understand resonance behavior and its relation to the nuclear energy levels
- Know where to find nuclear data

Microscopic Cross section

Consider a beam of mono-energetic neutrons of intensity I incident on a very thin material such that there are Na atoms/cm^2.s

The collision rate of neutrons is proportional to the neutron beam intensity and the nuclei density Na. The constant of proportionality is defined as the neutron microscopic cross-section."

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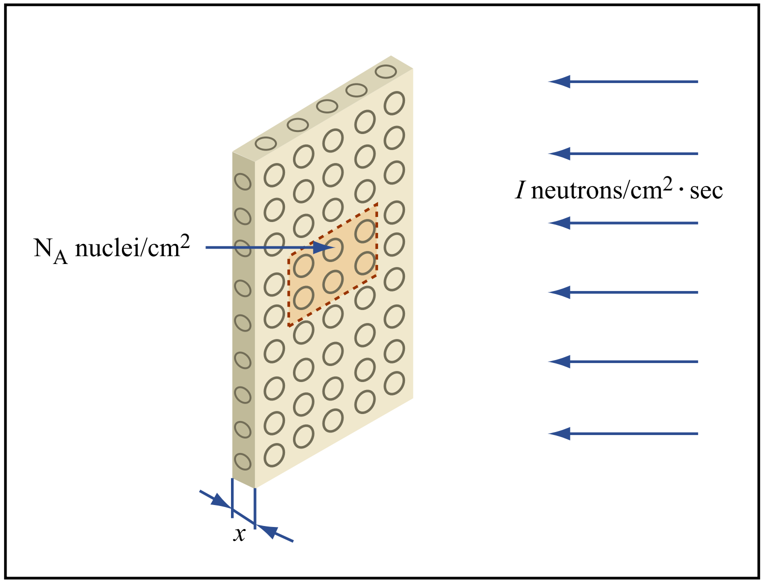

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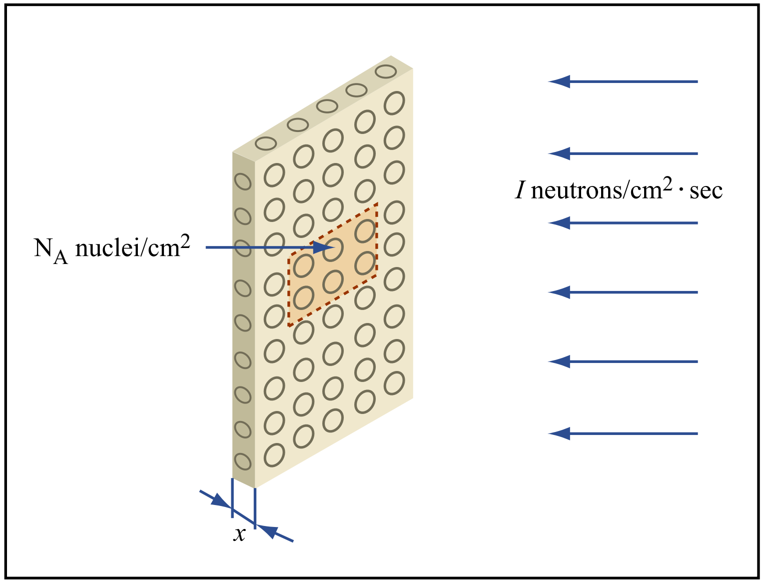
Microscopic Cross section

The microscopic cross-section characterizes the probability of a neutron interaction.

$$R = \sigma \quad I \quad N_A$$

$$\left[\frac{\#}{cm^2 s}\right] \quad \left[cm^2\right]\left[\frac{\#}{cm^2 s}\right]\left[\frac{\#}{cm^2}\right]$$



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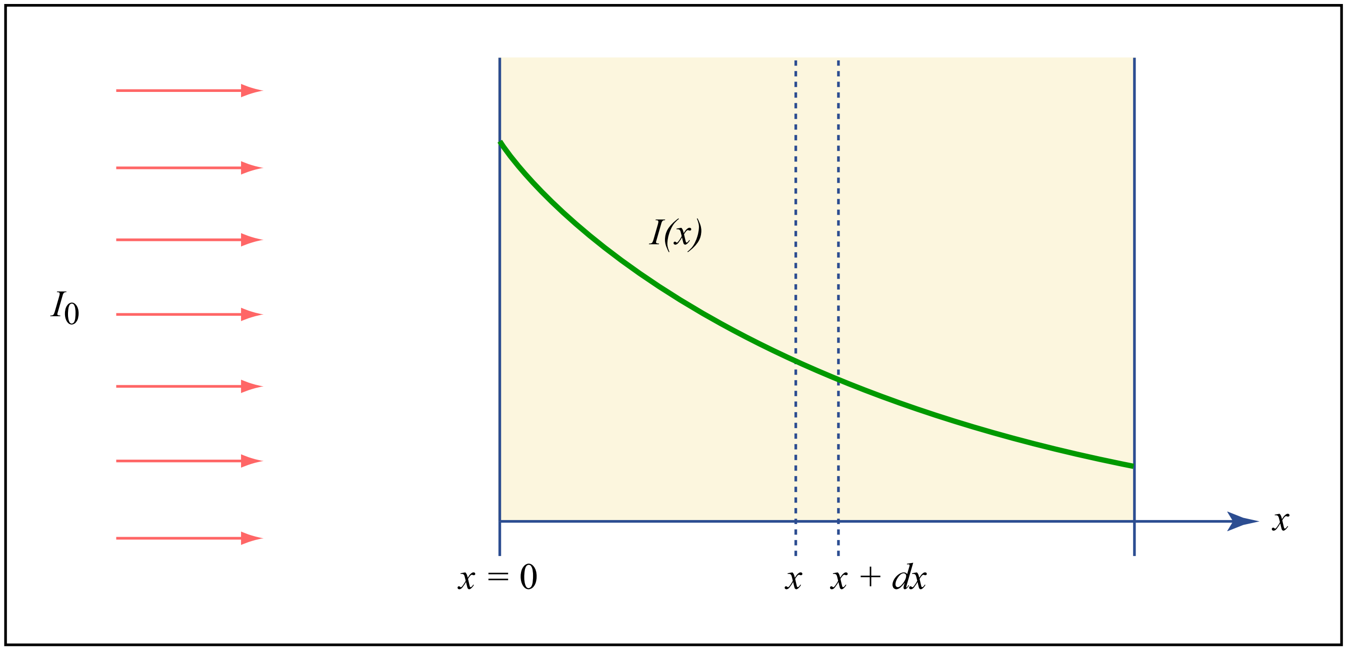
Cross Section - Microscopic

