# Massachusetts Institute of Technology - Physics Department 

Physics - 8.01
Assignment \#7
November 1, 1999.
It is strongly recommended that you read about a subject before it is covered in lectures.

| Lecture Date | Material Covered | Reading |
| :--- | :--- | :--- |
| \#21 Mon 11/1 | Torques - Oscillating Bodies - Hoops | Page 325-334, 394-396 |

\#22 Wed 11/3 Kepler's Laws - Elliptical Orbits Page 218-229
Satellites - Change of Orbits - Ham Sandwich Lecture Supplement of 11/3
See the Home Page
\#23 Fri 11/5 Doppler Effect - Binary Stars Page 446-450
Neutron Stars and Black Holes Take Notes!
\#24 Mon 11/8 Rolling Motion-Gyroscopes Page 335-336, 339-345
Very Non-Intuitive! Lecture Supplement of $11 / 8$
See the Home Page
Due Monday, Nov 8, before 4 PM in 4-339B.
This is not an easy assignment; start early!
7.1 Multiple-Stage Rocket - page 271, problem 55

### 7.2 Slingshot Encounters

Spacecrafts can gain in mechanical energy as they encounter a planet. This may appear as a violation of the conservation of mechanical energy, but it is not. The gained energy is at the expense of the orbital energy of the planet. The easiest way to see how this works in principle is to treat the problem as a one-dimensional collision. Let the spacecraft have a mass $m$ and just before the encounter a velocity $v$, the planet a mass $M$ and velocity $V$. Both velocities are relative to the sun and they are in opposite directions. Thus the angle between $v$ and $V$ is $180^{\circ}$. Assume that the spacecraft rounds the planet and departs in the opposite direction. Thus, after the encounter the velocity of the spacecraft is in the same direction as $V$.
a) What is the speed of the spacecraft after the encounter in terms of $m, M$ and the speed of the spacecraft before the encounter and the speed of the planet before the encounter?
b) The speed of the spacecraft just before the encounter is $10 \mathrm{~km} / \mathrm{sec}$ and the speed of the planet $13 \mathrm{~km} / \mathrm{sec}$ (this is the orbital speed of Jupiter). What then is the speed of the spacecraft just after the encounter?
c) If the spacecraft has a mass of 2000 kg , by how much has its energy increased?
7.3 Figure Skater - page 320, problem 23
7.4 Parallel Axis Theorem - page 320, problem 26 PIVoT
7.5 Pulsars - page 322, problem 41
7.6 Perpendicular Axis Theorem - page 322, problem 45 PIVoT
7.7 Change of Angular Momentum due to a Torque - page 324, problem 59 PIVoT
7.8 Spin Up of Disk due to a Torque - page 348, problem 11 PIVoT
7.9 A Classic! - Translation and Rotation PIVoT - Look under "non-conservation of angular momentum" (you will also see some demonstrations there) and under "conservation of angular momentum". A rod is lying at rest on a perfectly smooth horizontal surface (no friction). We give the rod a short impulse (a hit) perpendicular to the length direction of the rod at $P$. The mass of the rod is 3 kg , its length is 50 cm , the impulse is $4 \mathrm{~kg} \cdot \mathrm{~m} / \mathrm{sec}$, the distance from the center C of the rod to P is 15 cm .
a) What is the translational speed of C after the rod is hit?
b) What is the angular velocity $\omega$ of the rod about C?
c) What is the position of the rod 8 sec after it was hit; how far did C move, and what is the angle between the direction of the rod and its direction before it was hit?
d) What is the total kinetic energy of the rod after it was hit?

### 7.10 The Amazing Yo-Yo! PIVoT

A yo-yo rests on the floor (the static friction coefficient with the floor is $\mu$ ). The inner (shaded) portion of the yo-yo has a radius $R_{1}$, the two outer disks have radii $R_{2}$. A string is wrapped around the inner part. Someone pulls on the string at an angle $\alpha$ (see sketch). The "pull" is very gentle and carefully increased until the yo-yo starts to roll. Try it at Home; it's Fun! You can watch the demo on PIVoT! For what angles of $\alpha$ will the yo-yo roll to the left and for what angles to the right?

7.11 This is a difficult problem - It too is a Classic!

A solid disk with radius $R_{1}$ is spinning about a horizontal axle $l$ at an angular velocity $\omega$ (it rotates freely; friction is ignored). The axle is perpendicular to the disk; it goes through the center $S$ of the disk. The circumference of this disk $(\# 1)$ is pushed against the circumference of another disk which is in all respects identical to $\# 1$ except that its radius is $R_{2}$, and it is at rest. It can rotate freely about a horizontal axle, $m$, through P; $m$ and $l$ are parallel. The friction coefficient between the two touching surfaces (disk circumferences) is $\mu$. We wait until an equilibrium situation is reached. At that time disk $\# 1$ is spinning with angular velocity $\omega_{1}$, and disk $\# 2$ with angular velocity $\omega_{2}$.

a) Is kinetic energy of rotation conserved? Give your reasons.

Now imagine that you are doing this "experiment" and that you hold one axle $m$ in your left hand and axle $l$ in your right hand; you keep them parallel.
b) Do you have to apply a torque while you are pushing the disks against each other?
c) Is the total angular momentum of the two disks conserved?
d) Calculate $\omega_{1}$ and $\omega_{2}$ in terms of $R_{1}, R_{2}$, and $\omega$. It is quite remarkable that $\omega_{1}$ and $\omega_{2}$ are independent of $\mu$ and independent of the time it takes for the equilibrium to be reached; i.e., independent of how hard one pushes the disks against each other.

