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MWG

22.38 PROBABILITY AND ITS APPLICATIONS TO RELIABILITY, QUALITY CONTROL AND RISK ASSESSMENT

Fall 2004

NOTE ON STRUCTURE FUNCTIONS

For "and" operation, T = A + B,

$$X_T = \varphi(X) = 1 - (1 - X_A)(1 - X_B).$$

For $\overline{T} = \overline{A} \cup \overline{B}$, complementary event, "or" operator,

$$Y_{\overline{T}} = \varphi^D(Y) = \varphi^D(1 - X_1,\ 1 - X_2, \ldots, 1 - X_n).$$

Note: $\varphi(\underline{X}) + \varphi^D(\underline{1 - X}) = 1$.

Then, $Y_{\overline{T}} = \varphi^D(\underline{Y}) = 1 - \varphi(\underline{1 - Y})$ [from $\varphi^D(X) = 1 - \varphi(\underline{1 - X})$]:

$$Y_{\overline{T}} = 1 - \Big[1 - \underbrace{\big(1 - (1 - Y_{\overline{A}})\big)}_{-Y_A}\underbrace{\big(1 - (1 - Y_{\overline{B}})\big)}_{-Y_B}\Big]$$

$$= 1 - \left[1 + Y_{\overline{A}} Y_{\overline{B}}\right]$$

$$= Y_{\overline{A}} Y_{\overline{B}}$$

Then, $\varphi(X) + \varphi^D(Y) = 1$ or $\left[1 - (1 - X_A)(1 - X_B)\right] + Y_{\overline{A}} Y_{\overline{B}} = 1$.