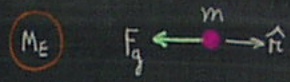
PE: Gravitational (General)

$\vec{F}_g = -\frac{G m M_E}{r^2}\hat{r}$ Earth-Object

$U(P) = -\int_{P_0}^{P} \vec{F}\cdot d\vec{r} + U(P_0)$

$U(r) = \int_{\infty}^{r} \frac{G m M_E \, dr'}{r'^2} + U(P_0)$



$U(r) = -\frac{G m M_E}{r'}\Big|_{\infty}^{r} + U(P_0)$

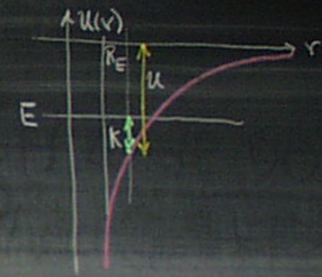
Let $U(P_0) = U(\infty) = 0$

$U(r) = -\frac{G m M_E}{r}$

$E = K + U$

$= \frac{1}{2} m v^2 - \frac{G m M_E}{r}$

Total ME is conserved!



2

PE: Spring Force

$U(P) = -\int_{P_0}^{P} \vec{F}\cdot d\vec{r} + U(P_0)$

$U(x=0) = 0$

$U(x) = -\int_{0}^{x} F_x(x')\,dx'$

$= -\int_{0}^{x} (-kx')\,dx'$

$U(x) = \frac{1}{2} k x^2$

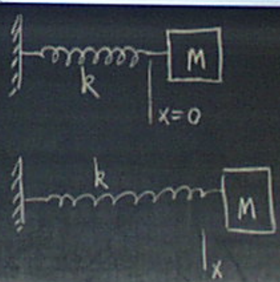
$E = K + U$

$= \frac{1}{2} m v^2 + \frac{1}{2} k x^2$

$=$ constant.

Conservative Force

Total ME is conserved!



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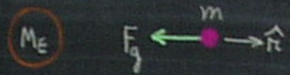
Lecture 15, Blackboard #1

PE Gravitational (General)

②

$\vec{F}_g = -\frac{GmM_E}{r^2}\hat{r}$ Earth-Object



$U(P) = -\int_{P_0}^{P} \vec{F}\cdot d\vec{r} + U(P_0)$

$U(r) = \int_{\infty}^{r} \frac{GmM_E}{r'^2}\,dr' + U(P_0)$

$U(r) = -\frac{GmM_E}{r'}\Big|_{\infty}^{r} + U(P_0)$

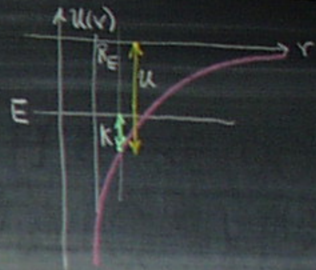
Let $U(P_0) = U(\infty) = 0$

$U(r) = -\frac{GmM_E}{r}$

$E = K + U$

$= \frac{1}{2}mv^2 - \frac{GmM_E}{r}$

Total ME is conserved!



Forces and Potential Energy

③

Force	$F(x)$	$U(x)$	X_0	$F(X_0)$
Gravity (near)	$mg\hat{j}$	mgy	$y=0$	$-mg\hat{j}$
Gravity (far)	$-\frac{GmM_E}{r^2}\hat{r}$	$-\frac{GmM_E}{r}$	$r=\infty$	0
Spring	$-kx\,\hat{i}$	$\frac{1}{2}kx^2$	$x=0$	0

Non-Cons Forces

$E_2 - E_1 = W_{Friction}$

$\Delta K + \Delta U = W_{Friction}$

$E_1 = K_1 + U_1$

$E_2 = K_2 + U_2$

$\left(\frac{1}{2}mv_2^2 + U_2\right) - \left(\frac{1}{2}mv_1^2 + U_1\right) = \int_{x_1}^{x_2} f\,dx$

Superposition

Several Forces: $\vec{F}_1, \vec{F}_2, \vec{F}_3$, etc

$W_{Total} = \int \vec{F}_1\cdot d\vec{r} + \int \vec{F}_2\cdot d\vec{r} + \int \vec{F}_3\cdot d\vec{r}$

$W_{Total} = \int \vec{F}_R\cdot d\vec{r}$ $\vec{F}_R \equiv$ Resultant Force.