Scattering	$\sigma_s = \sigma_e + \sigma_{in}$
Absorption	$\sigma_a = \sigma_\gamma + \sigma_f$
Total	$\sigma_t = \sigma_s + \sigma_a$

$$\sigma = \frac{\textit{Number of reactions /nucleus / s}}{\textit{Number of incident neutrons / cm}^2\textit{s}} = \frac{(R / N_A)}{1}$$

Incident Beam (Neutron) on a Thick Target



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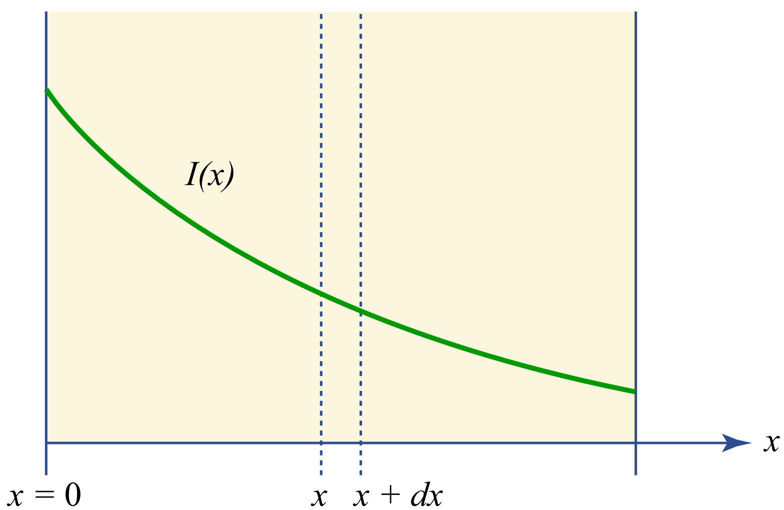
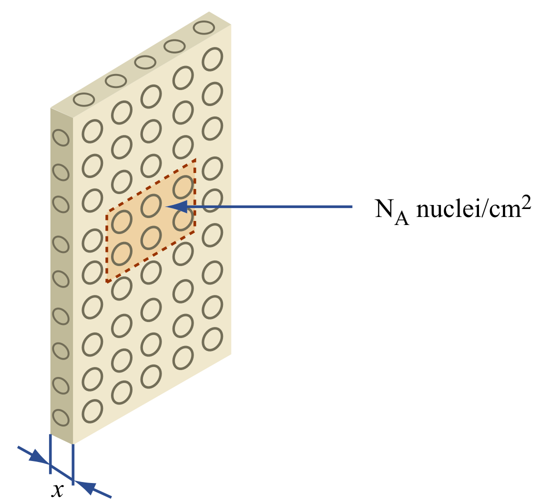
Now consider the case of a thick target with an incident beam I_0 for which we want to know the unattenuated beam intensity as a function of position I(x).

Unattentuated beam in target

- Taking an infinitesimally thin portion of the target, dx, allows us to use the previous analysis on dx between x and x + dx.





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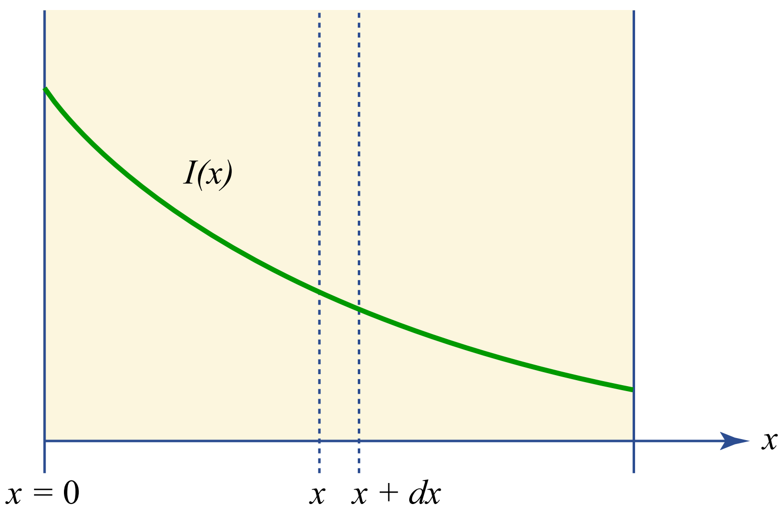
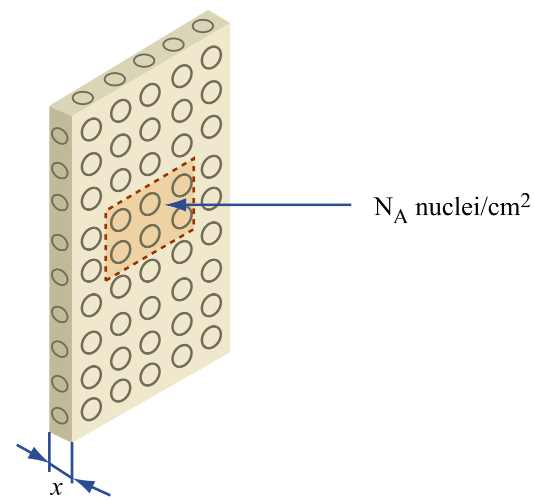
Unattentuated beam in target





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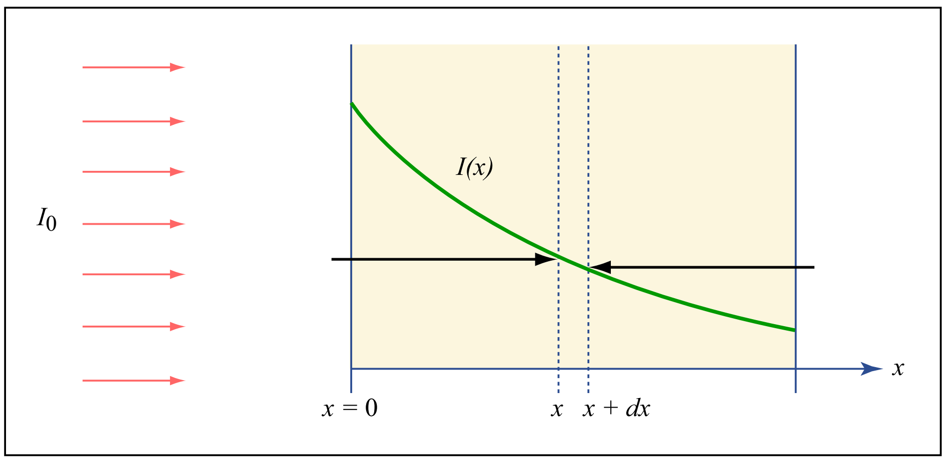
Number of target nuclei per cm^2 in dx is

$dN_A = N\,dx$ where

N = number density of the target nuclei in units cm^{-3}.

