Time Dilation

Classical: Time absolute
Length absolute

Special Relativity:

Abandon absolute T, L

Abandon simultaneity

Light Clock

- laser emits pulse
- mirror reflects pulse
- detector counts pulse

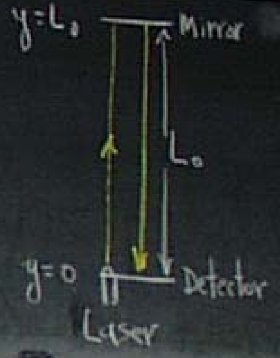
System-S′

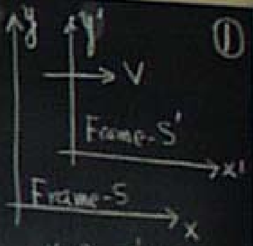
· Relative Velocity, V

· Pulse emitted $x_1'=0$, $y_1'=0$, $z_1'=0$, $t_1'=0$

· Pulse reflected
$x_2'=0$
$y_2'=L_0$
$z_2'=0$
$t_2'=L_0/c$

· Pulse detected.
$x_3'=y_3'=z_3'=0$
$t_3'=2L_0/c$



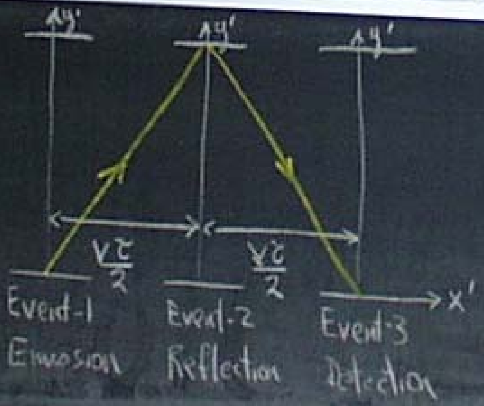


$x=0$ $x'=0$
$t=0$ $t'=0$

①

System-S

· Observes experiment in S′
· As light travels, S′ moves to right, velocity V.
· Light follows triangular path.



· Pulse emitted:
$x_1=0$ $y_1=0$ $z_1=0$
$t_1=0$

· Clock moves to right a distance $v\tau/2$

τ = total travel time in S!

· Pulse reflected:
$x_2=\frac{v\tau}{2}$ $y_2=L_0$ $z_2=0$
$t_2=\tau/2$.

· Clock continues to move

· Pulse detected
$x_3=v\tau$ $y_3=0$ $z_3=0$
$t_3=\tau$

Path length of light in S along 1-arm:

$$\sqrt{L_0^2+\left(\frac{v\tau}{2}\right)^2}$$

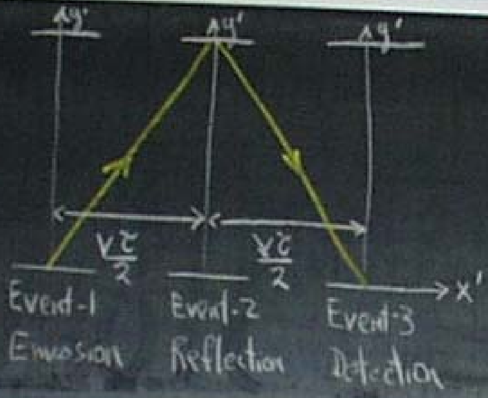
Light travels this distance with speed c.

②

Lecture 34 Blackboard #1

System-S

- Observes experiment in S'
- As light travels, S' moves to right, velocity V.
- Light follows triangular path.



- Pulse emitted:

$x_1=0 \quad y_1=0 \quad z_1=0$

$t_1=0$

- Clock moves to right a distance $v\tau/2$

τ = total travel time in S!

- Pulse reflected:

$x_2=\frac{v\tau}{2} \quad y_2=L_0 \quad z_2=0$

$t_2=\tau/2$.

- Clock continues to move
- Pulse detected

$x_3=v\tau \quad y_3=0 \quad z_3=0$

$t_3=\tau$

Path length of light in S along 1-arm:

$$\sqrt{L_0^2+\left(\frac{v\tau}{2}\right)^2}$$

Light travels this distance with speed c.

