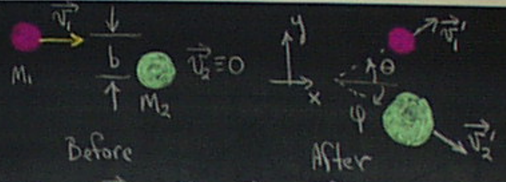
2D: Elastic Collisions

(1)

- One particle initially stationary. $\vec{v}_2 = 0$
- Any forces act along line of particles $\Rightarrow$ 2D
- Cons of $\vec{P}$ (x, y: 2-Eq's) } 4-Unknowns $(v_1', v_2', \theta, \varphi)$ } 3-Eq's
- Cons of E (1-Eq)



Cons of $\vec{P}$: $m_1\vec{v}_1 = m_1\vec{v}_1' + m_2\vec{v}_2'$

Cons of E: $\frac{1}{2}m_1 v_1^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2$

P_x: $m_1 v_1 = m_1 v_1'\cos\theta + m_2 v_2'\cos\varphi$

P_y: $0 = m_1 v_1'\sin\theta - m_2 v_2'\sin\varphi$

(2)

Special Case: $m_1 = m_2 = m$

$v_1^2 = v_1'^2 + v_2'^2$ (1) E-Cons

$v_2'\cos\varphi = v_1 - v_1'\cos\theta$ (2) P_x

$v_2'\sin\varphi = v_1'\sin\theta$ (3) P_y

$v_2'^2 = v_1^2 - 2v_1 v_1'\cos\theta + v_1'^2$ $(2)^2 + (3)^2$

Use (1) to eliminate v_2'

$v_1' = v_1\cos\theta$

$\frac{1}{2}m v_1'^2 = \frac{1}{2}m v_1^2\cos^2\theta$ KE of m_1 After Scattering

Cons of $\vec{P}$: $\vec{v}_1 = \vec{v}_1' + \vec{v}_2'$ (4)

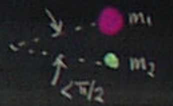
Sq (4): $v_1^2 = v_1'^2 + v_2'^2 + 2v_1'v_2'\cos(\theta+\varphi)$ (4')

Compare (4') with (1)

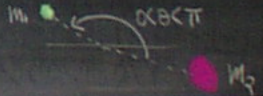
$\theta + \varphi = \pi/2$

Scattering angle between two equal masses $= 90^\circ$!!

If $m_1 > m_2$ $\theta < \pi/2$



If $m_1 < m_2$ $0 < \theta < \pi$



Lecture 20, Blackboard #1

Special Case: $m_1 = m_2 = m$

$v_1^2 = v_1'^2 + v_2'^2$ ① : E-Cons.

$v_2'\cos\varphi = v_1 - v_1'\cos\theta$ ② : P_x

$v_2'\sin\varphi = v_1'\sin\theta$ ③ : P_y

$v_2'^2 = v_1^2 - 2v_1 v_1'\cos\theta + v_1'^2$ ②^2 + ③^2

Use ① to eliminate v_2'

$v_1' = v_1\cos\theta$

$\frac{1}{2}m v_1'^2 = \frac{1}{2}m v_1^2\cos^2\theta$ KE of m_1 After Scattering

Cons of $\vec{P}$: $\vec{v}_1 = \vec{v}_1' + \vec{v}_2'$ ④

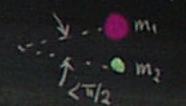
Sq ④ : $v_1^2 = v_1'^2 + v_2'^2 + 2v_1'v_2'\cos(\theta+\varphi)$ ④'

Compare ④' with ①

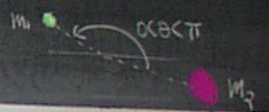
$\theta + \varphi = \pi/2$

Scattering angle between two equal masses $= \underline{90^\circ}$!!

If $m_1 > m_2$ $\theta < \pi/2$



If $m_1 < m_2$ $0 < \theta < \pi$



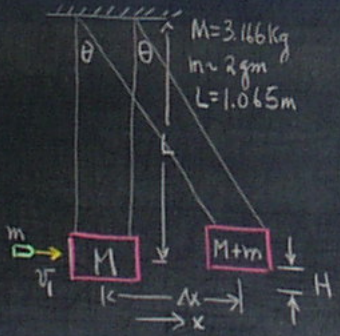
②

Ballistic Pendulum

Collision : $\vec{P}_{\text{Projectile}} \longrightarrow \vec{P}_{\text{Proj.+Block}}$

Totally Inelastic

Recoil : M+m move to a max height H.



