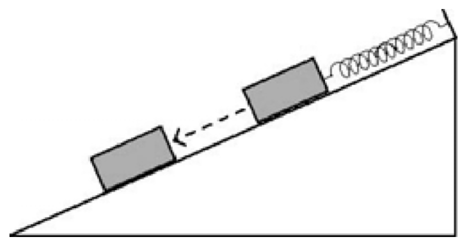
Conservation of Energy
Concept Questions

Question 1: A block of inertia m is attached to a relaxed spring on an inclined plane. The block is allowed to slide down the incline, and comes to rest. The coefficient of kinetic friction of the block on the incline is μ_k. For which definition of the system is the change in total energy (after the block is released) zero?



1. block
2. block + spring
3. block + spring + incline
4. block + spring + incline + Earth

Answer: 4 . Since there is dissipative forces acting between the block and the incline plane some gravitational potential energy is dissipated into the block and incline plane increasing the thermal energy of those objects.

Question 2: (Note the slight notation change, using $h_{\max}$ instead of h_m.)

A spring-loaded toy dart gun is used to shoot a dart straight up in the air, and the dart reaches a maximum height $h_{\max}$. The same dart is shot straight up a second time from the same gun, but this time the spring is compressed only half as far before firing. How far up does the dart go this time, neglecting friction and assuming an ideal spring?

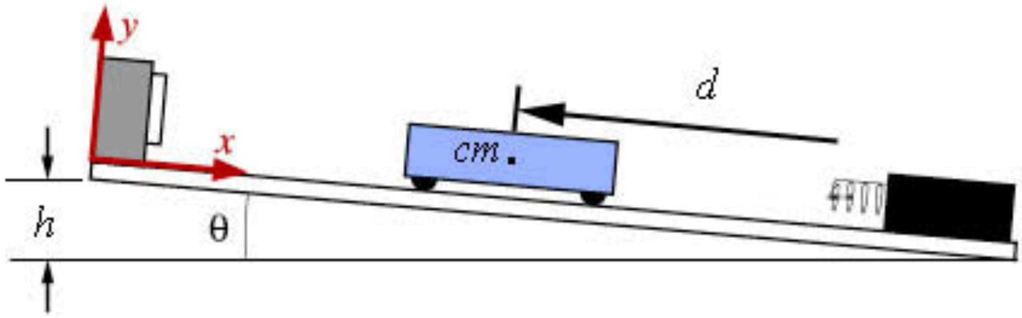
a) $h_{\max}$
b) $h_{\max}/2$
c) $h_{\max}/4$
d) $2h_{\max}$
e) $4h_{\max}$
f) The dart escapes to infinity.

Question 2 Solution:

Answer c) is correct. This problem is most simply solved using energy considerations. Specifically, if the spring is compressed only half as much, the potential energy stored in the spring, which is the energy delivered to the dart, is one-fourth as much. In the absence of friction, all of this energy becomes potential energy when the dart is at its maximum height.
Note that the intermediate calculation of the dart's speed when leaving the gun did not need to be calculated.

Question 3

An object of mass m slides down a plane that is inclined at an angle θ from the horizontal. The object starts out at rest. The center of mass of the cart is an unknown distance d from an unstretched spring with spring constant k that lies at the bottom of the plane. Assume the inclined plane to be frictionless. The spring compress a distance x when the mass first comes to rest? Find an expression for the distance d.



