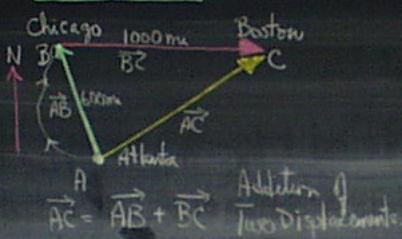


Vectors

- Mathematical Objects \rightarrow Phys. Quant.
- Magnitude
- Direction

Displacement: Generic Vector



Vector: Any quantity with magnitude and direction which behaved like the displacement vector ①

- displacement
- velocity
- acceleration
- force
- momentum
- torque
- angular momentum
- impulse

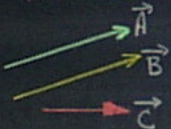
Class. Mech. Physic. Vectors

Add
Subtract
Multiply (Scalar/Vector) } Geometrical
Algebraic

Scalar: Any quantity with magnitude but NO direction:

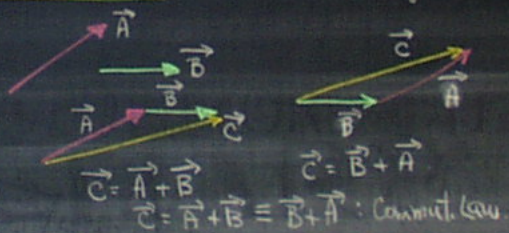
- length
- volume
- time
- density
- mass
- energy
- area
- temperature

$$\vec{A} = \vec{B}$$



$|\vec{A}| = |\vec{B}|$ magnitude
 = direction
 = units } Same!!

Vector Addition: Must be same kind of vectors!



Complete Parallelogram ②

$$\vec{R} = (\vec{A} + \vec{B}) + \vec{C}$$

$$= \vec{A} + (\vec{B} + \vec{C})$$

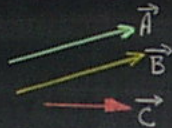
= Assoc. Law



Scalars: Any quantity with magnitude but NO direction:

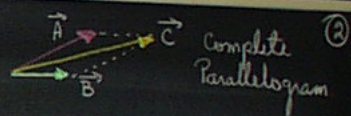
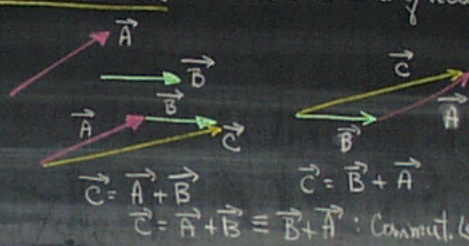
- length
- volume
- time
- density
- mass
- energy
- area
- temperature

$$\vec{A} = \vec{B}$$



$|\vec{A}| = |\vec{B}|$ magnitude
 = direction
 = units
 } Same!!

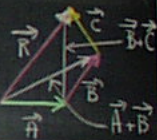
Vector Addition: Must be same kind of vectors!



$$\vec{R} = (\vec{A} + \vec{B}) + \vec{C}$$

$$= \vec{A} + (\vec{B} + \vec{C})$$

= Assoc. Law

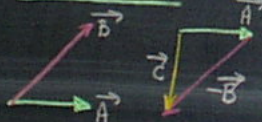


Negative of a Vector



- same magnitude
 - opposite in direction
- $$\vec{A} + (-\vec{A}) \equiv 0$$

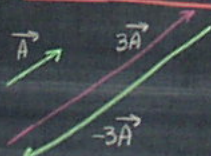
Subtraction:



$$\vec{C} = \vec{A} - \vec{B}$$

$$= \vec{A} + (-\vec{B})$$

Scalar \otimes Vector



$$\vec{F} = m \vec{a}$$

\uparrow scalar

\vec{F} (units) \neq \vec{a} (units)

Vector Components \Rightarrow Algebra of Vectors

- A vector is completely described by its components.
- Choose a coordinate system.
- Choose origin at foot of vector.

Position Vector (2D)

$$\vec{r} = r_x \hat{i} + r_y \hat{j}$$

$$r_x = r \cos \theta = r \sin \phi$$

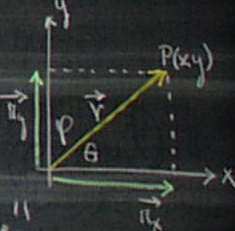
$$r_y = r \sin \theta = r \cos \phi$$

$$|\vec{r}| = \sqrt{r_x^2 + r_y^2}$$

$$= \sqrt{r^2 \cos^2 \theta + r^2 \sin^2 \theta}$$

$$= r$$

"Use components instead of vectors!"

