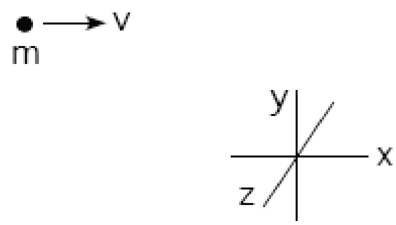
Angular Momentum
Concept Questions

Question 1: Angular Momentum



In the above situation where a particle is moving in the x-y plane with a constant velocity, the magnitude of the angular momentum $\left|\vec{\mathbf{L}}_0\right|$ about the origin

1. decreases then increases,
2. increases then decrease,
3. is constant,
4. is zero because this is not circular motion.

Solution 3. As the particle moves in the positive x-direction, the perpendicular distance from the origin to the line of motion does not change and so the magnitude of the angular momentum about the origin is constant. Recall that for the motion of the particle $x(t)$ varies with time but (y,z) are fixed. The angular momentum about the origin is

$$\vec{\mathbf{L}}_0 = (x(t)\;\hat{\mathbf{i}} + y\;\hat{\mathbf{j}} + z\hat{\mathbf{k}}) \times mv\hat{\mathbf{i}} = -ymv\;\hat{\mathbf{k}} + zmv\hat{\mathbf{j}}$$

which is constant. In the above, the relations

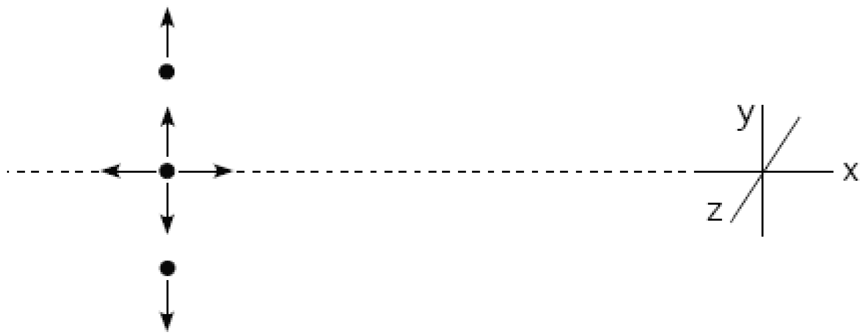
$$\vec{\mathbf{i}} \times \vec{\mathbf{j}} = \vec{\mathbf{k}},\;\; \vec{\mathbf{j}} \times \vec{\mathbf{i}} = -\vec{\mathbf{k}},\;\; \vec{\mathbf{i}} \times \vec{\mathbf{i}} = \vec{\mathbf{j}} \times \vec{\mathbf{j}} = \vec{\mathbf{0}}$$

were used. Note that the magnitude of $\left|\vec{\mathbf{L}}_0\right|$ is given by

$$\left|\vec{\mathbf{L}}_0\right| = ((ymv)^2 + (zmv)^2)^{1/2} = mv(y^2 + z^2)^{1/2} = mvr_\perp$$

where $r_\perp = (y^2 + z^2)^{1/2}$ is constant.

Question 2: Angular Momentum



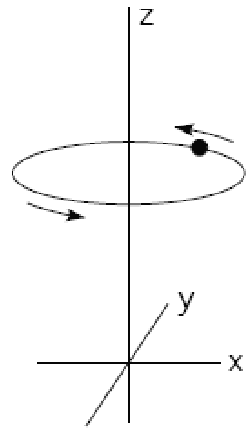
The diagram above shows six possible combinations of position and velocity for a particle of mass m and speed v moving in the x-y plane. How many distinct values of the angular momentum $\vec{\mathbf{L}}_0$ relative to the origin does this represent?

1) 1
2) 2
3) 3
4) 4
5) 5
6) 6

Answer 3. When the particle is moving towards or directly away from the origin the angular momentum about the origin is zero. The two upward pointing velocities have the same moment arm about the origin so they have the same angular momentum about the origin which points in the negative z-direction. The two downward pointing velocities have the same moment arm about the origin so they have the same angular momentum about the origin which points in the positive z-direction. So there are three distinct values of the angular momentum $\vec{\mathbf{L}}_0$ relative to the origin.