1. $d = \dfrac{1}{2mg\sin\theta}k\,x^2 - x$

2. $d = \dfrac{1}{2mg\sin\theta}k\,x^2$

3. $d = \dfrac{1}{2mg}k\,x^2 - x\sin\theta$

4. $d = \dfrac{1}{2mg}k\,x^2$

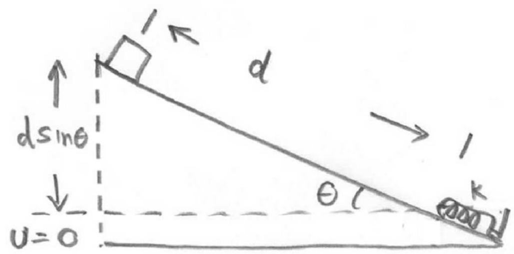
5. $d = \dfrac{1}{2mg\sin\theta}k\,x^2 + x$

6. $d = \dfrac{1}{2mg}k\,x^2 + x\sin\theta$

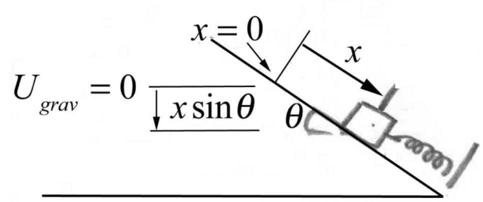
Solution 2: Let x denote the displacement of the spring from the equilibrium position. Choose the zero point for the gravitational potential energy $U_{\text{grav}}(x=0)=0$ not at the very bottom of the inclined plane, but at the location of the end of the unstretched spring. Choose the zero point for the spring potential energy when the spring is at its equilibrium position, $U_{\text{spring}}(x=0)=0$.

Choose for the initial state the instant the object is released. The initial kinetic energy is $K_0 = 0$. The initial potential energy is non-zero, $U_0 = mg\,d\sin\theta$. The initial mechanical energy is then

$$E_0 = K_0 + U_0 = mg\,d\sin\theta \qquad \textbf{(3.1)}$$



Choose for the final state the instant when the object first comes to rest and the spring is compressed a distance x at the bottom of the inclined plane. The final kinetic energy is $K_f = 0$ since the mass is not in motion. The final potential energy is non-zero, $U_f = \frac{1}{2}k\,x^2 - x\,mg\sin\theta$. Notice that the gravitational potential energy is negative because the object has dropped below the height of the zero point of gravitational potential energy.



The final mechanical energy is then

$$E_f = K_f + U_f = \frac{1}{2}k\,x^2 - x\,mg\sin\theta\,. \qquad \textbf{(3.2)}$$

Since we are assuming the track is frictionless, there is no non- conservative work. The change in mechanical energy is therefore zero,

$$0 = W_{\text{nc}} = \Delta E_{\text{mechanical}} = E_f - E_0\,. \qquad \textbf{(3.3)}$$

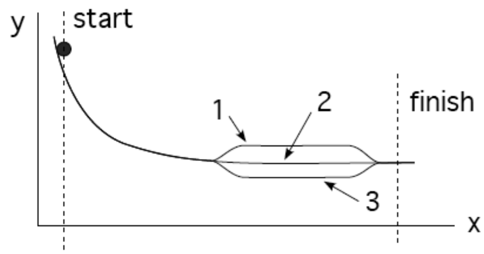
Thus mechanical energy is conserved, $E_f = E_0$, or

$$d\,mg\sin\theta = \frac{1}{2}k\,x^2 - x\,mg\sin\theta\,. \qquad \textbf{(3.4)}$$

Thus

$$d = \frac{1}{2mg\sin\theta}k\,x^2 - x\ . \qquad \textbf{(3.5)}$$

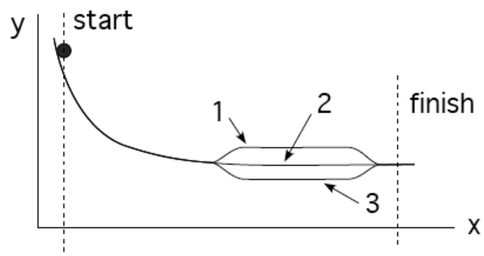
Question 4 Marble Run A marble starts from rest and slides down hill. Which path leads to the highest speed at the finish?



1) 1
2) 2
3) 3
4) all result in the same final speed

Answer 4. We are assuming that there is no friction on the paths so the mechanical energy is constant. The change in potential energy is the same for all three paths therefore the change in kinetic energy is also the same for all three paths.

