Massachusetts Institute of Technology - Physics Department
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Solutions for Assignment # 2

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Problem 2.1

(a) We need to mathematically describe the motion of the first stone. This is one-dimensional motion, in the vertical direction, with a constant acceleration due to the Earth's gravity. Label the vertical direction x with the x-axis pointing up from the surface of Earth and with the origin ($x = 0$) at the surface of Earth. If we set $v_0 = 15.0$ m/s and $g = 9.8$ m/s^2 and choose the time $t = 0$ to coincide with the throwing of the stone, then the motion is

$$x = v_0 t - \frac{1}{2} g t^2$$

Now we can ask, when is the first stone at the height $h = 11.0$ m? This question leads to the equation

$$h = v_0 t - \frac{1}{2} g t^2$$

which is a quadratic equation in t with the *two* solutions

$$t_\pm = \frac{v_0 \pm \sqrt{v_0^2 - 2gh}}{g} \qquad \Longrightarrow \qquad t_- = 1.22 \text{ s and } t_+ = 1.84 \text{ s}$$

If we label the quantities for the second stone x', t', and v_0', then similarly

$$x' = v_0' t' - \frac{1}{2} g t'^2$$

But you must be careful here: t' is the amount of time elapsed since the second stone was thrown; and the second stone is thrown 1.00 s after the first stone is thrown. So there is a delay of $\Delta t = 1.00$ s between the throwing of the stones, and thus we can write the relationship between t and t' as

$$t' = t - \Delta t$$

Now at the times $t = t_\pm$, or $t' = t_\pm - \Delta t$, the two stones are required to hit, implying that $x' = h$. This leads to the equation

$$h = v_0'(t_\pm - \Delta t) - \frac{1}{2}g(t_\pm - \Delta t)^2$$

with the solution

$$v_0' = \frac{h + \frac{1}{2}g(t_\pm - \Delta t)^2}{t_\pm - \Delta t}$$

So for the two possible values $t_\pm$, there are two possible velocities v_0':

$t_- = 1.22$ s with $v_0' = 51.1$ m/s = 114 mph

and

$t_+ = 1.84$ s with $v_0' = 17.2$ m/s = 38.5 mph

The speed of 114 mph would be a very good fast ball and almost as good for a tennis serve (which could be nearly 140 mph), therefore the plausible answer is 17.2 m/s = 38.5 mph.

(b) Now you must wait 1.30 s before throwing the second stone. The equation above, $v_0' = (h + \frac{1}{2}g(t_\pm - \Delta t)^2)/(t_\pm - \Delta t)$ still works, but now $\Delta t = 1.30$ s. The first thing you should notice is that $t_- = 1.22$ s is less than $\Delta t = 1.30$ s. So the first ball reaches the height h, for the first time, before you even throw the second ball. So the only possibility is to strike the first ball at the second time

$t_+ = 1.84$ s with $v_0' = 23.0$ m/s = 51 mph

Problem 2.2

If an object experiences free fall for a length of time t then the distance it falls is given by $d = \frac{1}{2}gt^2$. Thus measuring both t and d provides a value for g:

$$g = \frac{2d}{t^2}$$

If the error for the measurement of d is Δd and the error for the measurement of t is Δt then, by our simple method, the error for g is

$$\frac{2(d + \Delta d)}{(t - \Delta t)^2} - g$$

where the first quantity represents the largest value of g consistent with Δd and Δt.

The data from lectures, the resulting values for g, with the experimental error, and the consistency with the value of $g = 9.80$ m/s^2 for Boston are given below. (Both lectures make the identical measurement for the 3.000 m drop.)