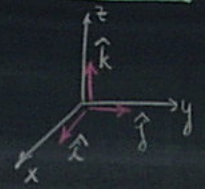


Rotated Coordinates



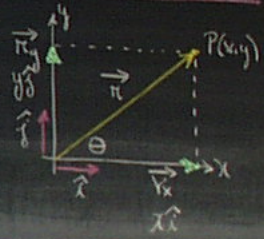
Decompose \vec{B} into components:
 $\vec{B}_{||}$: Parallel to x
 \vec{B}_{\perp} : Perpendicular to x
 $B_{||} = |\vec{B}| \cos 30^\circ$
 $B_{\perp} = |\vec{B}| \sin 30^\circ$

Unit Vectors: Most Important for Vector Algebra



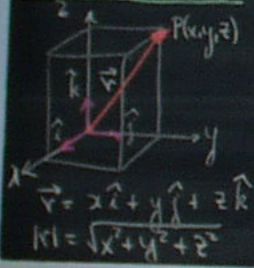
$\hat{i}, \hat{j}, \hat{k}$ along x, y, z
 \equiv Unit but dimensionless
 - Any vector can be written in terms of unit vectors
 - carry no dimensions.

Position Vector: (2D)

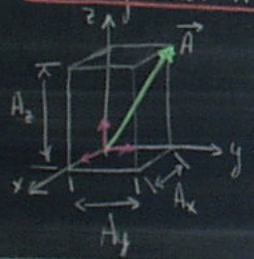


(4)
 $\vec{r} = \vec{r}_x + \vec{r}_y$
 $= x\hat{i} + y\hat{j}$
 $x = r \cos \theta$
 $y = r \sin \theta$
 $r = \sqrt{x^2 + y^2}$

Position Vector (3D)

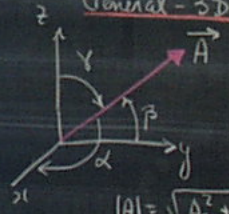


Arbitrary Vector: A



$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$
 - Drop \perp from tip of \vec{A} to each axis
 - Intercepts give components A_x, A_y, A_z

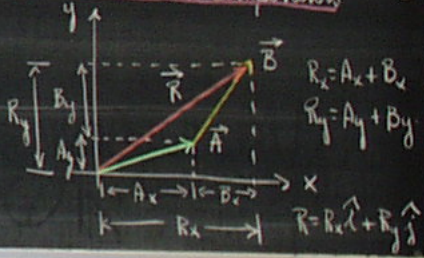
General - 3D Vector:



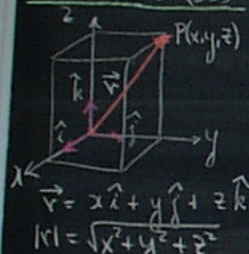
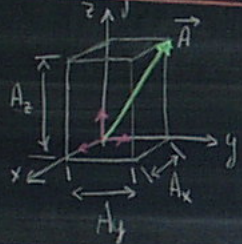
$A_x = A \cos \alpha$
 $A_y = A \cos \beta$
 $A_z = A \cos \gamma$

$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$
 $\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$

Vector Addition: Components



(5)
 $R_x = A_x + B_x$
 $R_y = A_y + B_y$
 $\vec{R} = R_x \hat{i} + R_y \hat{j}$

Position Vector (3D)Arbitrary Vector: \vec{A} 

$$\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$$

- Drop \perp from tip of \vec{A} to each axis
- Intercepts give components along $\hat{i}, \hat{j}, \hat{k}$

General - 3D Vector

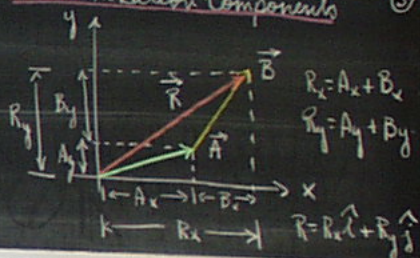
$$A_x = A \cos \alpha$$

$$A_y = A \cos \beta$$

$$A_z = A \cos \gamma$$

$$|\vec{A}| = \sqrt{A_x^2 + A_y^2 + A_z^2}$$

$$\cos^2 \alpha + \cos^2 \beta + \cos^2 \gamma = 1$$

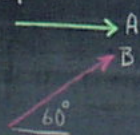
Vector Addition: Components (5)

