Massachusetts Institute of Technology - Physics Department
Physics-8.01 Fall 1999

Solutions for Assignment # 9

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Problem 9.1

(a) John does no work on the computer monitor. John does exert a force on the computer monitor, but the monitor does not move; hence there is no work done by John on the monitor. (To understand why John becomes exhausted if he does no work, please see the last paragraph on page 162.)

(b) When the fly flies in the jar with constant speed the net force on the fly must be zero. Thus the air pushes upwards on the fly, and the upward force is the weight of the fly. According to Newton's third law, the fly must therefore push on the air with the same force. Since the air is enclosed in the jar, this downward force is felt by the scale. Thus the scale reading remains the same.

There is a transient period while the fly takes off (it pushes against the bottom of the jar) as it is being accelerated. During that period the scale will read a higher value than before.

(c) When the jar is open, the situation is very different. Imagine that the fly is sitting on a tray, and the tray is on the scale, and then it starts to fly. Clearly the scale will read a lower value as the fly is "gone".

In an open jar, with the fly flying in the jar, the scale will read a lower value. The fly is still pushing down on the air with a force equal to its own weight, but now this force is not 100% transferred to the bottom of the jar. There is "leakage" to the outside world (outside the jar).

(d) We can fire the rocket with such an impulse as to bring our speed to zero. Then earth's gravity will bring us back without any further effort. If our speed in the orbit is v then the impulse required to come to a temporary stop is $I = mv$; thus a smaller orbital speed requires a smaller impulse which requires less fuel. Our speed is largest when we are closest to earth, and our speed is smallest when we are farthest from earth; therefore we should fire our rockets when we are farthest from earth.

(e) Let M_R be the mass of the rock, M_B be the mass of the boat and anything else in it, ρ_W be the density of water, and ρ_R be the density of the rock. With the rock in the boat, the amount of water displaced is

$$V_1 = \frac{M_B + M_R}{\rho_W} = \frac{M_B}{\rho_W} + \frac{M_R}{\rho_W}$$

With the rock at the bottom of the pool, the amount of water displaced is

$$V_2 = \frac{M_B}{\rho_W} + \frac{M_R}{\rho_R}$$

Since the rock sinks, we know that $\rho_R > \rho_W$; hence $V_1 > V_2$ and the water level will go down.

(f) Let M be the mass of ice and ρ_W be the density of water. With the ice initially floating, the amount of water displaced is

$$V_1 = \frac{M}{\rho_W}$$

The ice melts, and the corresponding volume of water is

$$V_2 = \frac{M}{\rho_W}$$

Thus $V_1 = V_2$ and the water level will remain the same.

Problem 9.2 (Ohanian, page 372, problem 16)

Let $m = 80$ kg, $l = 0.6$ m, and $h = 1.2$ m. We only need to concern ourselves with a cross-section of the box as shown in Figure 14.34 on page 372.

(a) Now imagine a line connecting the edge of the box which remains in contact with the floor with the center of mass. The angle between this line and the bottom of the box is given by

$$\tan\phi = \frac{\frac{h}{2}}{\frac{l}{2}} = \frac{h}{l} \qquad \Longrightarrow \qquad \phi = \tan^{-1}\left(\frac{h}{l}\right) \approx 63.4°$$

The angle of this line relative to the floor is $\theta + \phi$. The length of this line is given by

$$L = \sqrt{\left(\frac{l}{2}\right)^2 + \left(\frac{h}{2}\right)^2} \approx 0.671 \text{ m}$$

Therefore the height of the center of mass is given by

$$H = L\sin(\theta + \phi) \approx 0.671 \cdot \sin(\theta + 63.4°)$$

Therefore the potential energy is given by

$$U = mgH \approx 5.26 \times 10^2 \cdot \sin(\theta + 63.4°)$$

where the units of U are J and θ must be measured in degrees. The height of the center of mass is measured relative to the ground. When $\theta = 0$ the center of mass is at a height $\frac{h}{2}$ and $U \neq 0$.

(b) The critical angle θ_C occurs when

$$\theta_C + \phi = 90^\circ \qquad \Longrightarrow \qquad \theta_C = 26.6^\circ$$

(U is an increasing function of θ for $\theta < \theta_C$ and a decreasing function of θ for $\theta > \theta_C$.)