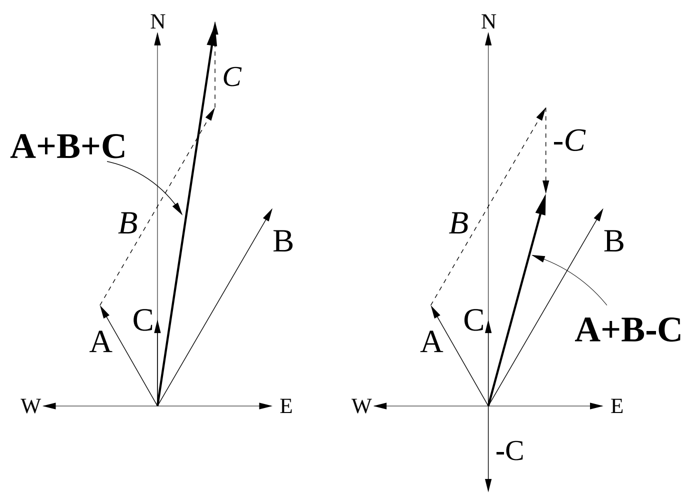
Distance Dropped	Time of Flight	Experimental Value for g	Consistency
$d = 3.000 \pm 0.003$ m	$t = 0.781 \pm 0.002$ s	$g = 9.84 \pm 0.06$ m/s^2	yes
$d = 1.500 \pm 0.003$ m	$t = 0.551 \pm 0.002$ s	$g = 9.88 \pm 0.09$ m/s^2	yes
$d = 1.500 \pm 0.003$ m	$t = 0.550 \pm 0.002$ s	$g = 9.92 \pm 0.09$ m/s^2	no

The last value for g is not consistent with the value of $g = 9.80$ m/s^2. This is most likely due to an under-estimate of the error for d. The way the apple hangs and rotation of the apple as it falls are factors that are hard to account for, so the error of 0.003 m is probably too small.

Problem 2.3

The sum $\vec{A} + \vec{B} + \vec{C}$ is graphically performed by: (1) drawing $\vec{A}$, (2) 'sliding' $\vec{B}$ so that the tail of $\vec{B}$ lies on the tip of $\vec{A}$, (3) 'sliding' $\vec{C}$ so that the tail of $\vec{C}$ lies on the tip of the 'new' $\vec{B}$, and (4) drawing the vector, $\vec{A} + \vec{B} + \vec{C}$, from the tail of $\vec{A}$ to the tip of the 'new' $\vec{C}$.

The vector $-\vec{C}$ is drawn along the direction opposite to $\vec{C}$ with the same length as $\vec{C}$. Then, the sum $\vec{A} + \vec{B} - \vec{C}$ is graphically performed as above but with the vector $-\vec{C}$ used in place of $\vec{C}$.



Problem 2.4

The vector $\vec{A}$ has the components: $A_x = 5.0$, $A_y = -3.0$, and $A_z = 1.0$. The magnitude is given by

$$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2} = \sqrt{(5.0)^2 + (-3.0)^2 + (1.0)^2} = \sqrt{35.0} = 5.9$$

The angle θ_x between $\vec{A}$ and the x-axis can be found from the relation

$$A_x = |A| \cos\theta_x$$

which gives

$$\theta_x = \cos^{-1}\left(\frac{A_x}{|A|}\right) = \cos^{-1}\left(\frac{5.0}{\sqrt{35.0}}\right) = 32.3^\circ$$

Similarly,

$$\theta_y = \cos^{-1}\left(\frac{A_y}{|A|}\right) = \cos^{-1}\left(\frac{-3.0}{\sqrt{35.0}}\right) = 120^\circ$$

and

$$\theta_z = \cos^{-1}\left(\frac{A_z}{|A|}\right) = \cos^{-1}\left(\frac{1.0}{\sqrt{35.0}}\right) = 80.3^\circ$$

Problem 2.5

You are given the vector

$$\vec{v} = 3\hat{x} - 6\hat{y} + 2\hat{z}$$

The magnitude is $|v| = \sqrt{49} = 7$, so the vector

$$\vec{v'} = \frac{1}{|v|}\vec{v} = 3/7\hat{x} - 6/7\hat{y} + 2/7\hat{z}$$

has the same direction as $\vec{v}$ but magnitude $|v'| = 1$. Finally the vector

$$\vec{v''} = 2\vec{v'} = 6/7\hat{x} - 12/7\hat{y} + 4/7\hat{z}$$

has the same direction as $\vec{v'}$, which has the same direction as $\vec{v}$, but now has magnitude $|v''| = 2$.