Relating reaction rate to beam intensity



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The total reaction rate in dx can be defined as

$$dR = \sigma_t I dN_A = \sigma_t I N dx$$

Each neutron that reacts decreases the unattenuated beam intensity, thus

$$-dI(x) = -\,[I(x + dx) - I(x)] = \sigma_t I N dx$$

Macroscopic cross-section

we can then solve this differential equation to get I(x)

$$\frac{dI(x)}{dx} = - N\sigma_t I(x)$$

we can then define the macroscopic cross-section such that

$$I(x) = I_0 e^{-N\sigma_t x}$$

Macroscopic cross section interpretation

$\Sigma_t \equiv$ Probability per unit path length that the neutron will interact with a nucleus in the target.

$\exp(-\Sigma_t x) \equiv$ Probability that a neutron will travel a distance x without making a collision.

$\Sigma_t \exp(-\Sigma_t x)dx \equiv$ Probability that a neutron will make its first collision in dx after traveling a distance x.

Mean free path of neutron

$$\bar{x} \equiv \int_0^\infty dx\, x\, p(x) = \Sigma_t \int_0^\infty dx\, x\, \exp(-\Sigma_t x) = \frac{1}{\Sigma_t}$$

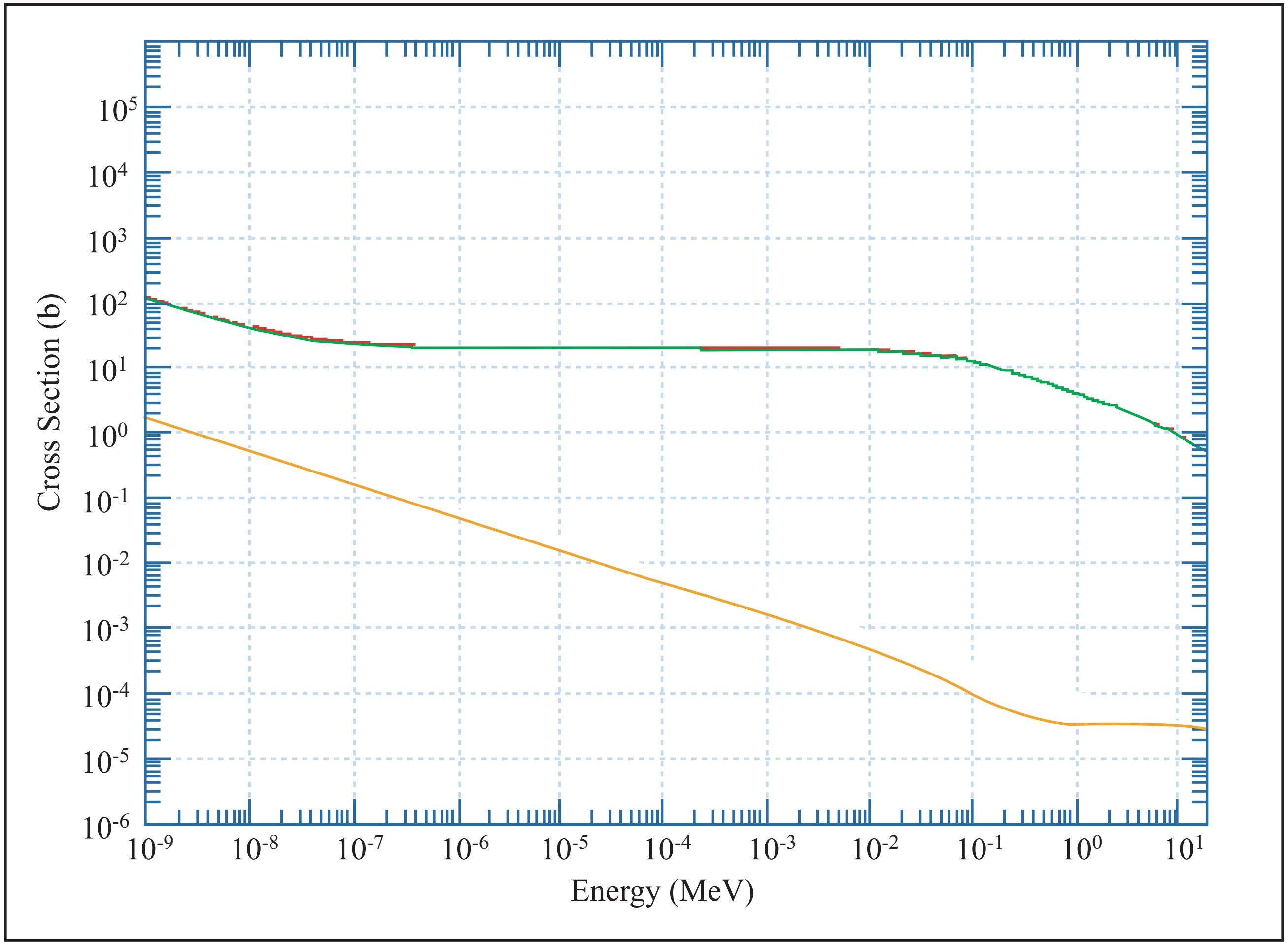
Interaction probability calculates the average distance a neutron travels before interacting with a nucleus

$\bar{x} = \Sigma_t^{-1} \equiv$ Average distance traveled by a neutron before making a collision

Two fundamental aspects of neutron cross sections

- Kinematics of two-particle collisions
  - Conservation of momentum
  - Conservation of energy
- Dynamics of nuclear reactions
  - Potential scattering
  - Compound nucleus formation

Hydrogen x.s.