(c) For a conservative force the work is the change in energy.

$$W = U(\theta_C) - U(0) = 5.26 \times 10^2 \cdot \sin(90^\circ) - 5.26 \times 10^2 \cdot \sin(63.4^\circ) \approx 55.7 \text{ J}$$

Problem 9.3 (Ohanian, page 372, problem 18)

Let w be the width of each book. We will use the scheme illustrated by Figure 14.36 on page 372. You first place the top book. Then you place the next book so that its right most edge is directly under the center of mass of the first book. Then you continue adding books at the bottom in this fashion: place each book so that its right most edge is directly under the center of mass for all the books above. We can explicitly work out the first five books in the sequence; the pattern will be clear. Let x_i denote the right most edge of the i^{th} book, and choose $x_1 = 0$. Then the shift between each book is $\Delta_i = x_{i+1} - x_i$ and the center of the i^{th} book is given by $x_i + \frac{w}{2}$. Then

$$x_1 = 0$$

$$x_2 = \frac{1}{1}\left(x_1 + \frac{w}{2}\right) = \frac{1}{2}w \qquad \Longrightarrow \qquad \Delta_2 = x_2 - x_1 = \frac{1}{2}w$$

$$x_3 = \frac{1}{2}\left(x_1 + \frac{w}{2} + x_2 + \frac{w}{2}\right) = \frac{3}{4}w \qquad \Longrightarrow \qquad \Delta_2 = x_3 - x_2 = \frac{1}{4}w$$

$$x_4 = \frac{1}{3}\left(x_1 + \frac{w}{2} + x_2 + \frac{w}{2} + x_3 + \frac{w}{2}\right) = \frac{11}{12}w \qquad \Longrightarrow \qquad \Delta_3 = x_4 - x_3 = \frac{1}{6}w$$

$$x_5 = \frac{1}{4}\left(x_1 + \frac{w}{2} + x_2 + \frac{w}{2} + x_3 + \frac{w}{2} + x_4 + \frac{w}{2}\right) = \frac{25}{24}w \qquad \Longrightarrow \qquad \Delta_4 = x_5 - x_4 = \frac{1}{8}w$$

The total protrusion is

$$x_5 - x_1 = x_5 - x_4 + x_4 - x_3 + x_3 - x_2 + x_2 - x_1 = \Delta_4 + \Delta_3 + \Delta_2 + \Delta_1 \approx 1.04 \cdot w$$

¿From the above it seems clear that the general rule is

$$\Delta_i = \frac{1}{2i} w = \frac{1}{i} \cdot \frac{w}{2}$$

For an infinite number of books the total protrusion is

$$\sum_{i=1}^{i=\infty} \Delta_i = \sum_{i=1}^{i=\infty} \frac{1}{i} \cdot \frac{w}{2} = \frac{w}{2} \cdot \sum_{i=1}^{i=\infty} \frac{1}{i}$$

The second piece above is the sum of the harmonic series, $\frac{1}{i}$,

$$\sum_{i=1}^{i=\infty} \frac{1}{i} \to \infty$$

which is known to diverge; therefore the maximum protrusion for an infinite number of books is an infinite distance.

Problem 9.4 (Ohanian, page 377, problem 50)

From Table 14.1 on page 366 we know that the Young's modulus for bone is $Y = 3.2 \times 10^{10}$ N/m^2. From Equation (27) on page 366 we know that

$$\frac{\Delta L}{L} = \frac{1}{Y} \frac{F}{A}$$

We know that $L = 38$ cm $= 38 \times 10^{-2}$ m, $A = 10$ cm$^2 = 10 \times 10^{-4}$ m^2, and $F = \frac{1}{2}\, 68 \cdot g$. (Each leg supports $\frac{1}{2}$ of the total weight.) This gives

$$\Delta L = \frac{1}{Y} \frac{F}{A} \cdot L \approx 4.0 \times 10^{-6} \text{ m}$$

Problem 9.5 (Ohanian, page 408, problem 31)

First we will calculate the initial moment of inertia I_1 and the initial angular frequency of the torsional pendulum ω_1. We will make the approximation that the balance wheel is a thin hoop. The moment of inertia for a thin hoop is given in Table 12.1 on page 309 as

$$I_1 = MR^2 = 1.5000 \times 10^{-8} \text{ kg} \cdot \text{m}^2$$

where $M = 0.6$ g $= 0.6 \times 10^{-3}$ kg and $R = \frac{1}{2} \cdot 1.0$ cm $= 5.0 \times 10^{-3}$ m. The period of the torsional pendulum is *supposed* to coincide with one second, but the watch makes $1.2 \cdot 60 = 72$ more periods per day than it should. The frequency is then given by

$$\nu_1 = \frac{24 \cdot 60 \cdot 60 + 72}{24 \cdot 60 \cdot 60} \approx 1 + 8.3333 \times 10^{-4} \text{ Hz}$$

and

$$\omega_1 = 2\pi\nu_1 = 2\pi + 5.2359 \times 10^{-3} \text{ radian/s}$$

We can use I_1 and ω_1 to calculate κ from Equation (75) on page 397.

$$\kappa = I_1\omega_1^2 \approx 5.9316 \times 10^{-7} \text{ m} \cdot \text{N/radian}$$