$U_{Total} = U_1 + U_2 + U_3 + \cdots$

$\therefore K_i + \sum U_i = K_f + \sum U_f$ Total ME Conserved!

Lecture 15, Blackboard #2

Example Simple Pendulum

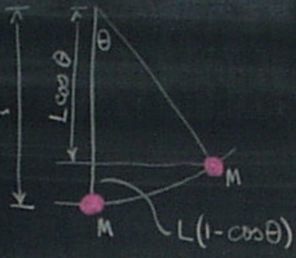
$U(\theta) = mgL(1-\cos\theta)$

$E = K + U$

$= \frac{1}{2}mv^2 + mgL(1-\cos\theta)$

For max angle $\theta = \theta_0 \Rightarrow v = 0$

$\therefore E_0 = mgL(1-\cos\theta_0)$



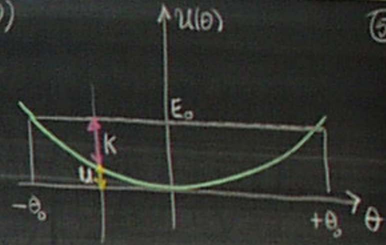
$\therefore mgL(1-\cos\theta_0) = \frac{1}{2}mv^2 + mgL(1-\cos\theta)$

$v^2 = 2gL(\cos\theta - \cos\theta_0)$

At bottom $\theta = 0$

$v_B = \sqrt{2gL(1-\cos\theta_0)}$

At $\theta = \theta_0$ $v \equiv 0$!



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Example

Drop block M from height H.

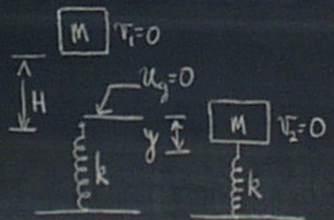
Q: What is max compression of spring?

All forces cons. (gravity + spring)

$E = K + U =$ conserved.

Pos 1: $v_1 = 0$; $K_1 = 0$

Pos 2: $v_2 = 0$; $K_2 = 0$



$K_1 + U_1 = K_2 + U_2$

$0 + mgH = 0 - mgy + \frac{1}{2}ky^2$

$y = \frac{1}{2}\left[\frac{2mg}{k} \pm \left(\frac{2mg}{k}\right)^2 + \frac{8mgh}{k}\right]$

???

4

5

Example: Simple Pendulum

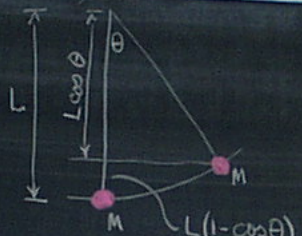
$U(\theta) = mgL(1-\cos\theta)$

$E = K + U$

$= \frac{1}{2}mv^2 + mgL(1-\cos\theta)$

For max angle $\theta = \theta_0 \Rightarrow v = 0$

$\therefore E_0 = mgL(1-\cos\theta_0)$



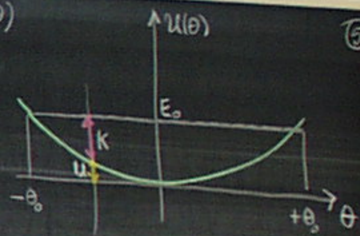
$\therefore mgL(1-\cos\theta_0) = \frac{1}{2}mv^2 + mgL(1-\cos\theta)$

$v^2 = 2gL(\cos\theta - \cos\theta_0)$

At bottom $\theta = 0$

$v_B = \sqrt{2gL(1-\cos\theta_0)}$

At $\theta = \theta_0$ $v \equiv 0$!



Example: Block + Spring + Friction

Mass released at x_A with $v_A = 0$

Spring is stretched.

Mass moves to left.

What is velocity v_B at $x = x_B$?

Force of friction $f = \mu_k N = \mu_k mg$.

Work done by 'f': $W_f = \int_{x_A}^{x_B} \vec{f}\cdot d\vec{x} = \mu_k mg(x_B - x_A)$

Energy Cons with non-cons forces.

