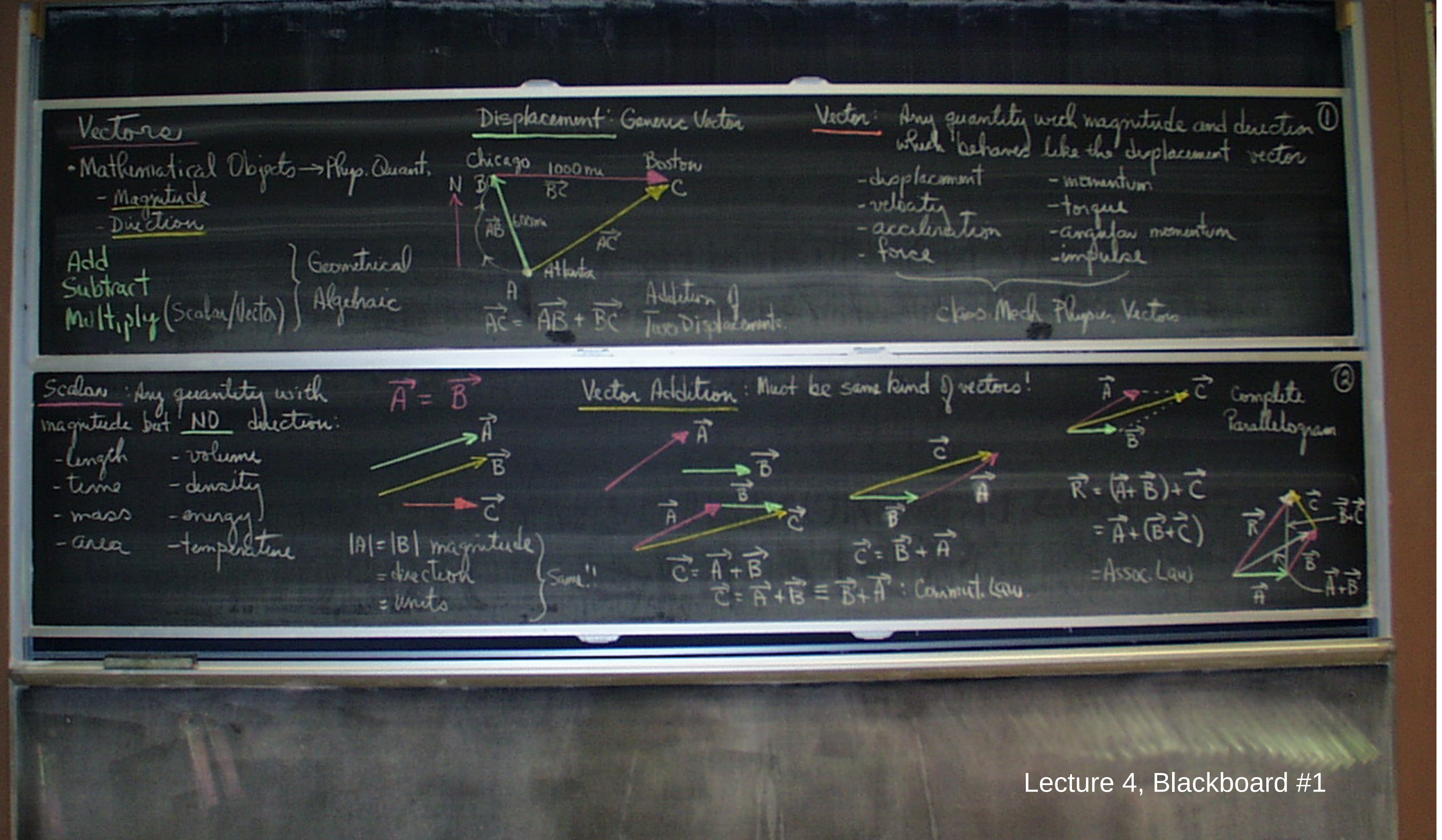