Problem 2.6

You are given the distances to Venlo (31 km) and Eindhoven (39 km), but you are not given any information concerning the directions to these cities. The greatest distance between the two cities is 31 km + 39 km = 70 km. This occurs when the two cities are in opposite directions. The shortest distance between the two cities is 39 km - 31 km = 8 km. This occurs when the two cities are in the same direction. The actual distance is 47 km, which is greater than 8 km and less than 70 km.

Problem 2.7

You are given the vectors

$$\vec{A} = 2\hat{x} - 3\hat{y} \qquad \text{and} \qquad \vec{B} = -1\hat{x} + a\hat{y} - 5\hat{z}$$

Now you must choose a value of a that makes $\vec{A}$ and $\vec{B}$ perpendicular. Mathematically you must satisfy

$$\begin{array}{rcl} \vec{A}\cdot\vec{B} & = & 0 \\ \Longrightarrow \quad A_xB_x + A_yB_y + A_zB_z & = & 0 \\ \Longrightarrow \quad 2(-1) + (-3)(a) + 0(-5) & = & 0 \\ a & = & -\frac{2}{3} \end{array}$$

Problem 2.8

You are given the vectors

$$\vec{A} = -5\hat{x} - 3\hat{y} + \hat{z} \qquad \text{and} \qquad \vec{B} = 2\hat{x} + 1\hat{y} - 3\hat{z}$$

The calculations for parts (a), (b), and (c) are all done 'component-wise.'

(a)
$$\begin{array}{rcl} \vec{A}+\vec{B} & = & (\ -5+2\ )\hat{x} + (\ -3+1\ )\hat{y} + (\ 1+-3\ )\hat{z} \\ & = & -3\hat{x} - 2\hat{y} - 2\hat{z} \end{array}$$

(b)
$$\begin{array}{rcl} \vec{A}-\vec{B} & = & (\ -5-2\ )\hat{x} + (\ -3-1\ )\hat{y} + (\ 1--3\ )\hat{z} \\ & = & -7\hat{x} - 4\hat{y} + 4\hat{z} \end{array}$$

(c)
$$\begin{array}{rcl} 2\vec{A}-3\vec{B} & = & (\ 2(-5)-3(2)\ )\hat{x} + (\ 2(-3)-3(1)\ )\hat{y} + (\ 2(1)-3(-3)\ )\hat{z} \\ & = & -16\hat{x} - 9\hat{y} + 11\hat{z} \end{array}$$

(d)
$$\begin{array}{rcl} \vec{A}\cdot\vec{B} & = & (-5)(2) + (-3)(1) + (1)(-3) \\ & = & -16 \end{array}$$

$$\begin{array}{rcl} \vec{B}\cdot\vec{A} & = & (2)(-5) + (1)(-3) + (-3)(1) \\ & = & -16 \end{array}$$

In fact it is true that $\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ for any two vectors.

(e)
$$\begin{array}{rcl} \vec{A}\times\vec{B} & = & (\ (-3)(-3)-(1)(1)\ )\hat{x} + (\ (1)(2)-(-5)(-3)\ )\hat{y} + (\ (-5)(1)-(-3)(2)\ )\hat{z} \\ & = & 8\hat{x} - 13\hat{y} + 1\hat{z} \end{array}$$

$$\begin{array}{rcl} \vec{B}\times\vec{A} & = & (\ (1)(1)-(-3)(-3)\ )\hat{x} + (\ (-3)(-5)-(2)(1)\ )\hat{y} + (\ (2)(-3)-(1)(-5)\ )\hat{z} \\ & = & -8\hat{x} + 13\hat{y} - 1\hat{z} \\ & = & -(8\hat{x} - 13\hat{y} + 1\hat{z}) \end{array}$$

In fact it is true that $\vec{A}\times\vec{B} = -\vec{B}\times\vec{A}$ for any two vectors.