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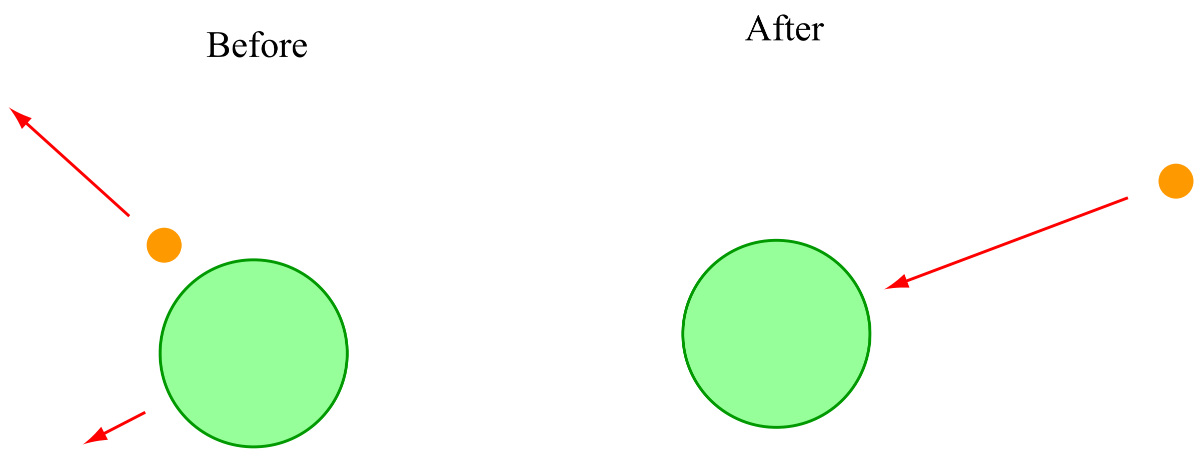
Potential scattering



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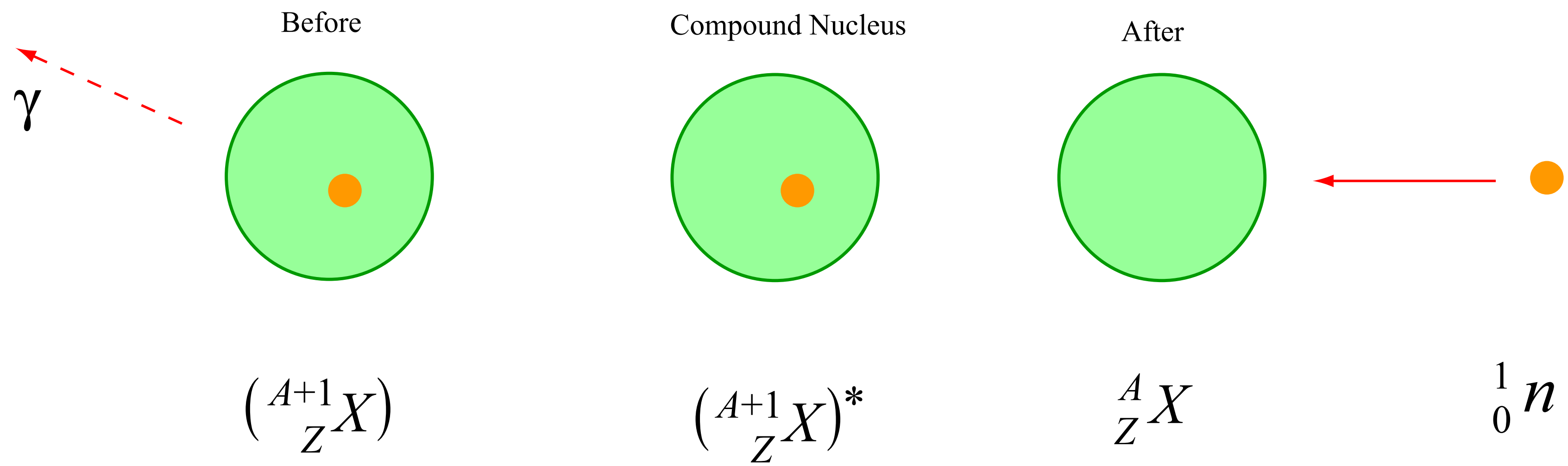
Hard sphere collision where the neutron bounces off the nucleus. The interaction time is approximately 10-17s.

Compound nucleus formation



Radiative capture

Image by MIT OpenCourseWare.

Neutron penetrates the nucleus and forms a compound nucleus (excited state). The compound nucleus regains stability by decaying. The interaction time is approximately 10-14s.

Compound nucleus decay processes

$${}^{1}_{0}n + {}^{A}_{Z}X \longrightarrow \left({}^{A+1}_{Z}X\right)^{*}$$

Resonance elastic scattering

$${}^{1}_{0}n + {}^{A}_{Z}X$$

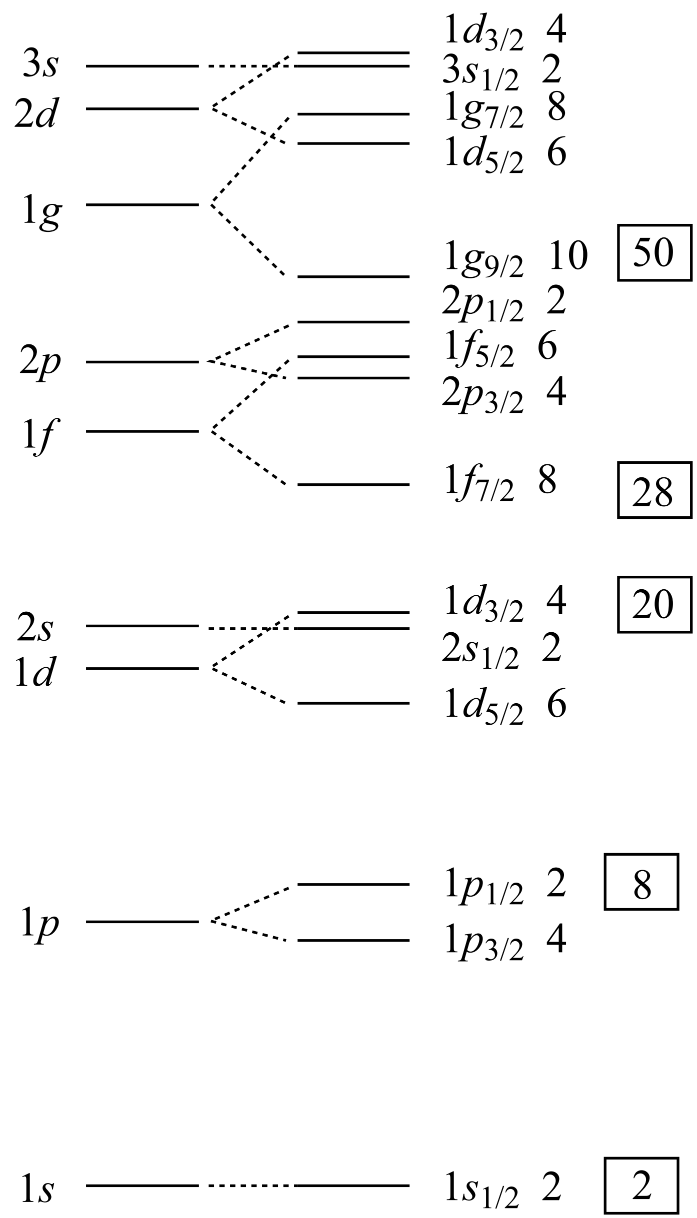
$${}^{1}_{0}n + \left({}^{A}_{Z}X\right)^{*}$$ Inelastic scattering

