①

Work

$$W=\int_{P_1}^{P_2}\vec{F}\cdot d\vec{r}$$

Kinetic Energy 1D

F = constant

$$W = F\,s = m a_x s$$

Assume a = const.

$$s=\tfrac{1}{2}(v_i+v_f)\,t$$

$$a=(v_f-v_i)/t$$

v_i: initial velocity

v_f = final velocity

$$W = m\left(\frac{v_f-v_i}{2}\right)(v_i+v_f)\,t$$

$$W=\tfrac{1}{2}mv_f^2-\tfrac{1}{2}mv_i^2$$

$$W = K_f - K_i$$

where $K=\tfrac{1}{2}mv^2$

↑ Kinetic Energy of particle with velocity v.

$W>0$ KE Increases

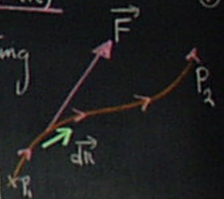
<0 KE Decreases

Kinetic Energy: $\vec{F}=\vec{F}(r)$

$\vec{F}$ = Resultant force acting on particle

· Particle moves from $P_1 \rightarrow P_2$

$$\boxed{\begin{aligned}\vec{F}&=F_x\hat{i}+F_y\hat{j}+F_z\hat{k}\\ d\vec{r}&=dx\,\hat{i}+dy\,\hat{j}+dz\,\hat{k}\end{aligned}}$$



②

$$W=\int_{P_1}^{P_2}\vec{F}\cdot d\vec{r}=\int_{P_1}^{P_2}(F_x\,dx+F_y\,dy+F_z\,dz)$$

$$F_x = m a_x = \frac{dv_x}{dt}$$

$$\int_{P_1}^{P_2}F_x\,dx=\int_{P_1}^{P_2}m\frac{dv_x}{dt}\,dx$$

$$\frac{dv_x}{dt}=\frac{dv_x}{dx}\cdot\frac{dx}{dt}=\frac{dv_x}{dx}\cdot v_x$$

$$\int_{P_1}^{P_2}F_x\,dx=\int_{P_1}^{P_2}m v_x\frac{dv_x}{\cancel{dx}}\,\cancel{dx}=\frac{1}{2}mv_x^2\Big|_{v_{x1}}^{v_{x2}}$$

$$=\tfrac{1}{2}m\left(v_{x1}^2-v_{x2}^2\right)$$

v_{x1} = velocity-x at P_1

v_{x2} = velocity-x at P_2

Repeat same for y and z:

$$W=\tfrac{1}{2}m\left[v_{x2}^2+v_{y2}^2+v_{z2}^2-\left(v_{x1}^2+v_{y2}^2+v_{z2}^2\right)\right]$$

$$W=\tfrac{1}{2}m\left(v_2^2-v_1^2\right)=K_2-K_1=\Delta K \Rightarrow \boxed{\text{Work-Energy Thm}}$$

KE: Potential for particle to do work by virtue of velocity!

W: Work done on particle by the net force equals the change in KE of the particle.

Lecture 13, Blackboard #1

$$W = \int_{P_1}^{P_2} \vec{F}\cdot\vec{dr} = \int_{P_1}^{P_2} (F_x dx + F_y dy + F_z dz)$$

$$F_x = m a_x = \frac{dv_x}{dt}$$

$$\int_{P_1}^{P_2} F_x dx = \int_{P_1}^{P_2} m \frac{dv_x}{dt} dx$$

$$\frac{dv_x}{dt} = \frac{dv_x}{dx}\cdot\frac{dx}{dt} = \frac{dv_x}{dx}\cdot v_x$$

$$\int_{P_1}^{P_2} F_x dx = \int_{P_1}^{P_2} m v_x \frac{dv_x}{\cancel{dx}} \cancel{dx} = \frac{1}{2} m v_x^2 \Big|_{v_{x1}}^{v_{x2}}$$

$$= \frac{1}{2} m (v_{x1}^2 - v_{x2}^2)$$

v_{x1} = velocity-x at P_1

v_{x2} = velocity-x at P_2

Repeat same for y and z:

$$W = \frac{1}{2} m \left[v_{x2}^2 + v_{y2}^2 + v_{z2}^2 - (v_{x1}^2 + v_{y2}^2 + v_{z2}^2)\right]$$

$$W = \frac{1}{2} m (v_2^2 - v_1^2) = K_2 - K_1 = \Delta K \Rightarrow \boxed{\text{Work-Energy Thm}}$$

KE: Potential for particle to do work by virtue of velocity!

W: Work done on particle by the net force equals the change in KE of the particle.

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Example:

- Move mass down to x_A
- Release with upward velocity v_0.
- What is velocity at x_B?