Problem 2.9

We are given the vectors

$$\vec{A} = 2\hat{x} - 3\hat{y} \qquad \text{and} \qquad \vec{B} = -\hat{x} + 4\hat{y} - 5\hat{z}$$

and we want to find a vector

$$\vec{V} = a\hat{x} + b\hat{y} + c\hat{z}$$

that has unit length ($|V| = 1$) and is perpendicular to both $\vec{A}$ and $\vec{B}$ ($\vec{V} \cdot \vec{A} = 0$ and $\vec{V} \cdot \vec{B} = 0$)

$$\begin{array}{rcrcl} & \vec{V} \cdot \vec{A} & = & 0 & \\ \Longrightarrow & 2a - 3b & = & 0 & \\ \Longrightarrow & a & = & \frac{3}{2}b & \\ & & & & \\ & \vec{V} \cdot \vec{B} & = & 0 & \\ \Longrightarrow & -a + 4b - 5c & = & 0 & \\ \Longrightarrow & -\frac{3}{2}b + 4b - 5c & = & 0 & (\text{ using } a = \frac{3}{2}b\ ) \\ \Longrightarrow & b & = & 2c & \\ \Longrightarrow & a & = & 3c & (\text{ using } a = \frac{3}{2}b\ ) \end{array}$$

So now we know that $\vec{V}$ has the form

$$\vec{V} = 3c\hat{x} + 2c\hat{y} + c\hat{z}$$

Now imposing the condition that $\vec{V}$ is normalized to unity gives the equation

$$\begin{array}{rcrc} & |\vec{V}| & = & 1 \\ \Longrightarrow & 9c^2 + 4c^2 + c^2 & = & 1 \\ \Longrightarrow & 14c^2 & = & 1 \\ \Longrightarrow & c & = & \pm 1/\sqrt{14} \end{array}$$

So this finally gives just two vectors

$$\vec{V} = \pm\frac{1}{\sqrt{14}}(3\hat{x} + 2\hat{y} + \hat{z})$$

Alternatively, we could form $\vec{A} \times \vec{B}$ which is perpendicular to both $\vec{A}$ and $\vec{B}$. Then we could normalize this new vector and include the two possible directions; this would give $\vec{V} = \pm\frac{1}{|\vec{A}\times\vec{B}|}\vec{A} \times \vec{B}$. You can check that this expression is identical to the result above.

Problem 2.10

The position as a function of time is given as

$$\vec{r} = (6 - 2t)\hat{x} + (3 + 4t - 6t^2)\hat{y} - (1 + 3t - 2t^2)\hat{z}$$

(a) The velocity vector at any time t is given by

$$\begin{aligned}
\vec{v} &= \frac{d\vec{r}}{dt} \\
&= \frac{d}{dt}(6 - 2t)\hat{x} + \frac{d}{dt}(3 + 4t - 6t^2)\hat{y} - \frac{d}{dt}(1 + 3t - 2t^2)\hat{x} \\
&= (-2)\hat{x} + (4 - 6(2)t)\hat{y} - (3 - 2(2)t)\hat{z} \\
&= -2\hat{x} + (4 - 12t)\hat{y} - (3 - 4t)\hat{z}
\end{aligned}$$

So in particular at $t = 3$,

$$\begin{aligned}
\vec{v} &= -2\hat{x} + (4 - 12(3))\hat{y} - (3 - 4(3))\hat{z} \\
&= -2\hat{x} - 32\hat{y} + 9\hat{z}
\end{aligned}$$

(b) The speed at $t = 3$ is given by the magnitude of $\vec{v}$ at $t = 3$ which is

$$|v| = \sqrt{(-2)^2 + (-32)^2 + (-9)^2} = \sqrt{1109} = 33.3$$

So the speed is 33.3 m/s.