$${}^{A+1}_{Z}X + \gamma$$ Radiative capture

Fission

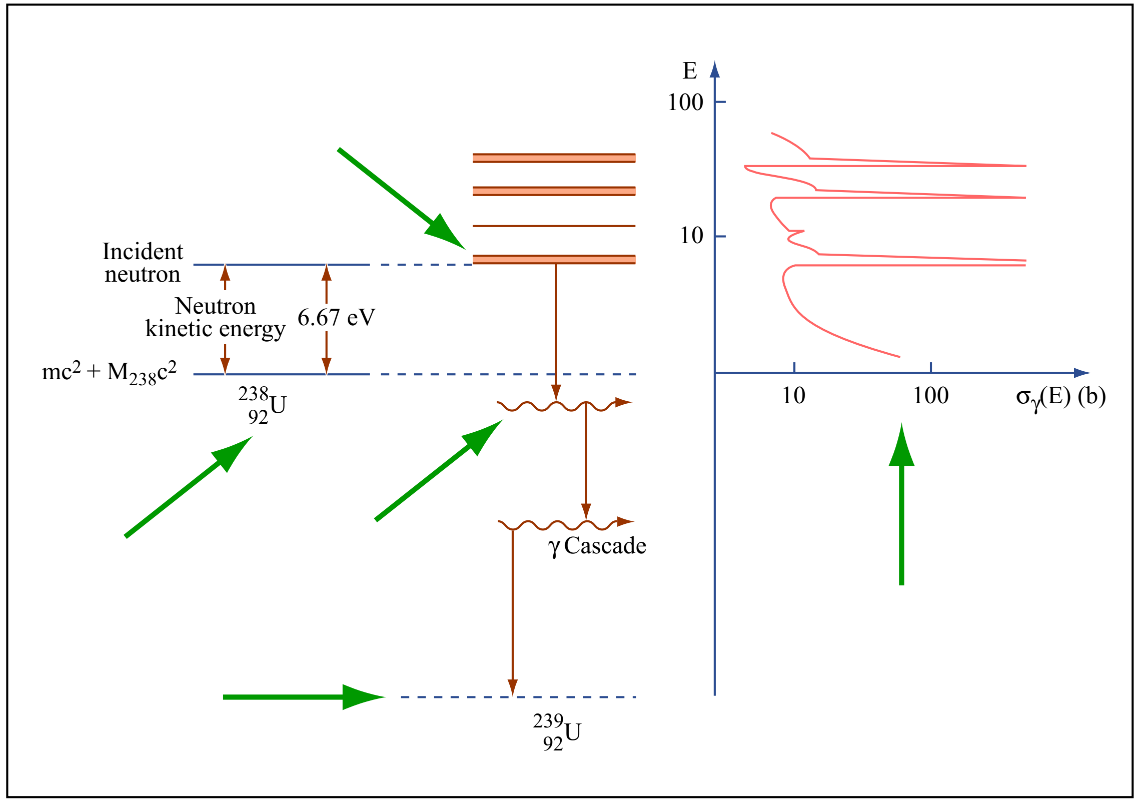
$${}^{A_1}_{Z_1}X + {}^{A_2}_{Z_2}X + 2 - 3\ {}^{1}_{0}n$$

Nuclear Shell Model



Radiative capture

The figure is for ^{238}U at E=6.67 eV.



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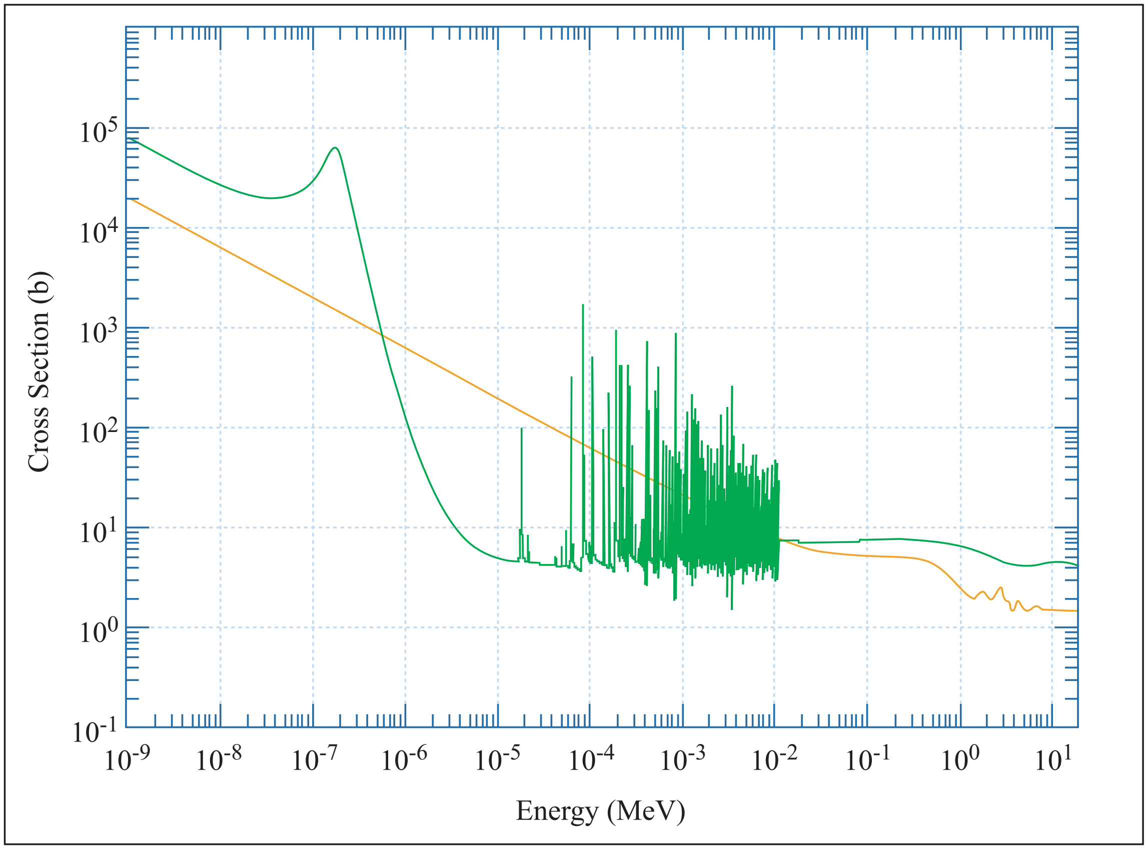
When the sum of the kinetic energy of the neutron in the CM and its binding energy correspond to an energy level of the compound nucleus, the neutron cross section exhibits a spike in its probability of interaction which are called resonances.