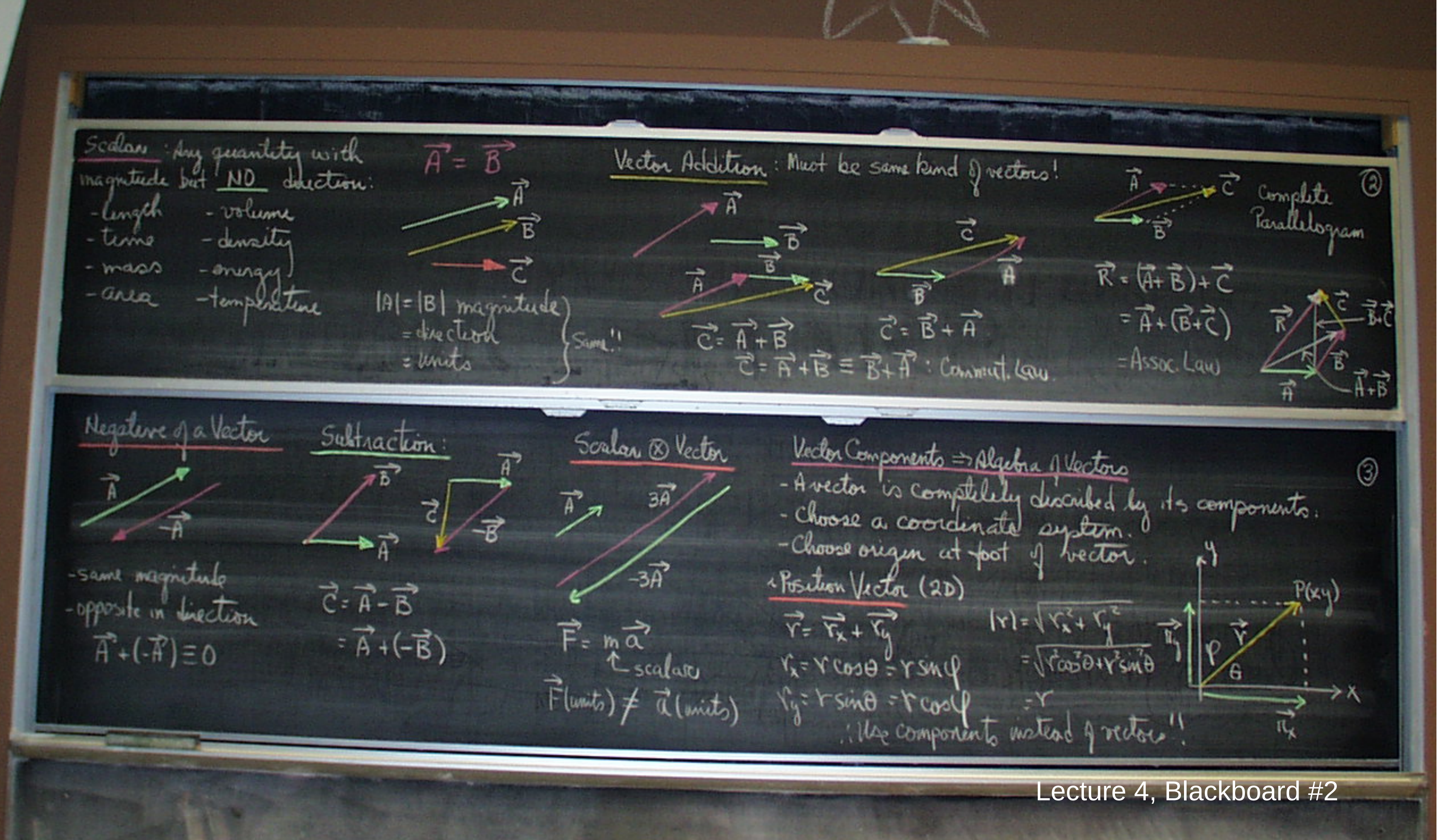