(c) The acceleration vector at any time t is given by

$$\begin{aligned}
\vec{a} &= \frac{d\vec{v}}{dt} \\
&= \frac{d}{dt}(-2)\hat{x} + \frac{d}{dt}(4 - 12t)\hat{y} - \frac{d}{dt}(3 - 4t)\hat{x} \\
&= (0)\hat{x} + (-12)\hat{y} - (-4)\hat{z} \\
&= -12\hat{y} + 4\hat{z}
\end{aligned}$$

So in particular at $t = 3$

$$\vec{a} = -12\hat{y} + 4\hat{z}$$

The magnitude of $\vec{a}$ at $t = 3$ is

$$|a| = \sqrt{(0)^2 + (-12)^2 + (4)^2} = \sqrt{160} = 12.6$$

So the magnitude of acceleration at $t = 3$ is 12.6 m/s^2.

Problem 2.11

Choose the z-axis pointing vertically up; the x-axis pointing horizontally along the direction of the car; and choose the x-z origin to be the corner of the ramp. Also choose the t origin to be the moment when the car is at the corner of the ramp, and let v_{0x} be the unknown speed of the car at that moment. The subsequent motion of the car is projectile motion.

The vertical motion of the car is given by

$$z = -\frac{1}{2}gt^2$$

So we can ask the question when does the car fall the distance $h = 2.0$ m? This gives the equation

$$-h = -\frac{1}{2}gt^2$$

with solution

$$t^\star = \sqrt{\frac{2h}{g}} = \sqrt{\frac{2(2.0\text{m})}{(9.8\text{m/s}^2)}} = 0.64 \text{ s}$$

The horizontal motion of the car is given by

$$x = v_{0x}t$$

so at $t = t^\star$ the horizontal position is

$$x^\star = v_{0x}t^\star$$

For the stunt driver to avoid crashing, $x^\star$ must be larger than the distance $d = 24.0$ m. So

$$\begin{aligned} & x^\star && > && d \\ \Longrightarrow\; & v_{0x}t^\star && > && d \\ \Longrightarrow\; & v_{0x} && > && \tfrac{d}{t^\star} \\ \Longrightarrow\; & v_{0x} && > && \frac{d}{\sqrt{\frac{2h}{g}}} \qquad (\text{ using } t^\star = \sqrt{\tfrac{2h}{g}}\ ) \end{aligned}$$

Therefore, to clear all the cars

$$\begin{aligned} v_{0x} &> \frac{24.0}{\sqrt{\frac{2*2.0}{9.8}}} \\ &> 37.6 \text{ m/s} \end{aligned}$$

Problem 2.12

We will need the same equation three times, so consider the case of arbitrary ball speed, v_0, arbitrary angle, θ, and arbitrary range d. The motion will be projectile. The usual quantities v_{0x} and v_{0z} are given by

$$v_{0x} = v_0 \cos\theta \qquad \text{and} \qquad v_{0z} = v_0 \sin\theta$$

It is simplest to choose a coordinate system with the origin on the ball and the usual labels for vertical and horizontal. The vertical motion is

$$z = v_0 \sin(\theta)t - \frac{1}{2}gt^2$$

We want to know the time $t^\star$ when the ball strikes the ground. This gives the equation

$$0 = v_0 \sin(\theta)t^\star - \frac{1}{2}gt^{\star 2}$$

which has the solutions

$$t^\star = 0 \qquad \text{and} \qquad t^\star = 2v_0 \sin(\theta)/g$$

of which the non-zero solution corresponds to the ball striking the ground.

The horizontal motion is given by

$$x = v_0 \cos(\theta)t$$

So the range is given by

$$d = v_0 \cos(\theta)t^\star = \frac{2v_0^2 \cos(\theta)\sin(\theta)}{g}$$

Solving that equation for v_0 gives

$$v_0 = \sqrt{\frac{gd}{2\cos(\theta)\sin(\theta)}}$$

(a) Now we want to determine v_0 when $\theta = 14°$ and $d = 240$ m.

$$v_0 = \sqrt{\frac{240(9.8)}{2\cos(14)\sin(14)}} = 70.8 \text{ m/s}$$

(b) Suppose the ball speed was greater by 0.6 m/s. We want to determine d when $\theta = 14°$ and $v_0 = 70.8$ m/s $+0.6$ m/s $= 71.4$ m/s.

$$d = \frac{2(71.4)^2 \cos(14) \sin(14)}{9.8} = 244.1 \text{ m}$$

The ball will travel 244.1 m -240.0 m $= 4.1$ m farther.

