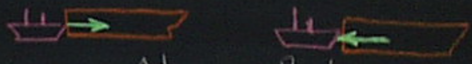
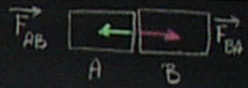
Newton's 3rd Law

- Forces in nature always act in pairs
- No single isolated force

"To every action there is an equal and opposite reaction. The mutual actions of two bodies upon each other are always equal, and directed to contrary parts."

Two forces in Action ⟷ Reaction act on different bodies!





$\vec F_{AB} = -\vec F_{BA}$ Action - Reaction

Body accelerated only by force acting on it!

$a_B = \frac{\vec F_{BA}}{M_B}$ $\qquad a_A = \frac{\vec F_{AB}}{M_A}$

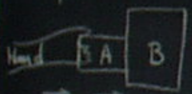
①

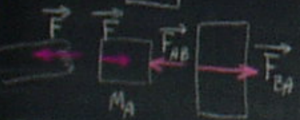
Example

$M_A = 1\,kg$

$M_B = 2\,kg$

$F = 2N$





$\vec a = ?$ $\quad F_{AB} = ?$

$F_{BA} = m_B a_B$ ①

$F - F_{AB} = m_A a_A$ ②

$a_A = a_B = a$ (Blocks in contact)

$F - F_{AB} + F_{BA} = (m_A + m_B)\, a$ ①+②

$|F_{AB}| = |F_{BA}|$ Newton's 3rd Law

$\therefore F = (m_A + m_B)\, a$

$a = \frac{F}{m_A + m_B} = \frac{2}{1+2} = \frac{2}{3}\, m/s^2$

Contact Force:

$F_{BA} = m_B a_B = m_B a$

$= 2 \times \frac{2}{3} = \frac{4}{3} N \neq F$

Gravitational Force

Newton: Every pair of particles exerts on one another a mutual gravitational force of attraction. Force prop. to masses and inversely prop. to square of distance between them.

$F_g = \frac{G m_1 m_2}{r^2}$

$G = 6.673 \times 10^{-11}\ N \cdot m^2/kg^2$

②

G

earth!

$F_g = F_g = N = w = mg$

Lecture 9, Blackboard #1

Example

$M_A = 1\,kg$
$M_B = 2\,kg$
$F = 2N$

$\vec{a} = ?$ $F_{AB} = ?$

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Gravitational Force ②

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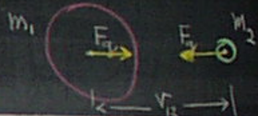
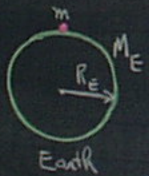


③

$G \equiv$ Universal Gravitational Const.
Cavendish Expt. (November)

Earth

$F_g = m\left(\dfrac{M_E G}{R_E^2}\right) = mg$
$\hookrightarrow g$



$M_E = \dfrac{g R_E^2}{G} \Rightarrow$ Weighing the earth!

Object on Earth

Object: $\vec{N} + \vec{F}_g = 0$
$\vec{N} = -\vec{F}_g$

Earth: $\vec{w} + \vec{f}_g = 0$
$\vec{w} = -\vec{f}_g$

But. $\left.\begin{array}{l}\vec{F}_g = -\vec{f}_g \\ \vec{N} = -\vec{w}\end{array}\right\}$ 3rd Law.

$\therefore F_g = f_g = N = w = mg$

Weight: Contact force $\vec{w}$ that an object exerts on whatever is supporting it.

$\left.\begin{array}{l}\vec{F}_g \text{ acts on object} \\ \vec{w} \text{ acts on earth}\end{array}\right\}$ $\vec{w} = m\vec{g}$

Example: Accelerating Elevator

m: $N - F_g = ma$

$N = F_g + ma$

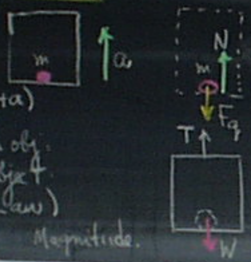
$= mg + ma = m(g+a)$

$N \Rightarrow$ Force elevator on obj.

F_g = Force earth on object

$\vec{N} = -\vec{w}$ (3rd Law)

$\therefore w = m(g+a)$ Magnitude.



Weight increased over object at rest.

If elevator accelerates down with a':

$w = m(g - a')$: Decreased

Free Fall: $a' = g$.

$\Rightarrow w \equiv 0$ No contact Force.

Example Astronaut/Satellite

At orbit radius: $g' = v^2/R$

Satellite: $F_s = F_g + w = Mg'$

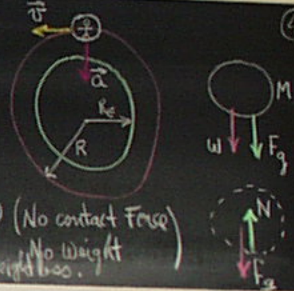
Astronaut $f_a = f_g - N = mg'$

$\left.\begin{array}{l} F_g = Mg' \\ f_g = m g' \end{array}\right\}$ Grav. Force Definition.