Question 5 Marble Run Shortest Time A marble starts from rest and slides down hill. Which path results in the shortest time to the finish?

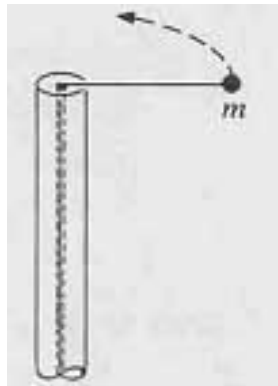


1) 1
2) 2
3) 3
4) all result in the same final speed

Answer 3. We are assuming that there is no friction on the paths so the mechanical energy is constant. Because the speed is greatest on path 3, the horizontal component of the velocity is greater on path 3 than at any point on path 2 or path 1, (included the starting and ending segments). Therefore it takes less time on path 3 to traverse the horizontal distance between the starting and ending points of each path.

Question 6 Conservation Laws 1

A tetherball of mass m is attached to a post of radius by a string. Initially it is a distance r_0 from the center of the post and it is moving tangentially with a speed v_0. The string passes through a hole in the center of the post at the top. The string is gradually shortened by drawing it through the hole. Ignore gravity and any dissipative forces. Until the ball hits the post,

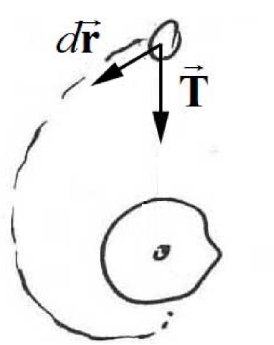


1. The kinetic energy of the ball is constant.

2. The kinetic energy of the ball changes.

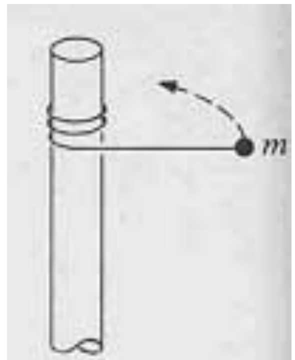
3. Not enough information is given to determine whether the kinetic energy of the ball changes or not.

Answer: 2. Since the path of the ball is not circular, a small displacement of the ball has a radial component inward so the dot product between the force by the rope on the ball with the displacement is non-zero hence the work done by the force is not zero. Therefore the kinetic energy of the ball changes.



Question 7 Conservation Laws 2

A tetherball of mass m is attached to a post of radius R by a string. Initially it is a distance r_0 from the center of the post and it is moving tangentially with a speed v_0. The string wraps around the outside of the post. Ignore gravity and any dissipative forces. Until the ball hits the post,

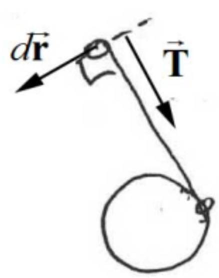


1. The kinetic energy of the ball is constant.

2. The kinetic energy of the ball changes.

3. Not enough information is given to determine whether the kinetic energy of the ball changes or not.

Answer 1. A small displacement of the ball is always perpendicular to string since at each instant in time the ball undergoes an instantaneous circular motion about the string contact point with pole. Therefore the dot product between the force by the rope on the ball with the displacement is zero hence the work done by the force is zero. Therefore the kinetic energy of the ball does not change.



Question 8 Trolley

A streetcar is freely coasting (no friction) around a large circular track. It is then switched to a small circular track. When coasting on the smaller circle its linear speed is

1. greater
2. less.
3. unchanged.

Answer: 3. Unchanged. Carefully draw a free body diagram for the streetcar while it is on the link of track connecting the two circular tracks. In the vertical direction, the force of gravity at all times balances the vertical component of the normal force of the track on the car, so there is no acceleration in the vertical direction and we may ignore it for the rest of this problem. In the horizontal direction, the train is guided by the horizontal component of the normal force of the track on the wheels. This force is always normal to the direction of motion, so no work is done by this force. Therefore the streetcar's total energy does not change, and its speed remains constant.