(2)

$$\therefore \frac{\tau}{2}=\frac{\sqrt{L_0^2+\left(\frac{v\tau}{2}\right)^2}}{c}$$

$$\frac{\tau^2}{4}=\frac{L_0^2+\left(\frac{v\tau}{2}\right)^2}{c^2}$$

Solve for τ:

$$\tau^2=\frac{4L_0^2/c^2}{1-v^2/c^2}$$

$$\tau=\frac{2L_0/c}{\sqrt{1-v^2/c^2}}=\gamma\tau_0$$

$\tau_0=\frac{2L_0}{c}$ = Proper Time

τ_0 = Time measured in S' in which clock is at rest!!

τ = time measured in S using different clocks along x-axis.

$\tau > \tau_0$ Longer Time Int.

Moving clock ticks more slowly than a clock at rest.

$\Longrightarrow$ Time Dilation.

Time Dilation – Lor. Fitz.

- Clock at rest in S'
- Two events, A, B.

$$\left.\begin{matrix} A: & x_0' & t_A' \\ B: & x_0' & t_B' \end{matrix}\right\} \tau_0 = t_B' - t_A'$$

Proper Time in Rest System.

Use $t=\gamma(t'+x'v/c^2)$

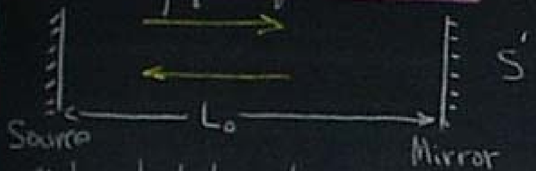
$t_A=\gamma(t_A'+vx_0'/c^2) \quad t_B=\gamma(t_B'+vx_0'/c^2)$

$T=t_B-t_A=\gamma(t_B'-t_A')=\gamma\tau_0$ Time Dilation!!

(3)

Lecture 34 Blackboard #2

Relativity of Length: Parallel



· Meter stick, L_0, at rest in S'

· Light pulse makes round trip.

$\Delta t_0 = \frac{2L_0}{c}$ Proper Time.

S-Frame: Ruler moves to right with velocity V. Length is L.

Time to travel distance in S is Δt_1

During this time mirror moves distance $v\Delta t_1$

Total path in S is

$d = L + v\Delta t_1$

Light travels with speed c

$\therefore d = c\Delta t_1$

$c\Delta t_1 = L + v\Delta t_1$

$\Delta t_1 = \frac{L}{c-v}$

Note: Light does not travel with speed $(c-v)$!! It travels a distance longer than L.

④

Return trip:

$\Delta t_2 = \frac{L}{c+v}$: Detector approaches light beam. Distance shorter.

Total Time:

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2L}{c\left[1 - v^2/c^2\right]}$$

$$\frac{\Delta t}{\gamma} = \Delta t_0 = \frac{2L_0}{c}$$ From Time Dilation

Eliminate Δt:

$L = L_0\sqrt{1 - v^2/c^2}$

$L = L_0/\gamma$

L measured in S is shorter than proper length L_0 in S'! Length Contraction

Length Contraction: Lor-Fitz

· Ruler at rest in S' (x'_A, x'_B)

· Length $L_0 = x'_B - x'_A$ Proper length

S' → Moves to right with V.

· Measure in S. Mark both ends at t.

$x'_B = \gamma(x_B - vt)$

$x'_A = \gamma(x_A - vt)$

$x'_B - x'_A = \gamma(x_B - x_A)$

$L_0 = \gamma L$

$L = L_0/\gamma$

$L < L_0$ Lorentz Contraction.

⑤

is longer !!

Lecture 34 Blackboard #3

Return trip

$\Delta t_2 = \frac{L}{c+v}$: Detector approaches light beam. Distance shorter

Total Time:

$$\Delta t = \Delta t_1 + \Delta t_2 = \frac{L}{c-v} + \frac{L}{c+v} = \frac{2L}{c\left[1 - v^2/c^2\right]}$$

$$\frac{\Delta t}{\gamma} = \Delta t_0 = \frac{2L_0}{c}$$ From Time Dilation

Eliminate Δt:

$$L = L_0\sqrt{1 - v^2/c^2}$$

$$L = L_0/\gamma$$

L measured in S is shorter than proper length L_0 in S'!