Net force on block:

$$F = F_g - F_s = mg - kx$$

Work done:

$$W = \int_{x_A}^{x_B} F dx = \int_{x_A}^{x_B} (mg - kx) dx$$

$$W = \left(mgx - \frac{1}{2}kx^2\right)\Big|_{x_A}^{x_B}$$

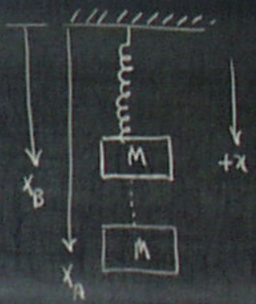
$$= mg(x_B - x_A) - \frac{1}{2}k(x_B^2 - x_A^2)$$

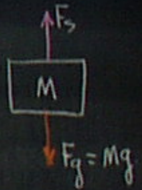
$$= 0.5\times 9.8(0.2-0.5) - \frac{1}{2}50(0.2^2 - 0.5^2)$$

$$= 3.78\ \text{J}$$

$$W = \frac{1}{2} m v_B^2 - \frac{1}{2} m v_A^2$$

$$\therefore v_B^2 = v_A^2 + \frac{2W}{m} = \left[(-2)^2 + \frac{2\times 3.78}{0.5}\right] = \pm 4.37\ \text{m/s}$$





3

$m = 0.5$ kg $\quad x_A = 0.5$ m

$k = 50$ N/m $\quad x_B = 0.2$ m

$v_0 = 2$ m/s

Example:

- Ball in vertical circle.
- Find minimum speed at B so at top, A, tension just vanishes.
- Energy and Dynamics.

At top. $T + mg = \frac{m v_T^2}{R}$

$T = 0$ in limit for min. v.

$\therefore\ v_T^2 = gR$

$W_T = 0 \quad T \perp$ displacement.

$W_g(A \to B) = mgH = K_B - K_A$

$mg(2R) = \frac{1}{2} m v_0^2 - \frac{1}{2} m v_T^2$

$v_0^2 = v_T^2 + 4gR$

$= gR + 4gR$

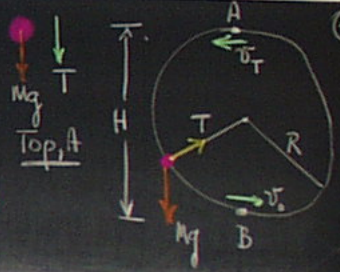
$= 5gR$

$v_0 = \sqrt{5gR}$

If $v < v_0$ will not reach top!

$> v_0 \quad T > 0$

④



Gravitational Potential Energy

$W = K_2 - K_1 = \Delta K$ W-E Theorem

$KE = \frac{1}{2} m v^2$ Capacity to do work due to its velocity.

$PE = U(\vec{r})$ Capacity to do work due to position in space

Potential Energy!

AND because of position.

$F_g = -mg$ constant.

Move from $(x_1, y_1, z_1) \to (x_2, y_2, z_2)$

$W_g = \int_{z_1}^{z_2} \vec{F} \cdot \vec{dz} = \int_{z_1}^{z_2} -mg\, dz$

$= -mgz \Big|_{z_1}^{z_2} = -mg(z_2 - z_1)$

$= -U(z_2) + U(z_1)$

Where

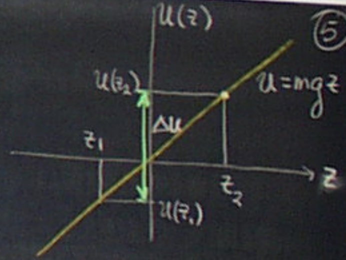
$U(z_2) = mgz_2$

$U(z_1) = mgz_1$

$U(z) \equiv$ Gravitational Pot. En.

= capacity to do work because it is at height z above surface of attracting earth!!

⑤



Lecture 13, Blackboard #3

Gravitational Potential Energy

$W = K_2 - K_1 = \Delta K$ W-E Theorem

$KE = \frac{1}{2}mv^2$ Capacity to do work due to its velocity.

$PE = U(\vec{r})$ Capacity to do work due to position in space

Potential Energy!

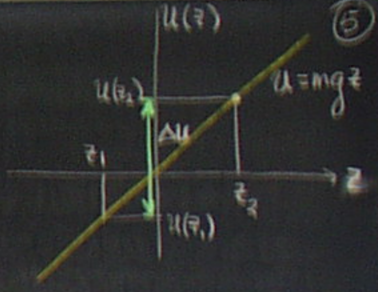
$F_g = -mg$ constant.

Move from $(x_1, y_1, z_1) \rightarrow (x_2, y_2, z_2)$

$$W_g = \int_{z_1}^{z_2} \vec{F} \cdot d\vec{z} = \int_{z_1}^{z_2} -mg\,dz$$
$$= -mgz\Big|_{z_1}^{z_2} = -mg(z_2 - z_1)$$
$$= -U(z_2) + U(z_1)$$

where
$U(z_2) = mgz_2$
$U(z_1) = mgz_1$

$U(z) \equiv$ Gravitational Pot. En.
= capacity to do work because it is at height z above surface of attracting earth!!