$\therefore w = N \equiv 0$ (No contact Force)

No Weight

Astronaut $\Rightarrow$ Weightless.



Gravitational/Inertial Mass

1. Inertial Mass:

$\vec{F} = M_I \vec{a}$ Force $\longleftrightarrow$ Acceleration

2'nd Law

2. Gravitational Mass:

$\vec{F} = \dfrac{G M_E M_G}{R_E^2} \hat{n} = \vec{g} M_G$

Q: Is $M_I = M_G$ for all bodies?

Q: Do all objects fall due to gravity with same $\vec{a}$?

Body 1: $M_I(1)\, a(1) = \dfrac{G M_E}{R^2} M_G(1) = g M_G(1)$

Body 2: $M_I(2)\, a(2) = g M_G(2)$

$$\frac{M_I(1)}{M_G(1)} = \frac{M_I(2)}{M_G(2)} \times \frac{a(2)}{a(1)}$$

Expt: $a(2)/a(1)$ = constant.

Galileo/Eötvös/Dicke: 1×10^{-12}

Ratio: $\dfrac{M_I}{M_G} = 1$ for choice of G.

Newtonian Theory: $M_I \sim M_G$ Not Needed

General Relativity: $M_I \sim M_G$ Absolutely Necessary

Principle of Equivalence

to the deformation.

R small: small force/unit disp.

Gravitational/Inertial Mass

1. Inertial Mass:

$\vec{F} = m_I \vec{a}$ Force ⟷ Acceleration

2'nd Law

2. Gravitational Mass:

$\vec{F} = \frac{G M_E M_G}{R_E^2} \hat{r} = \vec{g} M_G$

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Newtonian Theory: $M_I \sim M_G$ Not Needed

General Relativity: $M_I \sim M_G$ Absolutely Necessary

(5)

Principle of Equivalence

Hooke's Law: Springs

Bodies Elastic: steel balls, rubber bands, springs.

Body resists deformation: ⇒ Restoring Force.

Hooke's Law: The magnitude of the restoring force is directly proportional to the deformation.

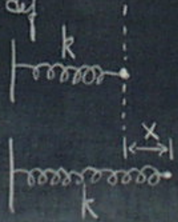
- approximate
- empirical
- good for small def

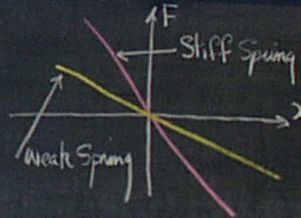
Coil Spring:

$F = -k\,x$

└ Spring Const.

└ opposes def.

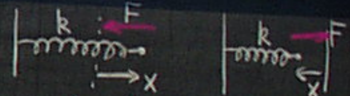




$[k] = N/m$

k large: large force/unit disp.

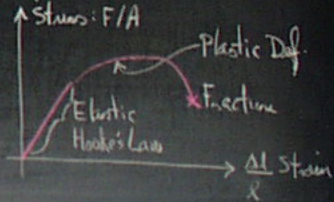
k small: small force/unit disp.



(6)

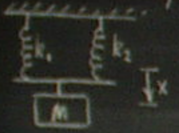
$+x$: elongation $F <$

$-x$: compression $F >$



Lecture 9, Blackboard #4

Parallel Springs



$$-k_1 x - k_2 x + Mg = 0$$

$$x(k_1+k_2) = Mg$$

$$x = \frac{Mg}{k_1+k_2} = \frac{Mg}{k_{eff}}$$

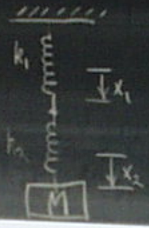
$$k_{eff} = k_1 + k_2$$

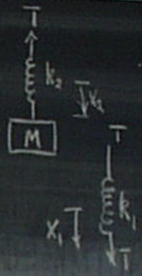
= Stiffer Spring

If $k_1 = k_2 = k$

$$k_{eff} = 2k$$

Series Springs





$$T = Mg$$

$$\therefore k_2 x_2 = Mg$$

$$k_1 x_1 = Mg \text{ (Also)}$$

$$x = x_1 + x_2 = Mg\left[\frac{1}{k_1} + \frac{1}{k_2}\right]$$

Let $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2}$

$\Rightarrow$ Weaker Spring

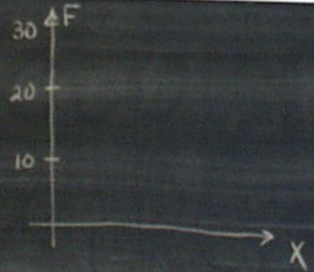
If $k_1 = k_2 = k$

$$k_{eff} = \frac{1}{2}k$$

⑦

Measure Spring k

M	X	Mg (N)
0		0
1 kg		9.81
2 kg		19.6
3 kg		29.4



⑧

Lecture 9, Blackboard #5