(c) Suppose the angle was greater by $0.5°$. We want to determine d when $\theta = 14° + 0.5° = 14.5°$ and $v_0 = 70.8$ m/s.

$$d = \frac{2(70.8)^2 \cos(14.5) \sin(14.5)}{9.8} = 248.0 \text{ m}$$

The ball will travel by 248.0 m -240.0 m $= 8.0$ m farther.

Problem 2.13

The motion of the skier is identical to the motion of the car in problem 2.11, if you choose a similar coordinate system, with $v_0 = 110$ km/h $= \frac{110 \times 10^3 \text{m}}{(60)(60)\text{s}} = 30.6$ m/s. So the motion is given by

$$x = v_0 t \qquad \text{and} \qquad z = -\frac{1}{2} g t^2$$

(a) The equations above descibe the motion of the skier, but the equations do not know where the ground is. We must supply that information. Normally we choose the coordinates so that the ground corresponds to $z = 0$, but that is not true in this case. The gound is really a hill. It starts at $x = 0$ and $z = 0$, but then it slopes down at an angle of $45°$, i.e. it is a line through the origin with slope -1. The mathematical description of the line is

$$z_{ground} = -x_{ground}$$

The skier landing on the slope is mathematically indicated by the intersection of the curve of the skier and the curve of the ground. Suppose this intersection occurs at the horizontal position x; because this is a point on the ground we know that the correspondingly $z = -x$ So if the skier lands at time $t = t^\star$, then

$$\begin{aligned} x &= -z \\ x &= v_0 t^\star \\ z &= -\tfrac{1}{2} g t^{\star 2} \\ \Longrightarrow \quad v_0 t^\star &= \tfrac{1}{2} g t^{\star 2} \\ \Longrightarrow \quad t^\star &= 0 \qquad \text{or} \\ t^\star &= \tfrac{2 v_0}{g} \end{aligned}$$

of which the non-zero solution is the one that corresponds to the skier landing. At that time,

$$x = v_0 t^\star = \frac{2v_0^2}{g} = \frac{2(30.6\text{m/s})^2}{9.8} = 191\text{m}$$

$$\text{and} \qquad z = -x = -191\text{m/s}$$

So the distance down the slope is

$$d = \sqrt{x^2 + z^2} = 270\text{m}$$

(b) The skier attains large speeds which make considerations of air resistance necessary. Air resistance makes the actual distance shorter than our calculated distance. This will be explored in more detail in lecture.

Problem 2.14

If the radius of the orbit is $R = 1.50 \times 10^{11}$ m, then the circumference of the orbit is

$$C = 2\pi R = (2)(3.14)(1.50 \times 10^{11}\text{m}) = 9.42 \times 10^{11}\text{m}$$

The Earth travels this distance once in the time $\tau = 1$ year $= (365)(24)(60)(60)$ s $= 3.15 \times 10^7$ s. So it's speed is

$$v = \frac{C}{\tau} = \frac{2\pi R}{\tau} = \frac{9.42 \times 10^{11}\text{m}}{3.15 \times 10^7\text{s}} = 2.99 \times 10^4\text{m/s}$$

So the centripetal acceleration is given by

$$a = \frac{v^2}{R} = \frac{\left(\frac{2\pi R}{\tau}\right)^2}{R} = \frac{4\pi^2 R}{\tau^2} = \frac{(2.99 \times 10^4\text{m/s})^2}{1.50 \times 10^{11}\text{m}} = 5.97 \times 10^{-3}\text{m/s}^2$$