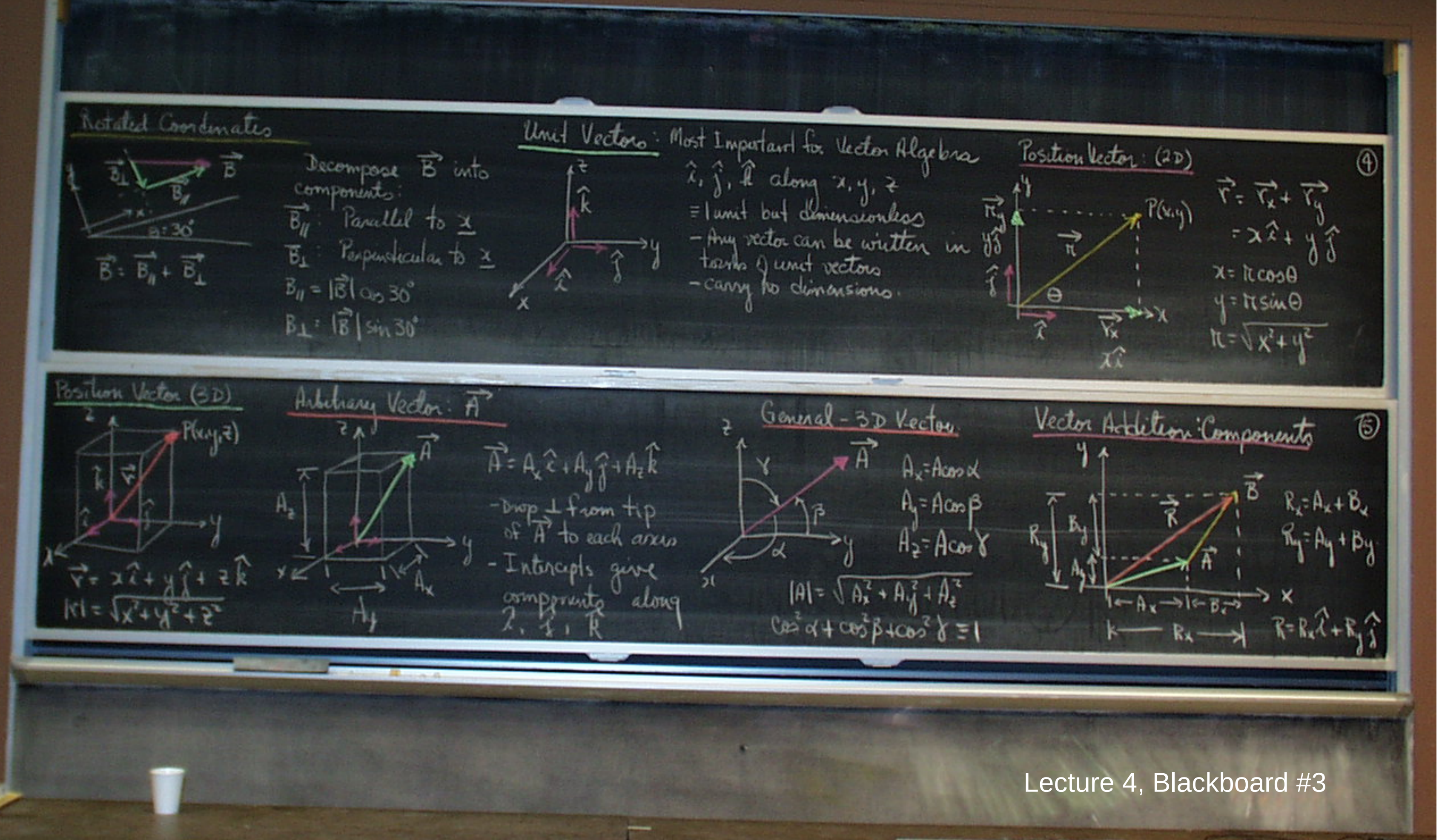