U-238



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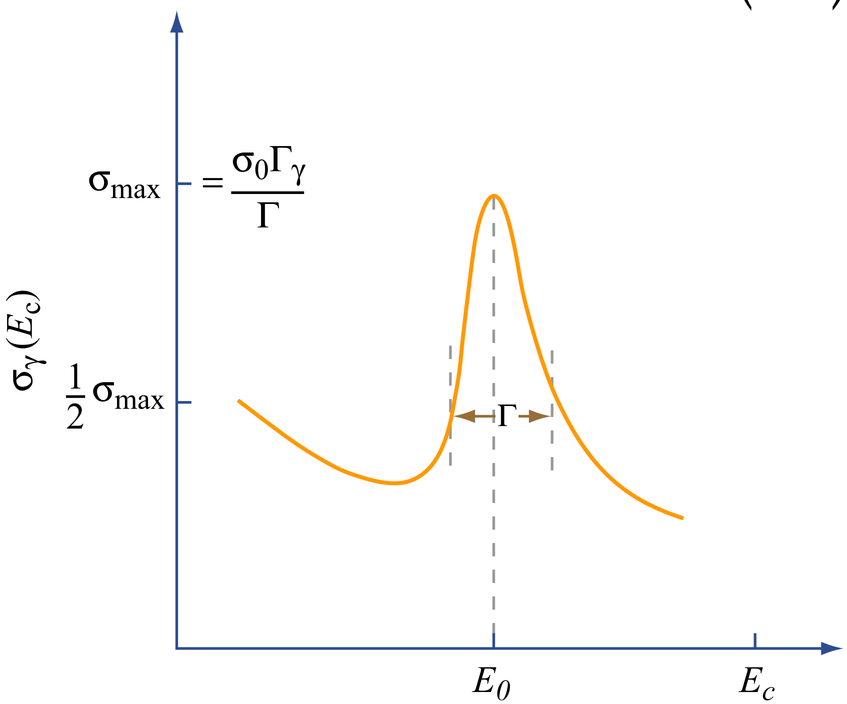
Cross Section Modeling

- Experimental data isn’t available at every energy
- Quantum mechanical models are used to provide cross section values around data points
- Simplest version is Single Level Breit-Wigner
  - Valid for widely spaced resonances

$$\sigma_\gamma(E_c) = \sigma_0 \frac{\Gamma_\gamma}{\Gamma}\left(\frac{E_0}{E_c}\right)^{1/2} \frac{1}{1+y^2}, \quad y = \frac{2}{\Gamma}(E_c - E_0)$$



Γ = Total line width (FWHM)

Γ_γ = Radiative line width

σ_0 = Total cross section at $E = E_0$

Breit-Wigner Formula for Resonance Capture Cross Section

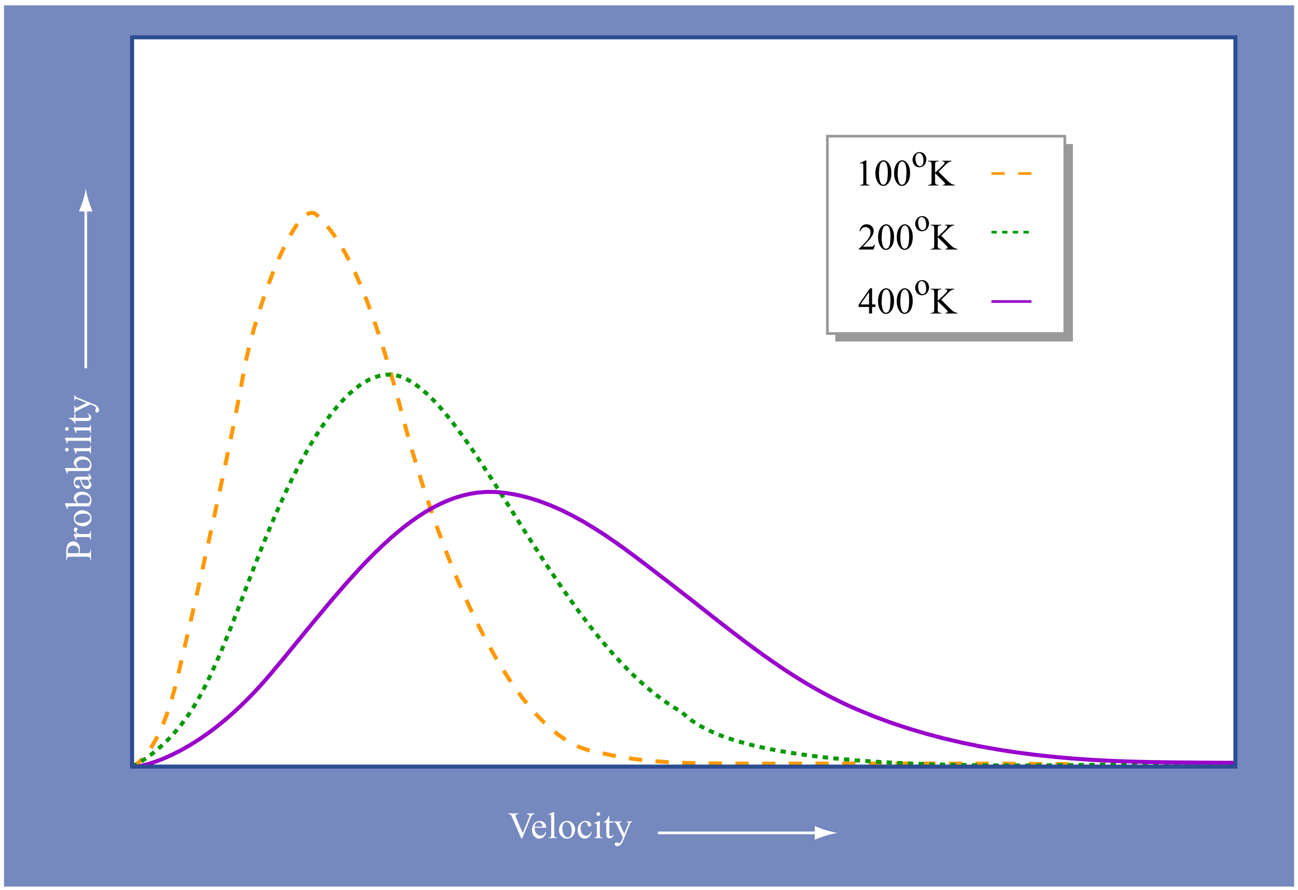
Doppler Effect

- Cross sections are functions of relative speed between neutron and target nucleus
  - Generally assumed that target is at rest
  - Valid for smooth cross sections
  - Not valid for resonances