Question 3: Angular Momentum

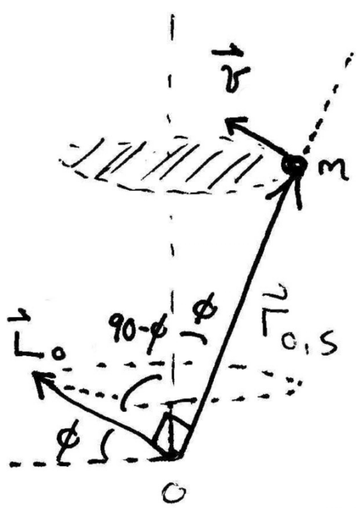
A particle of mass m moves in a circle of radius R at an angular speed ω about the z axis in a plane parallel to but above the x-y plane. Relative to the origin

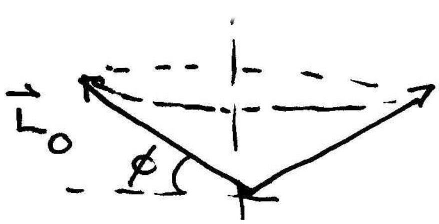


1. $\vec{\mathbf{L}}_0$ is constant.
2. $\left|\vec{\mathbf{L}}_0\right|$ is constant but $\vec{\mathbf{L}}_0 / \left|\vec{\mathbf{L}}_0\right|$ is not.
3. $\vec{\mathbf{L}}_0 / \left|\vec{\mathbf{L}}_0\right|$ is constant but $\left|\vec{\mathbf{L}}_0\right|$ is not.
4. $\vec{\mathbf{L}}_0$ has no z-component. .

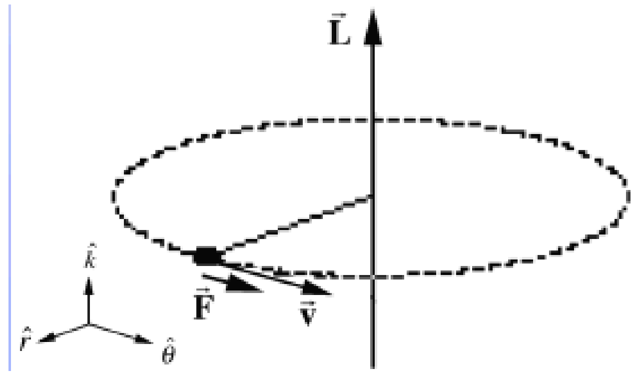
Answer 2.

The vector $\vec{\mathbf{L}}_0 / \left|\vec{\mathbf{L}}_0\right|$ represents a unit vector in the direction of $\vec{\mathbf{L}}_0$. The angular momentum about the origin is shown in the figure to the right (see Table Problem 2 W09D2-1). The magnitude $\left|\vec{\mathbf{L}}_0\right|$ is constant. As the particle moves in a circle, the angular momentum sweeps out a cone (shown in the figure below) and so the direction of $\vec{\mathbf{L}}_0$ is changing and hence $\vec{\mathbf{L}}_0 / \left|\vec{\mathbf{L}}_0\right|$ is not a constant unit vector.





Question 4 Change in Angular Momentum: A person spins a tennis ball on a string in a horizontal circle with velocity $\vec{\mathbf{v}}$ (so that the axis of rotation is vertical). At the point indicated below, the ball is given a sharp blow (force $\vec{\mathbf{F}}$) in the forward direction.



This causes a change in angular momentum $\Delta\vec{\mathbf{L}}$ about the center of the circle in the

1. $\hat{\mathbf{r}}$ - direction
2. $\hat{\boldsymbol{\theta}}$ - direction
3. $\hat{\mathbf{k}}$ -direction

Answer 3. The torque about the center of the circle points in the $\hat{\mathbf{k}}$ -direction since

$$\vec{\tau}_0 = \vec{\mathbf{r}}_{0,F} \times \vec{\mathbf{F}} = r\hat{\mathbf{r}} \times F\hat{\mathbf{e}} = rF\hat{\mathbf{k}}$$

The change in the angular momentum about the center of the circle is proportional to the angular impulse about the center of the circle

$$\int \vec{\tau}_0 dt = \int rFdt\hat{\mathbf{k}} = \Delta\vec{\mathbf{L}}$$

Therefore the change in angular momentum $\Delta\vec{\mathbf{L}}$ about the center of the circle is in the $\hat{\mathbf{k}}$ -direction.