Lecture 4, Blackboard #1

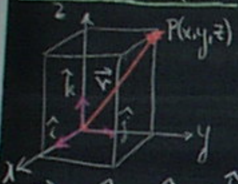


Lecture 4, Blackboard #2



Lecture 4, Blackboard #3

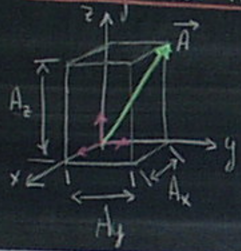
Position Vector (3D)



$\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$

$|r| = \sqrt{x^2 + y^2 + z^2}$

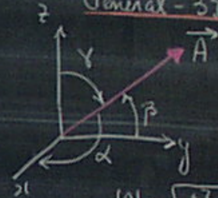
Arbitrary Vector: $\vec{A}$



$\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

- Drop ⊥ from tip of $\vec{A}$ to each axis
- Intercepts give components along $\hat{i}, \hat{j}, \hat{k}$

General - 3D Vector



$A_x = A\cos\alpha$

$A_y = A\cos\beta$

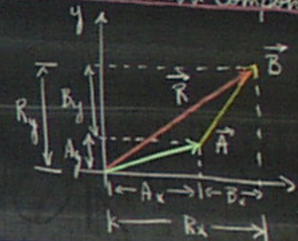
$A_z = A\cos\gamma$

$|A| = \sqrt{A_x^2 + A_y^2 + A_z^2}$

$\cos^2\alpha + \cos^2\beta + \cos^2\gamma \equiv 1$

Vector Addition: Components

⑤



$R_x = A_x + B_x$

$R_y = A_y + B_y$

$\vec{R} = R_x\hat{i} + R_y\hat{j}$

---

⑥

Let $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} + B_z\hat{k}$

$(\vec{A}+\vec{B}) = (A_x+B_x)\hat{i} + (A_y+B_y)\hat{j} + (A_z+B_z)\hat{k}$

$(\vec{A}-\vec{B}) = (A_x-B_x)\hat{i} + (A_y-B_y)\hat{j} + (A_z-B_z)\hat{k}$

Do any number the same way!!

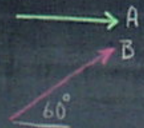
Example

Two vectors: $\vec{A}, \vec{B}$

Want $\vec{R} = \vec{A} + \vec{B}$

$|A| = 3$

$|B| = 4$



$\vec{A} = A_x\hat{i} + A_y\hat{j} = A\hat{i} + 0\hat{j}$

$\vec{B} = B_x\hat{i} + B_y\hat{j} = B\cos 60°\,\hat{i} + B\sin 60°\,\hat{j}$

$\vec{R} = \vec{A} + \vec{B} = (A_x+B_x)\hat{i} + (A_y+B_y)\hat{j}$

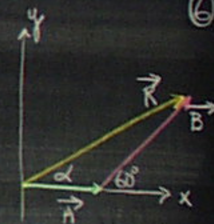
$= (A + B\cos 60°)\hat{i} + (0 + B\sin 60°)\hat{j}$

$= (3 + 4\cos 60°)\hat{i} + 4\sin 60°\,\hat{j}$

$\vec{R} = 5\hat{i} + 3.46\hat{j}$

$|R| = \sqrt{5^2 + 3.46^2} = 6.08$

$\tan\alpha = \frac{R_y}{R_x} = \frac{3.46}{5} \Rightarrow 34.7°$

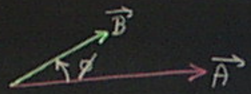


Lecture 4, Blackboard #4

Vector Multiplication

1. Dot Product

- scalar product
- inner product



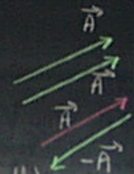
$\vec{A}\cdot\vec{B} = \underbrace{AB\cos\varphi}_{\text{scalar/number}} \qquad \varphi \leq 180^\circ$

$\vec{A}\cdot\vec{B} = \vec{B}\cdot\vec{A}$ Commutative

$\vec{A}\cdot\vec{A} = AA\cos 0^\circ = A^2$ (parallel)

$\vec{A}\cdot(-\vec{A}) = AA\cos 180^\circ$

$= -A^2$ (anti-parallel)



$\vec{A}\cdot\vec{B} = A(B\cos\varphi)$

↳ Proj. of $\vec{B}$ on $\vec{A}$

$= B(A\cos\varphi)$

↳ Proj of $\vec{A}$ on $\vec{B}$

⑦

$0 < \varphi < 90^\circ \quad (\vec{A}\cdot\vec{B}) > 0$

$90^\circ < \varphi < 180^\circ \quad (\vec{A}\cdot\vec{B}) < 0$

$\varphi = 90^\circ \quad \vec{A}\cdot\vec{B} \equiv 0 \quad \vec{A}$ and $\vec{B}$ are $\perp$

Good Test!!