Doppler Effect

- Resonances must be averaged over atom velocity
  - Assume target nuclei have Maxwell-Boltzmann energy distribution
- As atom temperature increases
  - Resonance becomes wider
  - Resonance becomes shorter
- Area stays approximately the same

Maxwell-Boltzmann Distribution



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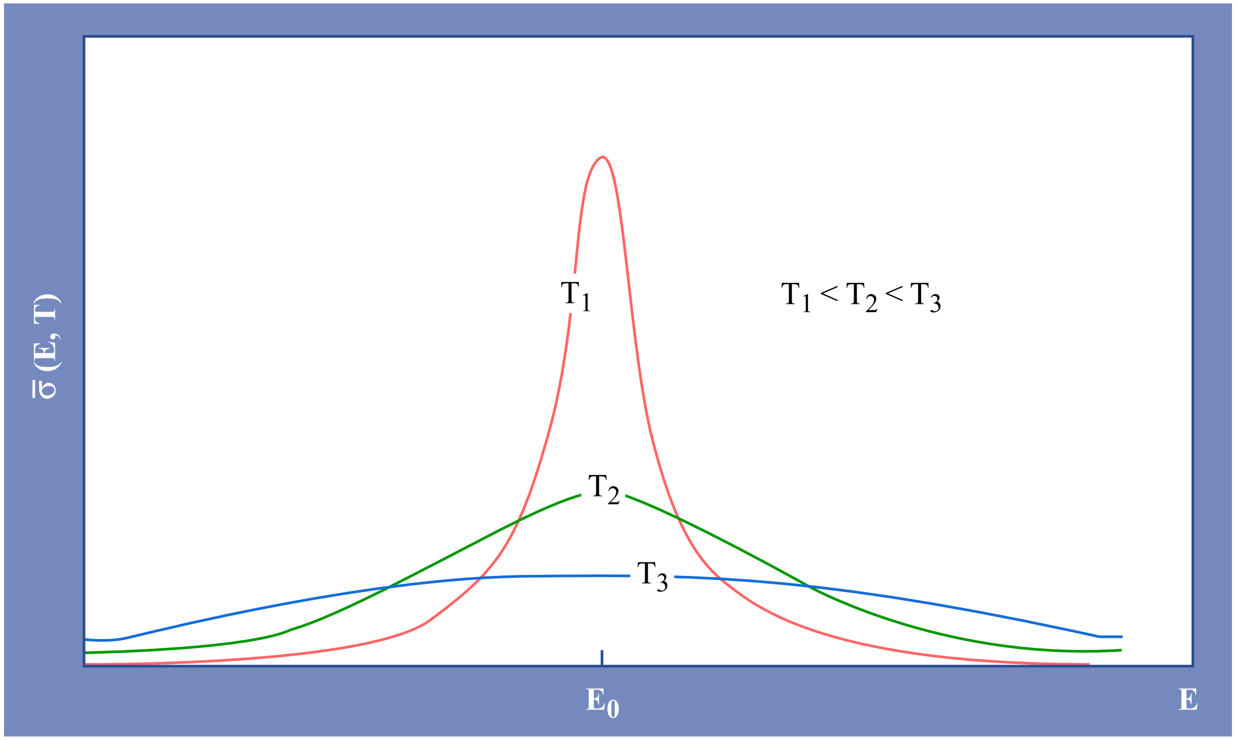
Doppler Effect

$$\bar{\sigma}(\mathrm{E})$$



E

$$\bar{\sigma}(\mathrm{E}) \rightarrow \sigma\left(\left|\vec{v} - \vec{V}\right|\right)$$

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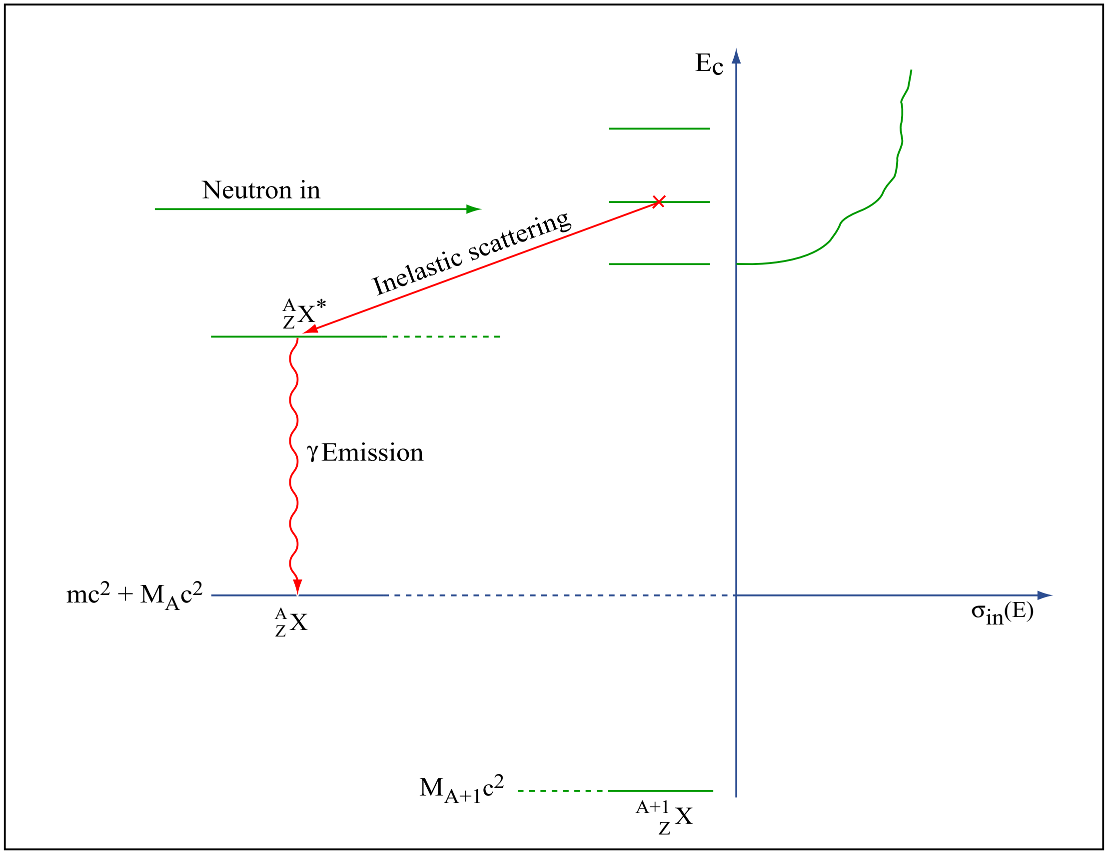
Scattering - Inelastic