Let: $\vec{A} = A_x \hat{i} + A_y \hat{j} + A_z \hat{k}$
 $\vec{B} = B_x \hat{i} + B_y \hat{j} + B_z \hat{k}$

Example:Two vectors: \vec{A}, \vec{B} Want $\vec{R} = \vec{A} + \vec{B}$

$|A| = 3$

$|B| = 4$



$\vec{A} = A_x \hat{i} + A_y \hat{j} = A \hat{i} + 0 \hat{j}$

$\vec{B} = B_x \hat{i} + B_y \hat{j} = B \cos 60^\circ \hat{i} + B \sin 60^\circ \hat{j}$

$$\vec{R} = \vec{A} + \vec{B} = (A + B_x) \hat{i} + (A_y + B_y) \hat{j}$$

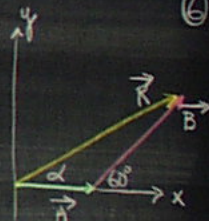
$$= (A + B \cos 60^\circ) \hat{i} + (0 + B \sin 60^\circ) \hat{j}$$

$$= (3 + 4 \cos 60^\circ) \hat{i} + 4 \sin 60^\circ \hat{j}$$

$$\vec{R} = 5 \hat{i} + 3.46 \hat{j}$$

$|\vec{R}| = \sqrt{5^2 + 3.46^2} = 6.08$

$$\tan \theta = \frac{R_y}{R_x} = \frac{3.46}{5} \Rightarrow \underline{\underline{34.7^\circ}}$$



$(\vec{A} + \vec{B}) = (A_x + B_x) \hat{i} + (A_y + B_y) \hat{j} + (A_z + B_z) \hat{k}$

$(\vec{A} - \vec{B}) = (A_x - B_x) \hat{i} + (A_y - B_y) \hat{j} + (A_z - B_z) \hat{k}$

Do any number the same way!!

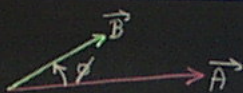
Vector Multiplication

1. Dot Product

- scalar product
- inner product

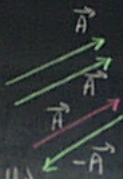
$$\vec{A} \cdot \vec{B} = AB \cos \varphi \quad \varphi \leq 180^\circ$$

$$\vec{A} \cdot \vec{B} = \vec{B} \cdot \vec{A} \quad \text{Commutative}$$



$$\vec{A} \cdot \vec{A} = AA \cos 0^\circ = A^2 \quad (\text{parallel})$$

$$\vec{A} \cdot (-\vec{A}) = AA \cos 180^\circ = -A^2 \quad (\text{anti-parallel})$$



$$\vec{A} \cdot \vec{B} = A(B \cos \varphi)$$

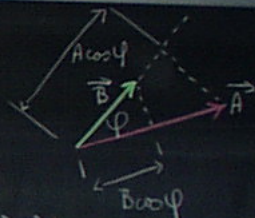
↑ Proj. of \vec{B} on \vec{A}

$$= B(A \cos \varphi)$$

↑ Proj. of \vec{A} on \vec{B}

$$0 < \varphi < 90^\circ \quad (\vec{A} \cdot \vec{B}) > 0$$

$$90^\circ < \varphi < 180^\circ \quad (\vec{A} \cdot \vec{B}) < 0$$



$\varphi = 90^\circ \quad \vec{A} \cdot \vec{B} = 0$ \vec{A} and \vec{B} are \perp
Good Test!!

$$\left. \begin{array}{l} \hat{i} \cdot \hat{i} = 1 \\ \hat{j} \cdot \hat{j} = 1 \\ \hat{k} \cdot \hat{k} = 1 \end{array} \right\} \begin{array}{l} \text{unit} \\ \text{vectors} \end{array}$$

$$\left. \begin{array}{l} \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = 0 \\ \hat{i} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0 \\ \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{j} = 0 \end{array} \right\} \begin{array}{l} \perp \\ \text{unit} \\ \text{vectors} \end{array}$$