$\left.\begin{array}{l}\hat{i}\cdot\hat{i} = 1\\ \hat{j}\cdot\hat{j} = 1\\ \hat{k}\cdot\hat{k} = 1\end{array}\right\}$ Unit Vectors

$\left.\begin{array}{l}\hat{i}\cdot\hat{j} = \hat{j}\cdot\hat{i} = 0\\ \hat{i}\cdot\hat{k} = \hat{k}\cdot\hat{i} = 0\\ \hat{j}\cdot\hat{k} = \hat{k}\cdot\hat{j} = 0\end{array}\right\}$ $\perp$ Unit Vectors

Can Show:

$(\vec{C}+\vec{D})\cdot\vec{E} = \vec{C}\cdot\vec{E} + \vec{D}\cdot\vec{E}$

Distr. Law

⑧

$\vec{A}\cdot\vec{B} = (A_x\hat{i} + A_y\hat{j} + A_z\hat{k})\cdot(B_x\hat{i} + B_y\hat{j} + B_z\hat{k})$

$= A_xB_x\,\hat{i}\cdot\hat{i} + A_xB_y\,\hat{i}\cdot\hat{j} + A_xB_z\,\hat{i}\cdot\hat{k}$

$+ A_yB_x\,\hat{j}\cdot\hat{i} + A_yB_y\,\hat{j}\cdot\hat{j} + A_yB_z\,\hat{j}\cdot\hat{k}$

$+ A_zB_x\,\hat{k}\cdot\hat{i} + A_zB_y\,\hat{k}\cdot\hat{j} + A_zB_z\,\hat{k}\cdot\hat{k}$

$\vec{A}\cdot\vec{B} = A_xB_x + A_yB_y + A_zB_z$ = A scalar.

Lecture 4, Blackboard #5

$\hat{i}\cdot\hat{i}=1$, $\hat{j}\cdot\hat{j}=1$, $\hat{k}\cdot\hat{k}=1$ — Unit Vectors

$\hat{i}\cdot\hat{j}=\hat{j}\cdot\hat{i}=0$, $\hat{i}\cdot\hat{k}=\hat{k}\cdot\hat{i}=0$, $\hat{j}\cdot\hat{k}=\hat{k}\cdot\hat{j}=0$ — $\perp$ Unit Vectors

Can Show:

$$(\vec{C}+\vec{D})\cdot\vec{E}=\vec{C}\cdot\vec{E}+\vec{D}\cdot\vec{E}$$

Distr. Law

(8)

$$\vec{A}\cdot\vec{B}=(A_x\hat{i}+A_y\hat{j}+A_z\hat{k})\cdot(B_x\hat{i}+B_y\hat{j}+B_z\hat{k})$$

$$=A_xB_x\,\hat{i}\cdot\hat{i}+A_xB_y\,\cancel{\hat{i}\cdot\hat{j}}+A_xB_z\,\cancel{\hat{i}\cdot\hat{k}}$$
$$+A_yB_x\,\cancel{\hat{j}\cdot\hat{i}}+A_yB_y\,\hat{j}\cdot\hat{j}+A_yB_z\,\cancel{\hat{j}\cdot\hat{k}}$$
$$+A_zB_x\,\cancel{\hat{k}\cdot\hat{i}}+A_zB_y\,\cancel{\hat{k}\cdot\hat{j}}+A_zB_z\,\hat{k}\cdot\hat{k}$$

$$\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z \quad = \text{A scalar.}$$

Example:

$$\hat{j}\cdot\vec{A}=\hat{j}\cdot[A_x\hat{i}+A_y\hat{j}+A_z\hat{k}]$$
$$=A_y\,!!$$

↳ Projects out y-comp.

$\hat{j}\ \vec{A}$

Work: $W=\vec{F}\cdot\vec{d}$

Example [Dot Product]

$$\vec{A}=3\hat{i}+7\hat{k}$$
$$\vec{B}=-\hat{i}+2\hat{j}+\hat{k}$$

$$\vec{A}\cdot\vec{B}=A_xB_x+A_yB_y+A_zB_z$$
$$=3(-1)+0(2)+7(1)$$
$$=+4$$

(9)

$$\vec{A}\cdot\vec{B}=|A||B|\cos\theta$$

$$\cos\theta=\frac{\vec{A}\cdot\vec{B}}{|A||B|}=\frac{4}{\sqrt{3^2+0^2+7^2}\ \sqrt{(-1)^2+2^2+1^2}}$$
$$=\frac{4}{\sqrt{58}\ \sqrt{6}}$$

$\theta=77.6^\circ$ Angle between $\vec{A}$, $\vec{B}$

Lecture 4, Blackboard #6