Simple Harmonic Oscillator
Challenge Problems

Problem 1: ***Dimensional Analysis*, *Estimation and Concepts***

Imagine that one drilled a hole with smooth sides straight through the center of the earth, of radius $R_e = 6.4 \times 10^6\ \text{m}$. If the air is removed from this tube (and the tube doesn't fill up with water, liquid rock or iron from the core), an object dropped into one end will have enough energy to just exit the other end after an interval of time. Use dimensional analysis to estimate that interval of time. Let $g = 9.8\ \text{m} \cdot \text{s}^{-2}$ be the gravitational constant.

Problem 2: Periodic Motion:

The motion of an object moving in one dimension is given by the function

$$x(t) = A\cos\left(\frac{2\pi}{T}t\right).$$

a) In your own words, describe the meaning of the constants T and A that appears in the above equation.

b) Find the velocity and acceleration of the object as functions of time.

c) Graph the position, velocity, and acceleration as functions of time. Be sure to indicate clearly on your graph the constants T and A.

Problem 3:

Imagine that one drilled a hole with smooth sides straight through the center of the earth. If the air is removed from this tube (and the tube doesn't fill up with water, liquid rock or iron from the core), an object dropped into one end will have enough energy to just exit the other end after an interval of time. Your goal is to find that interval of time. The steps outlined below show a way of finding this time interval. Make the assumption that the earth has uniform mass density.

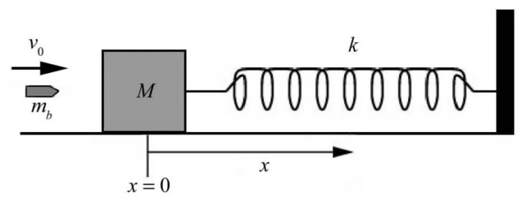
a) The gravitational force on an object of mass m located inside the earth a distance $r < r_e$ from the center (r_e is the radius of the earth) is due only to the mass of the earth that lies within a solid sphere of radius r. What is the gravitational force as a function of the distance r from the center of the earth? Express your answer in terms of the gravitational acceleration at the surface of the earth g and r_e Note: you do not need the mass of the earth m_e or the universal gravitation constant G to answer this question but you will need to find an expression relating m_e and G to g and r_e. You only need to assume that the earth is of uniform mass density. (You can neglect the amount of mass you drilled out.)

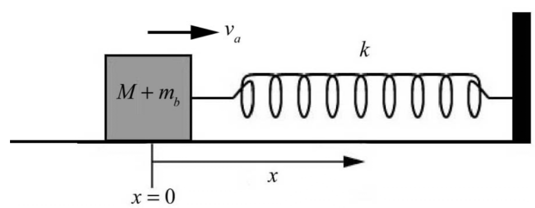
b) Use your result of part a) to explain why the object of mass m should oscillate (analogous to an object attached to a spring). In particular, how long would it take for this object to reach the other side of the earth?

c) What is the potential energy inside the earth as a function of r for the object-earth system? Can you think of a natural point to choose a zero point for the potential energy? Be careful because you will need to do a work integral to determine the change in potential energy when the object moves inside the earth and the gravitation force is no longer an inverse square when the object is inside the earth. Use energy considerations to find the velocity of the object when it passes through the center of the earth.

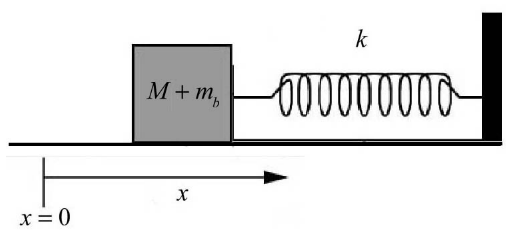
Problem 4

A massless spring with spring constant k is attached at one end of a block of mass M that is resting on a frictionless horizontal table. The other end of the spring is fixed to a wall. A bullet of mass m_b is fired into the block from the left with a speed v_0 and comes to rest in the block. (Assume that this happens instantaneously). The block and bullet are moving immediately after the bullet comes to rest with speed $v_a = m_b v_0 / (m_b + M)$.