Cons of $\vec{P}$: $m v_1 = (m+M)v_f$ Along-x ①

Cons of E : After collision $\frac{1}{2}(m+M)v_f^2 = (m+M)gH$

$\therefore v_f = \sqrt{2gH}$ ②

From ① $v_1 = \frac{(m+M)}{m}\sqrt{2gH}$

For $\theta \ll 1$

$\Delta x = L\sin\theta \simeq L\theta \Rightarrow \theta = \frac{\Delta x}{L}$

③

$H = L(1-\cos\theta)$

$\sim \frac{L\theta^2}{2} \sim \frac{(\Delta x)^2}{2L}$

$v_1 = \left(\frac{m+M}{m}\right)\sqrt{2g\left(\frac{\Delta x^2}{2L}\right)}$

$= \left(\frac{m+M}{m}\right)\sqrt{\frac{g}{L}}\,(\Delta x)$

$= \frac{2+3166}{2}\sqrt{\frac{9.81}{1.065}} \times$

Lecture 20 Blackboard #2

Kinetic Energies

$K_i = \frac{1}{2} m v_1^2$ bullet.

$K_f = \frac{1}{2}(m+M) v_f^2 = \frac{m^2}{2(m+M)} v_1^2$ Immed. after Collision

$\frac{K_f}{K_i} = \frac{m}{m+M} \sim \frac{2}{2+3166} \ll 1$

Most of KE is lost !!!

Collision Time

Assume uniform decelleration of bullet.

$v_f^2 = v_1^2 + 2aS$

$v_f \sim 0$

$S = 10cm$ (measured in can)

$a = -\frac{v_1^2}{2S} = \frac{-(300)^2}{2\times 0.1}\ m/s^2$

$v_f = v_1 + at$

$t = \frac{-v_1}{a} = \frac{300}{\left(\frac{300^2}{2\times 0.1}\right)} \sim 0.00067sec$ very short time.

Period of Pendulum:

$T = 2\pi\sqrt{\frac{L}{g}} = 2\pi\sqrt{\frac{1.06}{9.81}} = 2.14s$

④

Collisions Summary

1. Conditions: Δt = coll. Time
ΔT = Obsn. Time

$\Delta t \ll \Delta T$

$|I_{ext}| \ll |I_{coll}|$ Impulses.

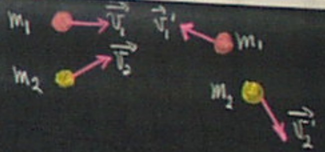
2. Classes of Collisions

Elastic: $\vec{P}$ Cons.
KE Cons.

Inelastic: $\vec{P}$ Cons.
KE NOT Cons

Comp. Inelastic: $\vec{P}$ Cons. Stick Together
$K_{INT} \equiv 0$

3. Notation



m_1, m_2 masses

$\vec{v}_1, \vec{v}_2$ initial vel's

$\vec{v}_1', \vec{v}_2'$ final vel's

$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1' + m_2\vec{v}_2'$ $\vec{P}$ Cons; All Coll.

$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2$ E Cons. Elastic Coll.

⑤

Lecture 20 Blackboard #3

Collisions Summary

1. Conditions: Δt = coll. Time
ΔT = Obsn. Time

$\Delta t \ll \Delta T$

$|I_{ext}| \ll |I_{coll}|$ Impulses.

2. Classes of Collisions

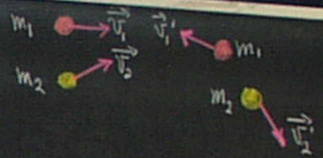
Elastic: $\vec{P}$ Cons.
KE Cons.

Inelastic: $\vec{P}$ Cons.
KE NOT Cons

Comp. Inelastic: $\vec{P}$ Cons. Stick Together
$K_{INT} \equiv 0$

3. Notation

m_1, m_2 masses
$\vec{v}_1, \vec{v}_2$ initial vel's
$\vec{v}_1{}', \vec{v}_2{}'$ final vel's



$m_1\vec{v}_1 + m_2\vec{v}_2 = m_1\vec{v}_1{}' + m_2\vec{v}_2{}'$ $\quad \vec{P}$ Cons; All Coll.