$\frac{1}{2}mv_B^2 + \frac{1}{2}kx_B^2 - \left[\cancel{\frac{1}{2}mv_A^2} + \frac{1}{2}kx_A^2\right] = W_f = \mu_k mg(x_B - x_A)$

$\frac{1}{2}mv_B^2 = \underbrace{\frac{1}{2}k(x_A^2 - x_B^2)}_{>0} + \underbrace{\mu_k mg(x_B - x_A)}_{<0}$

$x_A > x_B$

Example: Spring Force

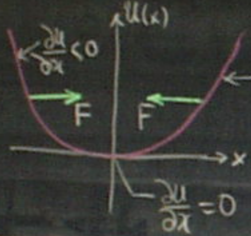
$\vec{F}_s = -kx\,\hat{x}$

$U(x) = \frac{1}{2}kx^2$

$F_x = -\frac{\partial U}{\partial x} = -kx$

$F_y = -\frac{\partial U}{\partial y} = 0$

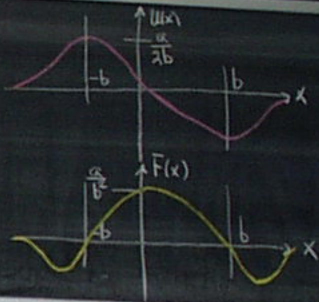
$F_z = -\frac{\partial U}{\partial z} = 0$



Example

$U(x) = \frac{-ax}{b^2+x^2}$

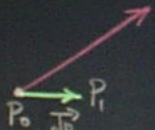
$F(x) = -\frac{\partial U}{\partial x} = \frac{a(b^2-x^2)}{(b^2+x^2)^2}$



⑧

Force ⟷ Potential Energy

$U(P_1) - U(P_0) = \int_{P_0}^{P_1} \vec{F}\cdot d\vec{r}$



Potential E → Force?

$dU = U(P_1) - U(P_0) = -\vec{F}\cdot d\vec{r}$

$= -F_x dx - F_y dy - F_z dz$

Assume $dy = 0,\ dz = 0$

Then $dU = -F\,dx$

$\therefore F_x = -\frac{dU}{dx}$ } Diff. $U(x,y,z)$ Keep y, z = constant.

Define: $\boxed{\text{Partial Derivatives}}$

$F_x = -\frac{\partial U}{\partial x} \quad F_y = -\frac{\partial U}{\partial y} \quad F_z = -\frac{\partial U}{\partial z}$

$\vec{F} = -\left[\frac{\partial U}{\partial x}\hat{i} + \frac{\partial U}{\partial x}\hat{j} + \frac{\partial U}{\partial z}\hat{k}\right] = -\vec{\nabla}U(x,y,z)$

$\vec{\nabla} = \left(\hat{i}\frac{\partial}{\partial x} + \hat{j}\frac{\partial}{\partial y} + \hat{k}\frac{\partial}{\partial z}\right)$

Gradient Vector Operator

⑦

Lecture 15, Blackboard #5

Example: Spring Force

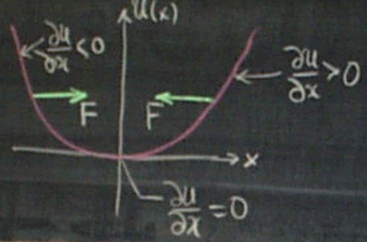
$\vec{F}_s = -kx\,\hat{x}$

$U(x) = \frac{1}{2}kx^2$

$F_x = -\frac{\partial U}{\partial x} = -kx$

$F_y = -\frac{\partial U}{\partial y} = 0$

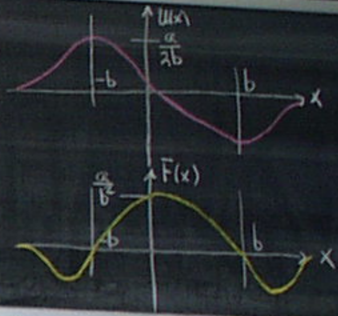
$F_z = -\frac{\partial U}{\partial z} = 0$



Example

$U(x) = \frac{-ax}{b^2+x^2}$

$F(x) = -\frac{\partial U}{\partial x} = \frac{a(b^2-x^2)}{(b^2+x^2)^2}$



Example

$U(x,y) = Ax^2y^2$

$F_x = -\frac{\partial U}{\partial x} = -2Axy^2$

$F_y = -\frac{\partial U}{\partial y} = -2Ax^2y$