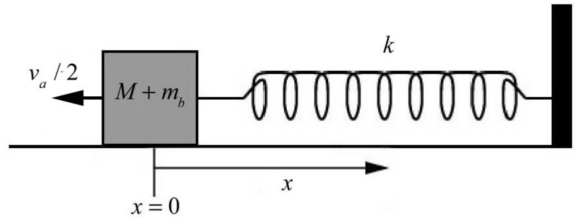
The resulting motion of the block and bullet is simple harmonic motion.



a) Find the amplitude of the resulting simple harmonic motion.

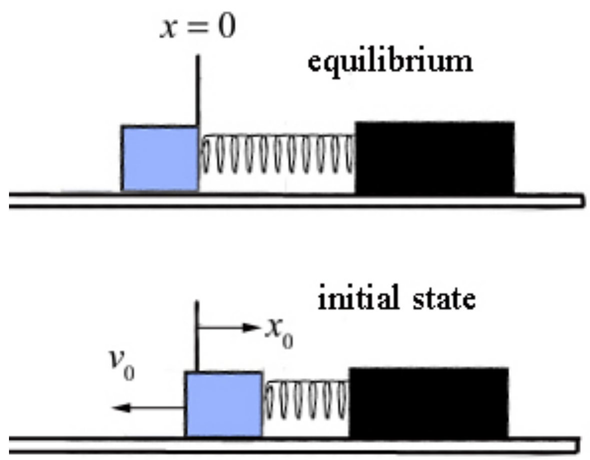
b) How long does it take the block to first return to the position $x = 0$?

c) Now suppose that instead of sliding on a frictionless table during the resulting motion, the block is acted on by the spring and a weak friction force of constant magnitude f. Suppose that when the block first returned to the position $x = 0$, the speed of the block was found to be $v_f = m_b v_0 / 2(m_b + M)$. How far did the block travel?



Problem 5 ***Simple Harmonic Motion***

Consider an ideal spring with spring constant k. The spring is attached to an object of mass m that lies on a horizontal frictionless surface. The spring-mass system is compressed a distance x_0 from equilibrium and then released with an initial speed v_0 toward the equilibrium position.



a) What is the period of oscillation for this system?

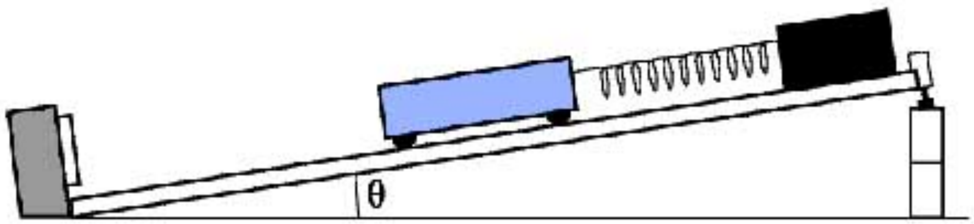
b) How long will it take for the object to first return to the equilibrium position?

c) What is the magnitude of the velocity of the object when it first returns to the equilibrium position?

d) Draw a graph of the position and velocity of the mass as a function of time. Carefully label your axes and clearly specify any special values.

Problem 6:

Consider an ideal spring that has an unstretched length l_0 and spring constant k. Suppose the spring is attached to a cart of mass m that lies on a frictionless plane that is inclined at an angle θ from the horizontal. The given quantities in this problem are l_0, m, k, θ and the gravitational constant g.



a) The spring stretches slightly to a new length $l > l_0$ to hold the cart in equilibrium. Find the length l in terms of the given quantities.

b) Now move the cart up along the ramp so that the spring is compressed a distance x_0 from the unstretched length l_0. Then the cart is released from rest. What is the speed of the cart when the spring has first returned to its unstretched length l_0?

c) What is the period of oscillation of the cart?