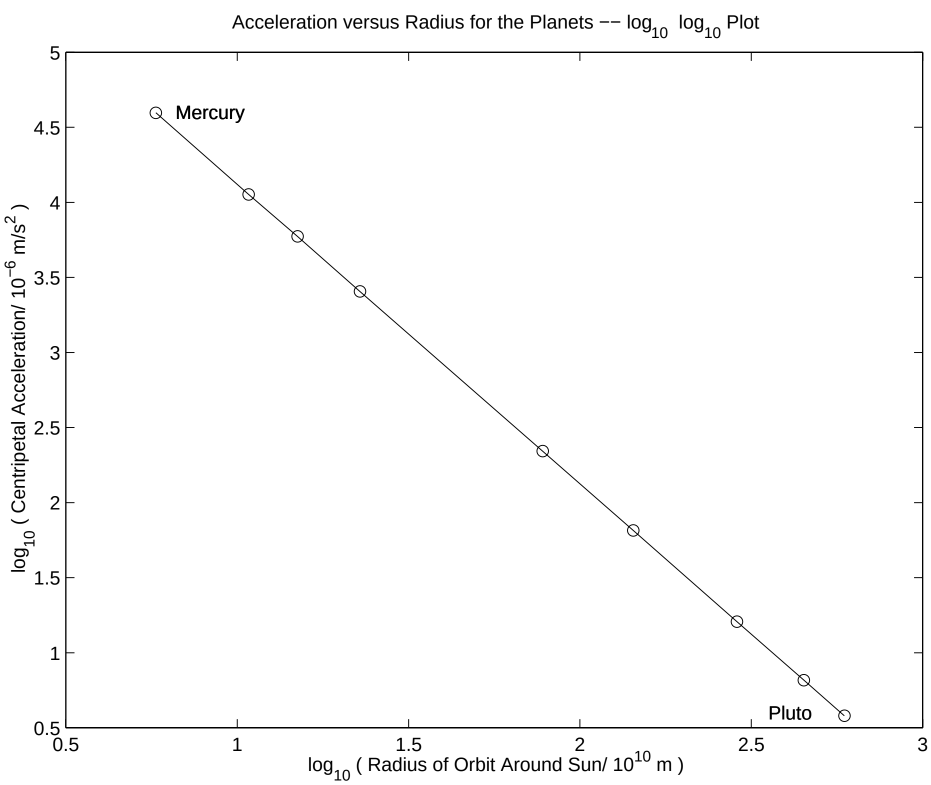
Problem 2.15

In problem 2.14 we actually calculated the centripetal acceleration in terms of the radius of the orbit and the period of the orbit. That result, $a = 4\pi^2 R/\tau^2$, is true for all the planets. This result does assume that all planets have circular orbits, but most orbits are elliptical, so the radius we use is the average distance of the planet to the sun. This was referred to as $\overline{R}$ in the lecture supplement for 9/17/99. Elliptical orbits will be discussed later in the course.

(a) Here is a table of information found by asking "what is the distance to out planets?" on the web site http://ask.com and by using the formula above for a.

Planets	Mean Radius (m)	Period (s)	Centripetal Acceleration (m/s^2)
Mercury	5.79×10^{10}	7.60×10^{6}	3.96×10^{-2}
Venus	1.08×10^{11}	1.94×10^{7}	1.13×10^{-2}
Earth	1.50×10^{11}	3.16×10^{7}	5.93×10^{-3}
Mars	2.28×10^{11}	5.94×10^{7}	2.55×10^{-3}
Jupiter	7.78×10^{11}	3.74×10^{8}	2.20×10^{-4}
Saturn	1.43×10^{12}	9.29×10^{8}	6.54×10^{-5}
Uranus	2.87×10^{12}	2.65×10^{9}	1.61×10^{-5}
Neptune	4.50×10^{12}	5.20×10^{9}	6.57×10^{-6}
Pluto	5.91×10^{12}	7.84×10^{9}	3.80×10^{-6}

(b) Below is a log-log plot of the centripetal acceleration versus the orbit radius. The dots show the data points, and the lines connect consecutive points. The axes of your plot may be different depending on the units chosen, but the slope of the curve should be the same. (I expressed the radii in units of 10^{10} m and the acceleration in units of 10^{-6} m/s^2.)