Length Contraction

Length Contraction: Lor-Fitz

- Ruler at rest in S' (x'_A, x'_B)
- Length $L_0 = x'_B - x'_A$ Proper length

$S' \rightarrow$ Moves to right with v.

- Measure in S. Mark both ends at t.

$$x'_B = \gamma(x_B - vt)$$

$$x'_A = \gamma(x_A - vt)$$

$$x'_B - x'_A = \gamma(x_B - x_A)$$

$$L_0 = \gamma L$$

$$L = L_0/\gamma$$

$L < L_0$ Lorentz Contraction.

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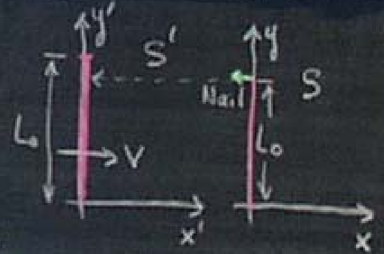
Relativity of Length: Transverse

- In Time Dilation used L_0 equal in both frames. Correct??
- Two identical meter sticks: L_0
- Place along y-axis in S and S'
- S' moves rel to S.
- Sharp nail on each stick.
- Sticks pass each other
- Assume moving stick ⊥ motion is longer!!

S-Frame

- Stick in S' longer.
- Nail in S makes mark on S'.
- Nail in S' misses S
- Stick in S' has a scratch
- Stick in S no scratch.

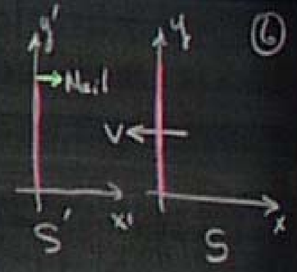


S'-Frame

- Stick in S moving and is longer
- S'-stick makes mark in S
- Nail on S misses S'
- Stick in S has mark
- Stick in S' no mark.

Results communicated Contradictory!!

Conclusion: Lengths measured ⊥ direction of motion are unchanged by motion.



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Lecture 34 Blackboard #4

Orietation of Moving Rod

S'-Frame - Velocity V

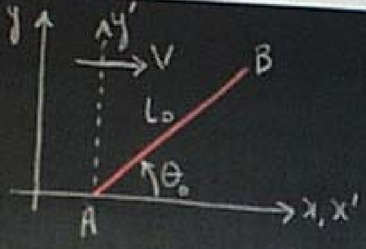
Rod length L_0

Angle θ_0 wrt x-axis

Ends of rod

A: $x'_A = 0 \quad y'_A = 0$

B: $x'_B = L_0 \cos\theta_0 \quad y'_B = L_0 \sin\theta_0$



S-Frame

$x'_A = 0 = \gamma(x_A - vt)$

$y'_A = 0 = y_A$

$x'_B = L_0 \cos\theta_0 = \gamma(x_B - vt)$

$y'_B = L_0 \sin\theta_0 = y_B$

$x_B - x_A = \dfrac{L_0}{\gamma}\cos\theta_0$

$y_B - y_A = L_0 \sin\theta_0$

The length in S is

$L = \sqrt{(x_B - x_A)^2 + (y_B - y_A)^2}$

$= L_0\left[\left(1 - \frac{v^2}{c^2}\right)\cos^2\theta_0 + \sin^2\theta_0\right]^{1/2}$

$L = L_0\left[1 - \frac{v^2}{c^2}\cos^2\theta_0\right]^{1/2}$

(7)

Angle of rod in S

$\theta = \arctan \dfrac{y_B - y_A}{x_B - x_A}$

$= \arctan \gamma \left(\dfrac{\sin\theta_0}{\cos\theta_0}\right)$

$= \arctan(\gamma \tan\theta_0)$

$\theta > \theta_0$

Rod is contracted and rotated.

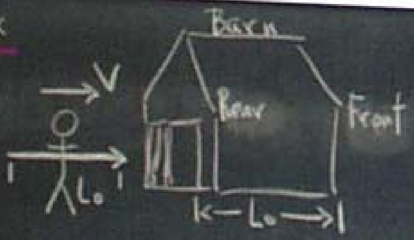
Pole-Vaulter Paradox

· Pole length L_0

· Barn length L_0

· Pole-Vaulter speed

$v = \frac{\sqrt{3}}{2} c \quad \gamma = 2.$



Farmer sees pole length $L_0/2$: Fits in barn.