$\frac{1}{2}m_1 v_1^2 + \frac{1}{2}m_2 v_2^2 = \frac{1}{2}m_1 v_1'^2 + \frac{1}{2}m_2 v_2'^2$ $\quad$ E Cons. Elastic Coll.

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Center-of-Mass Frame Collisions

2-Particles

$$\vec{r}_{cm} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2}{m_1+m_2}$$

$$\vec{V}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \frac{d\vec{r}_{cm}}{dt}$$

$\vec{v}_1, \vec{v}_2$: velocities in Lab Frame
$\vec{V}_{cm}$: velocity of cm rel Lab
$\vec{u}_1, \vec{u}_2$: velocities in CM Frame

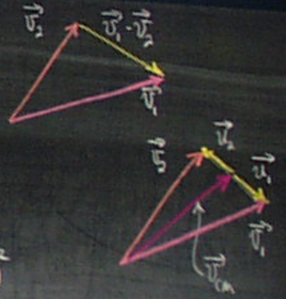
$$K = \frac{1}{2}MV_{cm}^2 + K\ INT$$
$$= \frac{1}{2}MV_{cm}^2 + \underbrace{\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2}_{\text{Int. KE}}$$

$$\vec{u}_1 = \vec{v}_1 - \vec{V}_{cm} = \vec{v}_1 - \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2} = \frac{m_2(\vec{v}_1 - \vec{v}_2)}{m_1+m_2}$$

$$\vec{u}_2 = \vec{v}_2 - \vec{V}_{cm} = \vec{v}_2 - \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1+m_2}$$
$$= \frac{-m_1(\vec{v}_1 - \vec{v}_2)}{m_1+m_2}$$

$$\therefore K = \frac{1}{2}MV_{cm}^2 + \frac{1}{2}\frac{m_1m_2}{m_1+m_2}(\vec{v}_1 - \vec{v}_2)^2$$

$M = m_1 + m_2$



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Lecture 20 Blackboard #4

2nd Term K = max energy available for a totally inelastic collision!

Let $\vec{v}_{rel} = \vec{u}_1 - \vec{u}_2 = \vec{v}_1 - \vec{v}_2$ rel. vel.

$$K = \frac{1}{2} M v_{cm}^2 + \frac{1}{2}\mu v_{rel}^2$$

$\mu = \frac{m_1 m_2}{m_1 + m_2}$ reduced mass of system.

Momenta in CM Frame

$$\vec{P}_{1cm} = m_1 \vec{u}_1 = \frac{m_1 m_2}{m_1+m_2}(\vec{v}_1 - \vec{v}_2) = \mu \vec{v}_{rel}$$

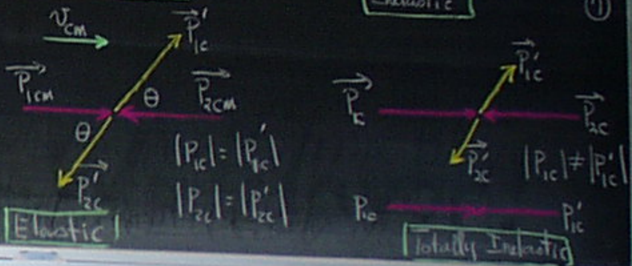
$$\vec{P}_{2cm} = m_2 \vec{u}_2 = \frac{-m_1 m_2}{m_1+m_2}(\vec{v}_1 - \vec{v}_2) = -\mu \vec{v}_{rel}$$

$$\text{Total } \vec{P}_{cm} = \vec{P}_{1cm} + \vec{P}_{2cm} = 0$$

$$\vec{P}_{Lab} = \vec{P}_{1L} + \vec{P}_{2L} = (m_1+m_2)\vec{v}_{cm} \neq 0$$

CM Collisions

⑦



$|P_{1c}| = |P'_{1c}|$

$|P_{2c}| = |P'_{2c}|$

$|P_{1c}| \neq |P'_{1c}|$

Example: Proton (m) + Deuteron (2m) Collisions

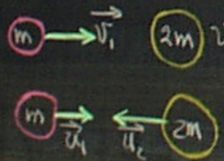
⑧

$m_2 = 2m_1$

$v_2 = 0$ [Target at rest initially]