We want to adjust a screw so that $\nu_2 = 1$ Hz and hence $\omega_2 = 2\pi$ radian/s. The required moment of inertia is

$$I_2 = \frac{\kappa}{\omega_2^2} \approx 1.5025 \times 10^{-8} \text{ kg} \cdot \text{m}^2$$

We must therefore increase the moment of inertia by

$$\Delta I = I_2 - I_1 \approx 2.5 \times 10^{-11} \text{ kg} \cdot \text{m}^2$$

Now let I_{hoop} be the moment of inertia of the wheel without the screw. Then

$$I_1 = I_{\text{hoop}} + mr_1^2 \quad \text{and} \quad I_2 = I_{\text{hoop}} + mr_2^2$$

and

$$\Delta I = m(r_2^2 - r_1^2)$$

where $m = 0.020$ g $= 2.0 \times 10^{-5}$ kg is the mass of one screw and r_1 and r_2 are the two radii of the screw. We will make the approximation that $r_1 \approx \frac{1}{2} \cdot 1.0$ cm $= 5.0 \times 10^{-3}$ m and then calculate r_2.

$$r_2 = \sqrt{r_1^2 + \frac{\Delta I}{m}} \approx 5.12 \times 10^{-3} \text{ m}$$

Therefore we need to move the screw out a distance

$$\Delta r = r_2 - r_1 \approx 1.2 \times 10^{-4} \text{ m} = 1.2 \times 10^{-2} \text{ cm}$$

Problem 9.6 (Ohanian, page 488, problem 17)

(a) We can use Bernoulli's equation to solve this problem.

$$\frac{1}{2}\rho v^2 + \rho g z + p = \text{constant}$$

where $\rho = 1055$ kg/ m^3. The flow of blood is slow enough that we can consider $v = 0$ throughout the body. The above equation becomes

$$\rho g z + p = \text{constant}$$

Now define the origin of z to coincide with the heart, and let $p_0 = 110$ mmHg $= 1.46 \times 10^4$ Pa. Then the constant is given by

$$p_0 = \text{constant}$$

and

$$\rho g z + p = p_0 \qquad \Longrightarrow \qquad p = p_0 - \rho g z$$

for other points in the body. For the feet, $z = -140$ cm $= -1.4$ m, and

$$p_{\text{feet}} = 1.46 \times 10^4 - 1055 \cdot 9.8 \cdot -1.4 \approx 2.91 \times 10^4 \text{ Pa} \approx 221 \text{ mmHg}$$

where $g = 9.8$ m/s^2. For the brain, $z = 40$ cm $= 0.4$ m, and

$$p_{\text{brain}} = p_0 - 1055 \cdot 0.4 \cdot g = 1.46 \times 10^4 - 1055 \cdot 0.4 \cdot 9.8 \approx 1.05 \times 10^4 \text{ Pa} \approx 79.8 \text{ mmHg}$$

where $g = 9.8$ m/s^2.

(b) The pressure in the brain is still given by the expression above

$$p_{\text{brain}} = p_0 - 1055 \cdot 0.4 \cdot g$$

Now $g = 61$ m/s^2, therefore

$$p_{\text{brain}} = p_0 - 1055 \cdot 0.4 \cdot 61 = p_0 - 2.57 \times 10^4 \text{ Pa} = p_0 - 196 \text{ mmHg}$$

The heart could at most create a pressure $p_0 = 190$ mmHg, so the pressure in the brain will be negative.

Problem 9.7 (Ohanian, page 489, problem 18)

The pressure at the surface of the liquid is $p = 1$ atm $= 1.013 \times 10^5$ Pa. This pressure must be able to support the mass of the column of water raised in the tube. Let A be the cross-section of the tube and $\rho = 1.0 \times 10^3$ kg be the density of water. Then

$$pA = F = Mg = \rho A h g \qquad \Longrightarrow \qquad h = \frac{p}{\rho g} \approx 10.3 \text{ m}$$

Problem 9.8 (Ohanian, page 489, problem 24)

Let $\rho_I = 920\ \mathrm{kg/m^3}$ is the density of ice and $\rho_W = 1025\ \mathrm{kg/m^3}$ is the density of sea water. Also let M be the mass of the ice. The volume of the ice is

$$V_I = \frac{M}{\rho_I}$$

The volume of water displaced by the ice is

$$V_W = \frac{M}{\rho_W}$$

The volume above the water is

$$\Delta V = V_I - V_W = M\left(\frac{1}{\rho_I} - \frac{1}{\rho_W}\right) = 30 \cdot 400 \cdot 400\ \mathrm{m^3} = 4.8 \times 10^6\ \mathrm{m^3}$$

