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Oscillations and Waves

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8-8 Rocket Propulsion

Rocket propulsion is a striking example of the conservation of momentum in action. The mathematical description of rocket propulsion can become quite complex because the mass of the rocket changes continuously as it burns fuel and expels exhaust gas. The easiest approach is to compute the change in the momentum of the total system (including the exhaust gas) for some time interval and use Newton's law in the form $F_{ext} = dP/dt$, where F_{ext} is the net force acting on the rocket.

Consider a rocket moving with speed v relative to the earth (Figure 8-44). If the fuel is burned at a constant rate, $R = |dm/dt|$, the rocket's mass at time t is

$$m = m_0 - Rt \tag{8-35}$$

where m_0 is the initial mass of the rocket. The momentum of the system at time t is

$$P_i = mv$$

At a later time $t + \Delta t$, the rocket has expelled gas of mass $R\ \Delta t$. If the gas is exhausted at a speed u_{ex} *relative to the rocket,* the velocity of the gas relative to the earth is $v - u_{ex}$. The rocket then has a mass $m - R\ \Delta t$ and is moving at a speed $v + \Delta v$ (Figure 8-45).

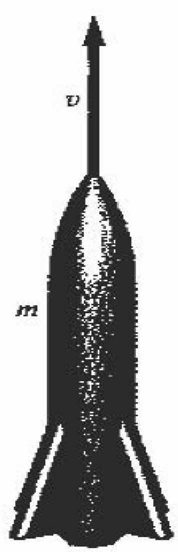

Figure 8-44

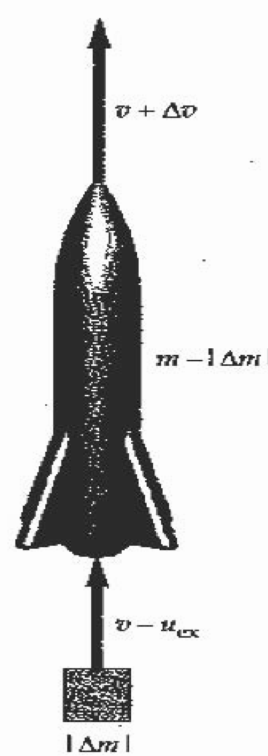

Figure 8-45

The momentum of the system at $t + \Delta t$ is

$$\begin{aligned} P_f &= (m - R\,\Delta t)(v + \Delta v) + R\,\Delta t(v - u_{ex}) \\ &= mv + m\,\Delta v - v\,R\,\Delta t - R\,\Delta t\,\Delta v + v\,R\,\Delta t - u_{ex}\,R\,\Delta t \\ &\approx mv + m\,\Delta v - u_{ex}\,R\,\Delta t \end{aligned}$$

where we have dropped the term $R\,\Delta t\,\Delta v$, which is the product of two very small quantities, and therefore negligible compared with the others. The change in momentum is

$$\Delta P = P_f - P_i = m\,\Delta v - u_{ex}\,R\,\Delta t$$

and

$$\frac{\Delta P}{\Delta t} = m\frac{\Delta v}{\Delta t} - u_{ex}R \tag{8-36}$$

As Δt approaches zero, $\Delta v/\Delta t$ approaches the derivative dv/dt, which is the acceleration. For a rocket moving upward near the surface of the earth, $F_{ext} = -mg$. Setting $dP/dt = F_{ext} = -mg$ gives us the **rocket equation:**

$$m\frac{dv}{dt} = Ru_{ex} + F_{ext} = Ru_{ex} - mg \tag{8-37}$$

Rocket equation

or

$$\frac{dv}{dt} = \frac{Ru_{ex}}{m} - g = \frac{Ru_{ex}}{m_0 - Rt} - g \tag{8-38}$$

The quantity Ru_{ex} is the force exerted on the rocket by the exhausting fuel. This is called the **thrust**:

$$F_{\mathrm{th}} = Ru_{\mathrm{ex}} = \left|\frac{dm}{dt}\right| u_{\mathrm{ex}} \tag{8-39}$$

Definition—Rocket thrust

Equation 8-38 is solved by integrating both sides with respect to time. For a rocket starting at rest at $t = 0$, the result is

$$v = -u_{\mathrm{ex}} \ln\left(\frac{m_0 - Rt}{m_0}\right) - gt \tag{8-40}$$

as can be verified by taking the time derivative of v. The **payload** of a rocket is the final mass, m_{f}, after all the fuel has been burned. The **burn time** t_{b} is given by $m_{\mathrm{f}} = m_0 - Rt_{\mathrm{b}}$, or

$$t_{\mathrm{b}} = \frac{m_0 - m_{\mathrm{f}}}{R} \tag{8-41}$$

Thus, a rocket starting at rest with mass m_0, and payload of m_{f}, attains a final speed

$$v_{\mathrm{f}} = -u_{\mathrm{ex}} \ln \frac{m_{\mathrm{f}}}{m_0} - gt_{\mathrm{b}} \tag{8-42}$$

Final speed of rocket

assuming the acceleration of gravity to be constant.