## Circular Motion Dynamics

## Strategy: Circular Orbits

i) Understand geometry
ii) From geometry determine acceleration
iii) Find combination of forces that give acceleration

## Strategy: <br> Applying Newton's Second Law for Circular Motion

- Always has a component of acceleration pointing radially inward
- May or may not have tangential component of acceleration
- Draw Free Body Diagram for all forces
- $\mathrm{mv}^{2} / \mathrm{r}$ is not a force but mass times acceleration and does not appear on force diagram
- Choose a sign convention for radial direction and check that signs for forces and acceleration are consistent


## Strategy: Applying Newton's Second Law for Circular Motion

- Always has a component of acceleration pointing radially inward
- May or may not have tangential component of acceleration
- Draw Free Body Diagram for all forces
- $m v^{2} / r$ is not a force but mass times acceleration and does not appear on force diagram
- Choose a sign convention for radial direction and check that signs for forces and acceleration are consistent


## Modeling Problems: Circular Motion

Always has a component of acceleration pointing radially inward

May or may not have tangential component of acceleration

Draw Free Body Diagram for all forces
$\mathrm{mv}^{2} / \mathrm{r}$ is not a force but mass times acceleration and does not appear on force diagram

Choose a sign convention for radial direction and check that signs for forces and acceleration are consistent

## Newton's Second Law: Equations of Motion for Circular Motion


$\hat{\mathbf{r}}$ : force $=-m v^{2} / r$ decomposition
$\hat{\boldsymbol{\theta}}: \quad$ from $\quad=m r d^{2} \theta / d t^{2}$
force diagram
$\hat{\mathbf{k}}$ :
$=0$

## Checkpoint Problem: horizontal circular motion no gravity

A point-like object of mass $m$ is attached to the end of a string and rotated in a circle of radius R in a horizontal plane with angular speed $\omega$. Assume that the string is massless and you may ignore the effect of gravitation. What is the tension in the string?


## Checkpoint Problem: Ball and spring

One end of a spring is attached to the central axis of a motor. The axis of the motor is in the vertical direction. A small ball of mass $m_{2}$ is then attached to the other end of the spring. The motor rotates at a constant frequency $f$. Neglect the gravitational force exerted on the ball. Assume that the ball and spring rotate in a horizontal plane. The spring constant is $k$. Let $r_{0}$ denote the unstretched length of the spring.

(i) How long does it take the ball to complete one rotation?
(ii) What is the angular frequency of the ball in radians per sec?
(iii) What is the radius of the circular motion of the ball?

## Checkpoint Problem: vertical circular motion

A point-like object of mass $m$ is attached to the end of a string of length $R$ and rotated in a vertical plane. How fast must the object move at the top of its orbit in order not to depart from a circular trajectory? For faster speeds, what is the tension in the string when the object is at the top and bottom of its trajectory? Assume that the string is massless and that gravity is acting on the object with constant g .


## Checkpoint Problem: Rotating Bucket

A pail of mass $m_{p}$ is full of water (mass $m_{w}$ ). A string is attached to the handle of the pail which is then whirled around a vertical circle at constant speed v. You may assume that the center of mass of the bucket and the water undergoes circular motion with radius $R$. What is the minimum speed that the pail must have at the top of its circular motion if the water is not to spill out of the upside-down pail? For faster speeds, find the tension in the string and the magnitude of the contact force between the water and the bucket.


## Checkpoint Problem: Bead inside surface of cone

A body of mass $m$ slides without friction on the inside of a cone. The axis of the cone is vertical and gravity is directed downwards. The apex half-angle of the cone is $\theta$ as shown in the figure. The path of the object happens to be a circle in a horizontal plane. The speed of the particle is $v_{0}$. Find the radius of the circular path and the time it takes to complete one circular orbit in terms
 of the given quantities and $g$.

## Demo: Rotating Bucket B104

http://tsgphysics.mit.edu/front/index.php?page= demo.php?letnum=B\%20104\&show=0

A bucket of balls is spun in a circle. No ball spills out.

## Demo: Centripetal vs. Centripetal

 B1021. http://tsgphysics.mit.edu/front/index.php?page=demo.php?। etnum=B\%20102\&show=0

A wooden ball is attached to the rim of a spinning wheel. The ball is held in place by a string. When the string is cut, the ball flies in a straight tangent to the wheel.

MIT OpenCourseWare
http://ocw.mit.edu

### 8.01SC Physics I: Classical Mechanics

For information about citing these materials or our Terms of Use, visit: http://ocw.mit.edu/terms.