Therefore the total mass of the ice is

$$M = \frac{\Delta V}{\left(\frac{1}{\rho_I} - \frac{1}{\rho_W}\right)} = \frac{4.8 \times 10^6\ \mathrm{m^3}}{\left(\frac{1}{920} - \frac{1}{1025}\right)} \approx 4.31 \times 10^{10}\ \mathrm{kg}$$

and the total volume of the ice is

$$V_I = \frac{M}{\rho_I} = \frac{4.31 \times 10^{10}}{920} \approx 4.68 \times 10^7\ \mathrm{m^3}$$

(b) The total mass of the ice is

$$M \approx 4.31 \times 10^{10}\ \mathrm{kg}$$

Problem 9.9 (Ohanian, page 491, problem 35)

The block will sink until it displaces an equal mass of oil ($\rho_O = 8.5 \times 10^2\ \mathrm{kg/m^3}$) and water ($\rho_W = 1.0 \times 10^3\ \mathrm{kg/m^3}$). (If the block is too heavy then it will sink to the bottom.) Let $H = 10\ \mathrm{cm} = 10^{-1}\ \mathrm{m}$ be the height of the box; $A = 30\ \mathrm{cm} \times 20\ \mathrm{cm} = 6 \times 10^{-2}\ \mathrm{m^2}$ be the area of the bottom of the box; and $M = 5.5\ \mathrm{kg}$ be the mass of the box. Also let h be the distance between the bottom of the box and the oil/water interface. The condition for equilibrium is

$$M = hA\rho_W + (H-h)A\rho_O \qquad \Longrightarrow \qquad h = \frac{M - HA\rho_O}{A\left(\rho_W - \rho_O\right)} \approx 4.4\ \mathrm{cm}$$

Problem 9.10 (Ohanian, page 493, problem 50)

Example 9 on page 481 illustrates a portion of this problem.

We can use Bernoulli's equation to relate the water at the top of the tank to the water emerging from the hole.

$$\frac{1}{2}\rho v^2 + \rho g z + p_{\text{atom}} = \rho g h + p_{\text{atom}} \qquad \Longrightarrow \qquad v = \sqrt{2g(h-z)}$$

The water at both the top of the tank and the hole is in contact with the atmosphere; hence the pressure is the same, p_{atom}. The velocity at the top of the tank is zero. Note that the velocity of the water emerging from the hole is given as if it fell the distance $h - z$. Now the water flows horizontally from the hole with speed v above. The time it takes to hit the floor is given by

$$\frac{1}{2}gT^2 = z \qquad \Longrightarrow \qquad T = \sqrt{\frac{2z}{g}}$$

The horizontal distance it moves is

$$d = vT = \sqrt{2g(h-z)} \cdot \sqrt{\frac{2z}{g}} = 2\sqrt{(h-z)z}$$

The maximum distance occurs when

$$\frac{dd}{dz} = 2\frac{1}{\sqrt{(h-z)z}} \cdot (h - 2z_{\text{max}}) = 0 \qquad \Longrightarrow \qquad z_{\text{max}} = \frac{1}{2}h$$

The maximum distance is

$$d_{\text{max}} = 2\sqrt{(h - z_{\text{max}})z_{\text{max}}} = 2\sqrt{\left(h - \frac{1}{2}h\right)\frac{1}{2}h} = h$$

Problem 9.11

(a) The ball displaces a volume V of the liquid of density ρ; therefore the mass of liquid which overflows is

$$M = \rho V$$

(b) At first, when the rod is approaching the water but when the ball is still above the water, the rod is carrying the full weight mg of the ball. However, as the ball is pushed into the water, the rod will carry less weight and when the ball is fully immersed, the weight even becomes negative; the rod is pushing down with a force $\rho V g - mg$ to keep the ball immersed. The water exerts a force (the buoyant force) of $\rho V g$ upwards on the ball. Since action equals minus reaction, the ball will exert a force downwards onto the water of $\rho V g$. The weight of the container with the remaining water is $W - \rho V g$. Hence the scale will read $W - \rho V g + \rho V g = W$. Notice that this is independent of the mass m of the ball. This is not surprising as the rod is carrying the weight of the ball.

(c) The buoyant force upwards on the ball is $\rho V g$ and the weight of the ball is mg. Thus the tension (pulling upwards at the scale) $T = \rho V g - mg$. The buoyant force $\rho V g$ is upwards on the ball, thus the ball will exert a force downwards onto the water of $\rho V g$. The weight of the container with the remaining water is $W - \rho V g$. Hence the scale will read $W - \rho V g - T + \rho V g = W - \rho V g + mg$. Notice that in the case that $\rho V g = mg$ (neutral buoyancy) the answer under part (b) and (c) should be the same. The tension in the string (c) is then zero, and the force needed for the rod to keep the ball immersed (b) is also zero. The scale reads W in both cases.