Later you'll learn how to determine the best curve for a set of data points, but these points seem to lie on such a straight line that we will just assume that the relationship is linear. Now we will use the slope between any two points to make a guess about the true slope. For my guess, I used the points for Mercury and Pluto: slope $\approx \frac{\log 39600 - \log 3.80}{\log 5.79 - \log 591.00} \approx -2.00$.

You should notice that all the planets, independent of their mass, lie on this curve. This indicates a relationship between radius and acceleration: $\log a = -2.00 \log R + C$ which implies that $a = K/R^2$, where C and K are constants independent of the mass of the planet. (As was shown in lecture.) Notice that the expression from problem 2.14 related a, τ, and R, but this expression is independent of τ indicating that there must be additional relationships between τ and R.

Here are a few lines of code that will produce graphs similar to the one above. First you need to find Mathlab. Log onto an Athena workstation and either (1) choose the 'Numerical/Math' option from the top, then choose 'Analysis and Plotting' and 'MATLAB' or (2) at the command line type 'add matlab; matlab'. Then at the prompt, which should look like '>>', type the following lines.

```
radii = [5.79, 10.80, 15.00, 22.80, 77.80, 143.00, 287.00, 450.00, 591.00]
acc = [39600, 11300, 5930, 2550, 220, 65.4, 16.1, 6.57, 3.80]
plot( log10(radii), log10(acc), 'o', log10(radii), log10(acc), '-' )
```

Problem 2.16

Let w be the speed of the wind blowing from A to B; v be the speed of the plane measured relative to the air; and d be the distance from A to B.

For the trip from A to B, the ground speed (speed of the plane measured relative to the ground) would be

$$v + w$$

and the time it would take to travel that distance would be

$$t_{A\to B} = \frac{d}{v+w}$$

For the trip from B to A, the ground speed would be

$$v - w$$

and the time of flight would be

$$t_{B\to A} = \frac{d}{v-w}$$

At this point you should notice that we could make $t_{B\to A}$ be very large by making $w \approx v$, which already indicates that the round trip with wind will be longer.

The total time of flight would be

$$t_w = t_{A\to B} + t_{B\to A} = d\left(\frac{1}{v+w} + \frac{1}{v-w}\right) = \frac{2d}{v}\left(\frac{1}{1 - w^2/v^2}\right)$$

The travel time without the wind ($w = 0$) is

$$t_0 = 2d/v$$

so we can write t_w as

$$t_w = t_0 \left(\frac{1}{1 - w^2/v^2} \right)$$

For $|w| < v$,

$$\left(\frac{1}{1 - w^2/v^2} \right) > 1$$

so $t_w > t_0$, and the round trip with wind is longer. But for $|w| > v$

$$\left(\frac{1}{1 - w^2/v^2} \right) < 0$$

which would make $t_w < 0$! So clearly we should be careful here.

The special points $w = \pm v$ cause problems: the above expression for t_w becomes infinite. In fact for $w = v$ the ground speed for the B to A portion is $v - w = 0$, indicating that the plane will never be able to leave point B. This means that someone standing on the ground at B will see the plane floating still in the air. (Where as a bird in the wind will see the plane as moving forward.) So all considerations of the trip from B to A are irrelevant: the plane never makes that portion of the trip. This is revealed in the expression for $t_{B \to A}$ which becomes infinite for $w = v$. So the analysis above breaks down: we can not talk about the round trip because it doesn't occur. In fact for $w > v$ the ground speed is $v - w < 0$, and the plane continues to be blown further in the A to B direction.

If the wind were to blow from B to A instead, the above results would work by considering $w < 0$. And when $w = -v$ (when the wind blows from B to A with speed v), the ground speed from A to B would be $v + w = 0$. The plane would never even begin its trip since it couldn't leave A. And for $w < -v$ the plane immediately is blown further away from B. So for this case the plane never even completes the first half of the round trip.