Pole vaulter sees barn length $\frac{L_0}{2}$. Does not fit!!

Farmer

· Close both doors at $t=0$

· Pole momentarily in barn.

$\left.\begin{array}{l} x_R = 0 \\ x_F = L_0/\gamma \end{array}\right\} t = 0$

Pole-Vaulter

$x'_R = 0$

$x'_F = L_0$

When do doors close in S'?

$t'_R = \gamma\left(0 - \frac{v0}{c^2}\right) = 0$

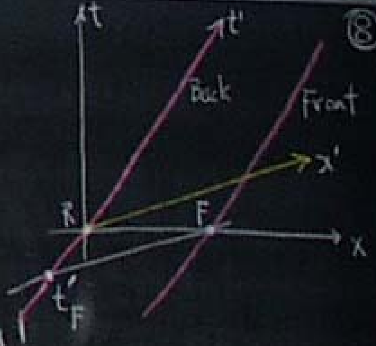
$t'_F = \gamma\left(0 - \frac{v}{c^2} L_0\right) = -\frac{\gamma v L_0}{c^2}$

Front open before rear closes.

Events not simultaneous in S'

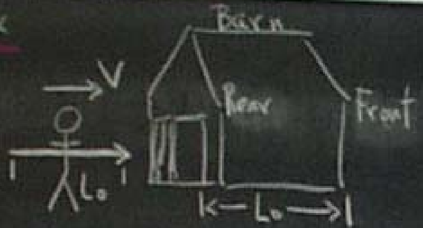
He sees shorter barn and runs through!

(8)



Pole-Vaulter Paradox

- Pole length L_0
- Barn length L_0
- Pole-Vaulter speed $v = \frac{\sqrt{3}}{2} c \quad \gamma = 2$.



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Farmer

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$$\left.\begin{array}{l} x_R = 0 \\ x_F = L_0/\gamma \end{array}\right\} t=0$$

Pole-Vaulter

$$x'_R = 0$$
$$x'_F = L_0$$

When do doors close in S'?

$$t'_R = \gamma\left(0 - \frac{v0}{c^2}\right) = 0$$

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Front open before rear closes.

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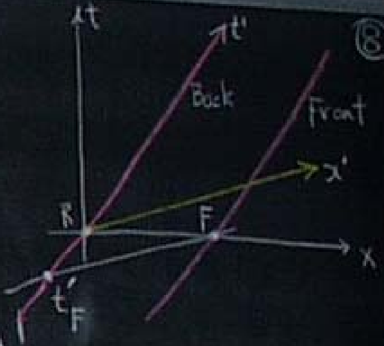
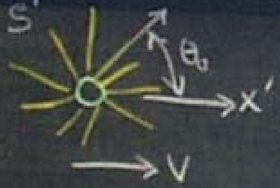


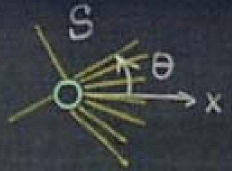
8

Headlight Effect

- Light beam in S' emitted at angle θ_0 wrt x'. What is θ in S?
- S' – After 1-sec.

$$x' = c\cos\theta_0$$
$$y' = c\sin\theta_0$$





S-Frame:

$$x = \gamma(x' + vt') = \gamma(c\cos\theta_0 + v)$$
$$y = y'$$
$$t = \gamma\left(t' + \frac{vx'}{c^2}\right) = \gamma\left(1 + \frac{v}{c}\cos\theta_0\right)$$

But in S

$$\cos\theta = \frac{x}{ct} = \frac{\gamma(c\cos\theta_0 + v)}{\gamma(c + v\cos\theta_0)} = \frac{\cos\theta_0 + v/c}{1 + \frac{v}{c}\cos\theta_0}$$

Assume in S' rays uniform

Half-light within $\theta_0 = \pm\pi/2$.

For $\theta_0 = \frac{\pi}{2} \Longrightarrow \cos\theta = v/c$

As $v \to c$ $\cos\theta \to 1$

$\therefore \theta \sim 0°$.

Radiation is very forward peaked.

$\beta = 0.9 \quad \theta = 25.8°$

$\beta = 0.99 \quad \theta = 8.1°$

9

Lecture 34 Blackboard #6