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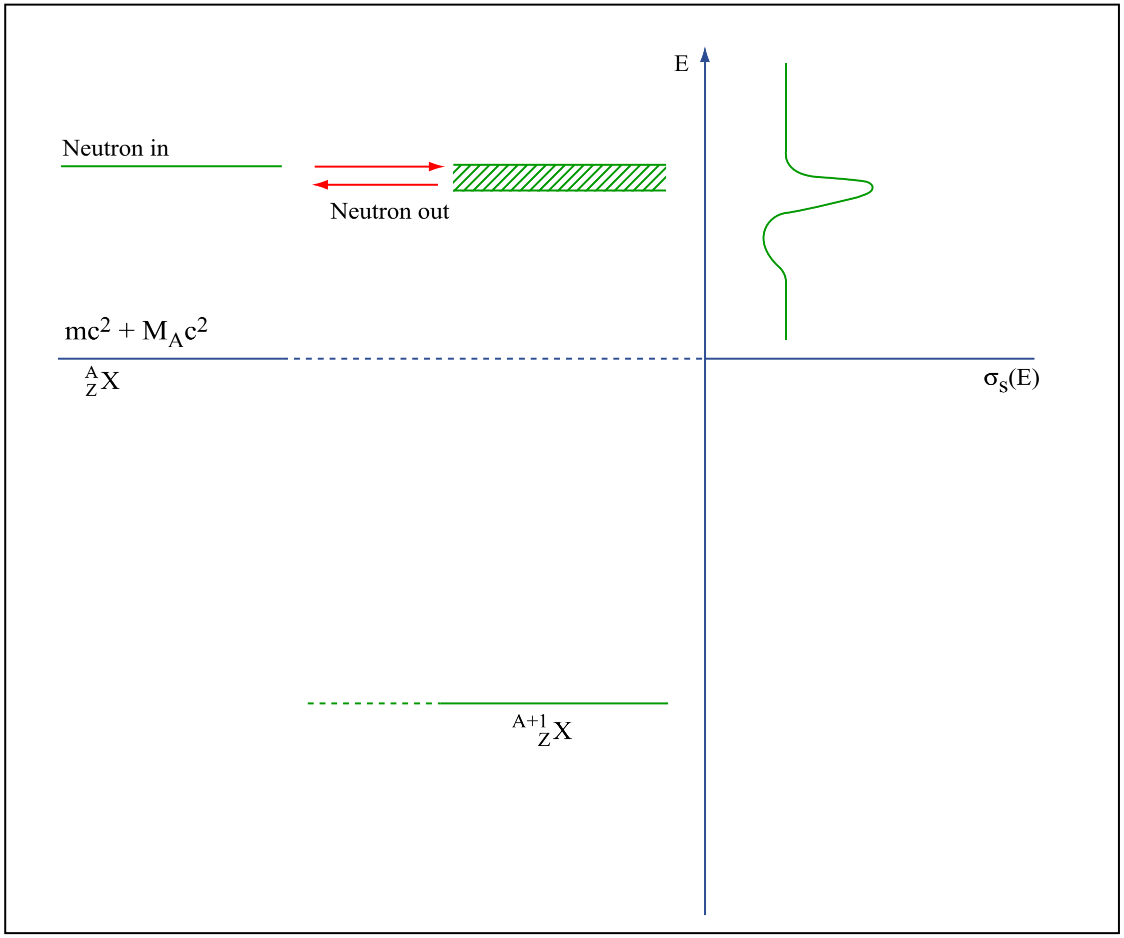
Inelastic scattering usually occurs for neutron energies above 10 keV. The excited state decays by gamma emission.

Resonance Scattering - Elastic



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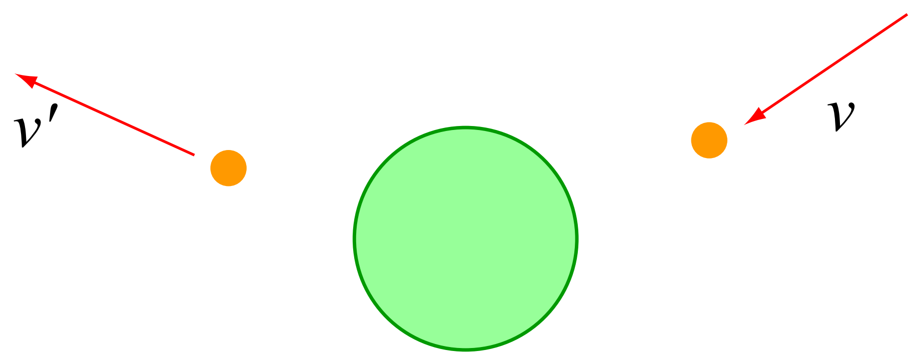
A compound nucleus is formed by the neutron and the nucleus. Peak and valley due to quantum mechanical interference term characterize the cross section. Kinetic energy is conserved.

Double-Differential Cross-Section



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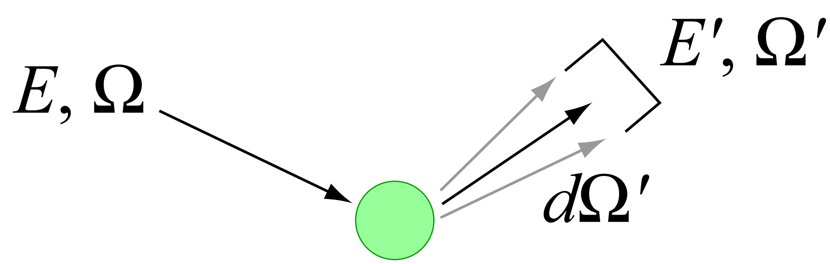
The scattering cross-section will depend on both the energy and angle.

Scattering cross section - Double Differential



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$$\sigma_s\left(E, \hat{\Omega} \rightarrow E', \hat{\Omega}'\right) dE' d\hat{\Omega}' \quad [cm^2]$$

$$\sigma_s\left(E, \hat{\Omega} \rightarrow E', \hat{\Omega}'\right) \qquad [cm^2 / eV \cdot sterradian]$$

This characterizes neutron scattering from an incident energy E and direction Ω to a final energy E' in the interval dE' and Ω' in a solid angle $d\Omega'$.

Neutron Scattering

- Lilley 5.5.2

Slowing Down Decrement

- Lilley 5.5.2

Neutron Moderators

- Lewis 3.3