Lab: m $\vec{v}_1$ 2m $v_2 = 0$

CM: m $\vec{u}_1$ $\vec{u}_2$ 2m



$$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + 2m(0)}{3m} = \frac{1}{3}\vec{v}_1$$

$$\vec{u}_1 = \frac{2}{3}\vec{v}_1 \qquad \vec{u}_2 = -\frac{1}{3}\vec{v}_1$$

$$\vec{P}_{T\,cm} = m_1 \vec{u}_1 + 2m\vec{u}_2 = m\frac{2}{3}\vec{v}_1 - 2m\frac{1}{3}\vec{v}_1 \equiv 0.$$

LAB. $K^L = \frac{1}{2} m v_1^2 = K_{cm} + K^{CM}_{INT}$

$$K_{cm} = \frac{1}{2} M v_{cm}^2 = \frac{1}{2}(m+2m)\left(\frac{1}{3}v_1\right)^2 = \frac{1}{3}\left(\frac{1}{2}m v_1^2\right) = \frac{1}{3}K^L$$

$$K^{CM}_{INT} = \frac{1}{2}m\left(\frac{2}{3}v_1\right)^2 + \frac{1}{2}(2m)\left(-\frac{1}{3}v_1\right)^2 = \frac{2}{3}\left(\frac{1}{2}m v_1^2\right) = \frac{2}{3}K^L$$

$$\therefore K^L = K^{CM} + K^{CM}_{INT}$$

Lecture 20 Blackboard #5

Example: Proton (m) + Deuteron (2m) Collisions

⑧

$m_2 = 2m_1$

$v_2 = 0$ [Target at rest initially]

Lab: (m) $\rightarrow \vec{v}_1$ (2m) $v_2 = 0$

CM: (m) $\rightarrow \vec{u}_1$ $\leftarrow \vec{u}_2$ (2m)

$$\vec{v}_{cm} = \frac{m_1 \vec{v}_1 + 2m(0)}{3m} = \frac{1}{3}\vec{v}_1$$

$$\vec{u}_1 = \frac{2}{3}\vec{v}_1 \qquad \vec{u}_2 = -\frac{1}{3}\vec{v}_1$$

$$\vec{P}_{T\,cm} = m_1\vec{u}_1 + 2m\vec{u}_2 = m\frac{2}{3}\vec{v}_1 - 2m\frac{1}{3}\vec{v}_1 \equiv 0.$$

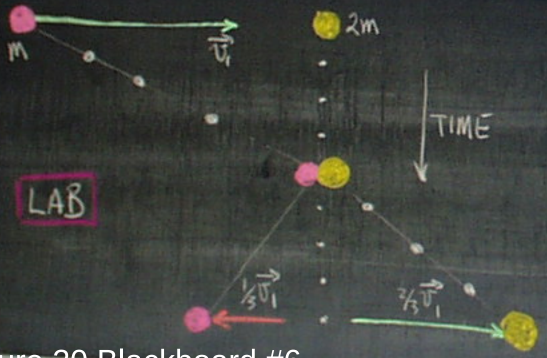
LAB: $K^L = \frac{1}{2} m v_1^2 = K_{cm} + K_{INT}^{CM}$

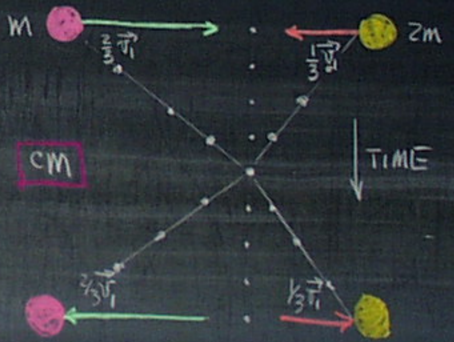
$$K_{cm} = \frac{1}{2} M v_{cm}^2 = \frac{1}{2}(m+2m)\left(\frac{1}{3}v_1\right)^2 = \frac{1}{3}\left(\frac{1}{2} m v_1^2\right) = \frac{1}{3}K^L$$

$$K_{INT}^{cm} = \frac{1}{2} m\left(\frac{2}{3}v_1\right)^2 + \frac{1}{2}(2m)\left(-\frac{1}{3}v_1\right)^2 = \frac{2}{3}\left(\frac{1}{2} m v_1^2\right) = \frac{2}{3}K^L$$

$$\therefore K^L = K^{cm} + K_{INT}^{CM}$$

⑨





Lecture 20 Blackboard #6

Lecture 20 Blackboard #7