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Suppose: Gravity is only force acting.

$$W_g = K_2 - K_1 = -U(z_2) + U(z_1)$$
$$\therefore K_2 + U(z_2) = K_1 + U(z_1)$$

$K + U(z) =$ Constant of the motion

Let $E = K + U(z) \equiv$ Mechanical Energy
= Capacity to do work because of velocity
AND because of position.

If only gravity,

$E = K + U(z) =$ Constant

$$\boxed{E = \tfrac{1}{2}mv^2 + mgz = \text{const.}}$$

Law of Conservation of Mech En.

z increases $\Rightarrow$ v decreases
z decreases $\Rightarrow$ v increases

Consider two different positions: z_1 and z_2

$$\tfrac{1}{2}mv_1^2 + mgz_1 = \tfrac{1}{2}mv_2^2 + mgz_2$$
$$v_1^2 + 2gz_1 = v_2^2 + 2gz_2$$
$$v_2^2 - v_1^2 = 2g(z_1 - z_2)$$
$$= -2g(z_2 - z_1)$$

Recall constant a kinematics!!

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Lecture 13, Blackboard #4

Gravity + Other Forces:

$W = W_g + W_{other} = K_2 - K_1 = \Delta K$

$W_{other} - (mgz_2 - mgz_1) = \frac{1}{2}mv_2^2 - \frac{1}{2}mv_1^2$

$W_{other} = \frac{1}{2}(mv_2^2 - mv_1^2) + mg(z_2 - z_1)$

$= \Delta K + \Delta U =$ change in KE + PE

$W_{other} = (\frac{1}{2}mv_2^2 + mgz_2) - (\frac{1}{2}mv_1^2 + mgz_1)$

$= (K_2 + U_2) - (K_1 + U_1)$

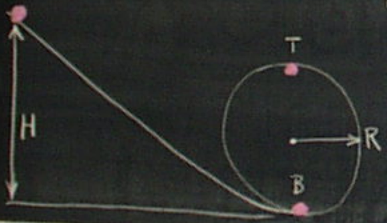
$= E_2 - E_1 = \Delta E$

If $W_{other} > 0$ ME increases.

< 0 ME decreases.

(7)

Example: Loop-the-Loop



What is H so ball does not fall off?

At top of loop velocity $= v_T$.

$F_T = mg + N = \frac{mv_T^2}{R}$

For min. $v_T \Rightarrow N \equiv 0$.

$\therefore v_T = \sqrt{gR}$

Conservation of Energy.

$mgH + 0 = mg(2R) + \frac{1}{2}mv_T^2$

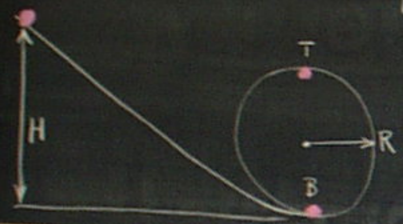
$v_T^2 = 2g(H - 2R)$

$v_T = \sqrt{2g(H-2R)} = \sqrt{gR} \Rightarrow H = \frac{5}{2}R$

(8)

Lecture 13, Blackboard #5

Example loop-the-loop



What is H so ball does not fall off?

At top of loop velocity $= v_T$.

$$F_T = mg + N = \frac{m v_T^2}{R}$$

For min. $v_T \Rightarrow N \equiv 0$.

$$\therefore v_T = \sqrt{gR}$$

Conservation of Energy.

$$mgH + 0 = mg(2R) + \tfrac{1}{2} m v_T^2$$

$$v_T^2 = 2g(H-2R)$$

$$v_T = \sqrt{2g(H-2R)} = \sqrt{gR} \Rightarrow H = \tfrac{5}{2}R$$

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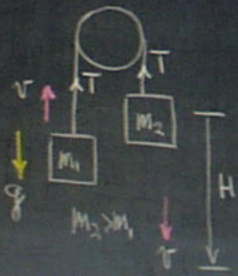
Atwood's Machine

- Release m_2 and let drop distance H
- $v=0$ at $t=0$

Final $v = ?$



$$W_{m_2} = m_2 gH - TH = \tfrac{1}{2} m_2 v^2 \quad (1)$$

$$W_{m_1} = -m_1 gH + TH = \tfrac{1}{2} m_1 v^2 \quad (2)$$

$$(1)+(2) \quad (m_2 - m_1) gH = \tfrac{1}{2}(m_2 + m_1) v^2$$

$$v = \sqrt{\frac{2(m_2-m_1)}{m_2+m_1} gH}$$

Calculate no Forces

No accelerations

No vectors

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