Can Show:

$$(\vec{C} + \vec{D}) \cdot \vec{E} = \vec{C} \cdot \vec{E} + \vec{D} \cdot \vec{E}$$

Distr. Law.

$$\vec{A} \cdot \vec{B} = (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k})$$

$$= A_x B_x \hat{i} \cdot \hat{i} + A_x B_y \hat{i} \cdot \hat{j} + A_x B_z \hat{i} \cdot \hat{k} \\ + A_y B_x \hat{j} \cdot \hat{i} + A_y B_y \hat{j} \cdot \hat{j} + A_y B_z \hat{j} \cdot \hat{k} \\ + A_z B_x \hat{k} \cdot \hat{i} + A_z B_y \hat{k} \cdot \hat{j} + A_z B_z \hat{k} \cdot \hat{k}$$

$$\vec{A} \cdot \vec{B} = A_x B_x + A_y B_y + A_z B_z = A \text{ scalar.}$$

$$\left. \begin{array}{l} \hat{i} \cdot \hat{i} = 1 \\ \hat{j} \cdot \hat{j} = 1 \\ \hat{k} \cdot \hat{k} = 1 \end{array} \right\} \text{unit Vectors}$$

$$\left. \begin{array}{l} \hat{i} \cdot \hat{j} = \hat{j} \cdot \hat{i} = 0 \\ \hat{i} \cdot \hat{k} = \hat{k} \cdot \hat{i} = 0 \\ \hat{j} \cdot \hat{k} = \hat{k} \cdot \hat{j} = 0 \end{array} \right\} \begin{array}{l} \perp \\ \text{unit} \\ \text{Vectors} \end{array}$$

Can Show:

$$(\vec{C} + \vec{D}) \cdot \vec{E} = \vec{C} \cdot \vec{E} + \vec{D} \cdot \vec{E}$$

Distr Law

$$\begin{aligned} \vec{A} \cdot \vec{B} &= (A_x \hat{i} + A_y \hat{j} + A_z \hat{k}) \cdot (B_x \hat{i} + B_y \hat{j} + B_z \hat{k}) \\ &= A_x B_x \hat{i} \cdot \hat{i} + A_x B_y \hat{i} \cdot \hat{j} + A_x B_z \hat{i} \cdot \hat{k} \\ &\quad + A_y B_x \hat{j} \cdot \hat{i} + A_y B_y \hat{j} \cdot \hat{j} + A_y B_z \hat{j} \cdot \hat{k} \\ &\quad + A_z B_x \hat{k} \cdot \hat{i} + A_z B_y \hat{k} \cdot \hat{j} + A_z B_z \hat{k} \cdot \hat{k} \\ \vec{A} \cdot \vec{B} &= A_x B_x + A_y B_y + A_z B_z = A \text{ scalar.} \end{aligned} \quad (8)$$

Example:

$$\begin{aligned} \hat{j} \cdot \vec{A} &= \hat{j} \cdot [A_x \hat{i} + A_y \hat{j} + A_z \hat{k}] \\ &= A_y!! \end{aligned}$$

↑ Projects out y-comp.

Wok: $W = \vec{F} \cdot \hat{j}$

Example [Dot Product]

$$\begin{aligned} \vec{A} &= 3\hat{i} + 7\hat{k} \\ \vec{B} &= -\hat{i} + 2\hat{j} + \hat{k} \end{aligned}$$

$$\begin{aligned} \vec{A} \cdot \vec{B} &= A_x B_x + A_y B_y + A_z B_z \\ &= 3(-1) + 0(2) + 7(1) \\ &= +4 \end{aligned}$$

$$\begin{aligned} \vec{A} \cdot \vec{B} &= |\vec{A}| |\vec{B}| \cos \theta \\ \cos \theta &= \frac{\vec{A} \cdot \vec{B}}{|\vec{A}| |\vec{B}|} = \frac{4}{\sqrt{3^2 + 0^2 + 7^2} \sqrt{(-1)^2 + 2^2 + 1^2}} \\ &= \frac{4}{\sqrt{58} \sqrt{6}} \\ \theta &= 77.6^\circ \text{ Angle between } \vec{A}, \vec{B} \end{aligned} \quad (9)$$