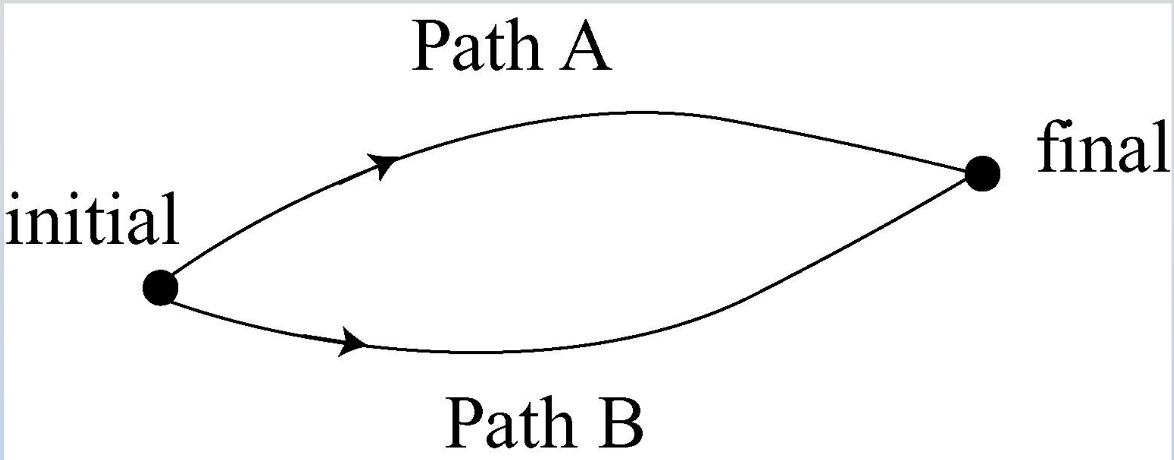
Potential Energy and Conservation of Energy

Conservative Forces



Definition: Conservative Force If the work done by a force in moving an object from an initial point to a final point is independent of the path (A or B),

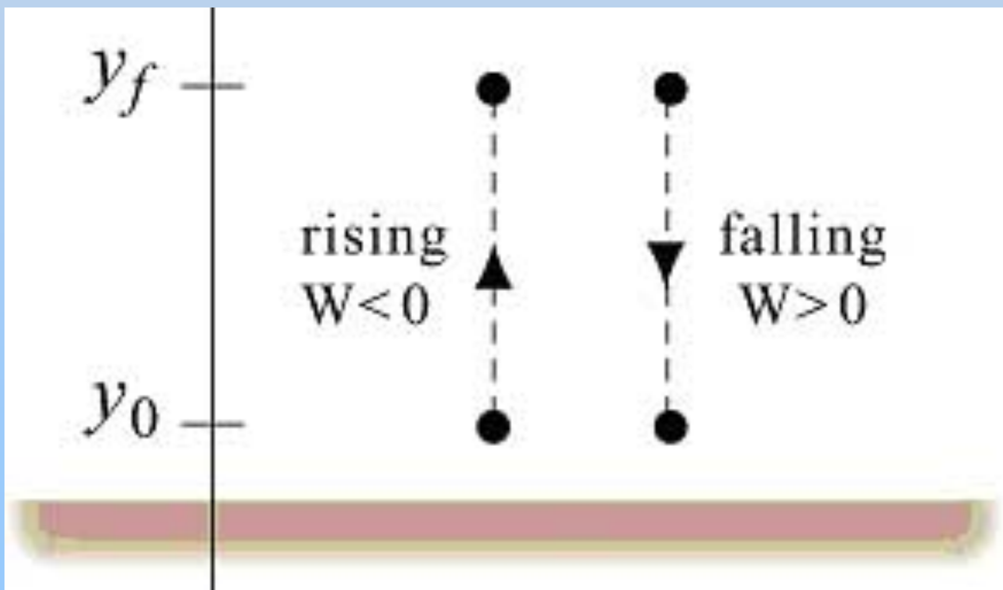
$$W_c \equiv \int_A^B \vec{\mathbf{F}}_{\mathrm{c}} \cdot d\vec{\mathbf{r}} \qquad \text{(path independent)}$$

then the force is called a *conservative force* which we denote by $\vec{\mathbf{F}}_{\mathrm{c}}$

Example: Gravitational Force

- Consider the motion of an object under the influence of a gravitational force near the surface of the earth

- The work done by gravity depends only on the change in the vertical position



$$\boxed{W_g = F_g \Delta y = -mg\Delta y}$$

Change in Potential Energy

Definition: Change in Potential Energy The *change in potential energy* of a body associated with a conservative force $\vec{\mathbf{F}}_{\mathrm{c}}$ is the negative of the work done by the conservative force in moving the body along any path connecting the initial and the final positions.

$$\boxed{\Delta U \equiv -\int_A^B \vec{\mathbf{F}}_{\mathrm{c}} \cdot d\vec{\mathbf{r}} = -W_c}$$

Work-Energy Theorem: Conservative Forces

The work done by the total force in moving an object from A to B is equal to the change in kinetic energy

$$W^{\text{total}} \equiv \int_{z_0}^{z_f} \vec{\mathbf{F}}^{\text{total}} \cdot d\vec{\mathbf{r}} = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_0^2 \equiv \Delta K$$

When the only forces acting on an object are **conservative forces**

$$\vec{\mathbf{F}}^{\text{total}} = \vec{\mathbf{F}}_c$$

then the change in potential energy is

$$\Delta U = -W_c = -W^{\text{total}}$$

Therefore

$$-\Delta U = \Delta K$$

Conservation of Energy for Conservative Forces

When the only forces acting on an object are conservative

$$\Delta K + \Delta U = 0$$

Definition: Mechanical Energy The *mechanical energy* is the sum of the kinetic and potential energies

$$E^{\text{mechanical}} \equiv K + U$$

Equivalently, the mechanical energy remains constant in time

$$E_f^{\text{mechanical}} = K_f + U_f = K_i + U_i = E_i^{\text{mechanical}}$$

Checkpoint Problem: Energy and Choice of System

You lift a ball at constant velocity from a height h_i to a greater height h_f. Considering the ball and the earth together as the system, which of the following statements is true?

1. The potential energy of the system increases.
2. The kinetic energy of the system decreases.
3. The earth does negative work on the system.
4. You do negative work on the system.
5. Two of the above.
6. None of the above.

Change in PE: Constant Gravity

Force: $$\vec{\mathbf{F}}_{\text{grav}} = m\vec{\mathbf{g}} = F_{\text{grav},y}\,\hat{\mathbf{j}} = -mg\,\hat{\mathbf{j}}$$

Work: $$W_{\text{grav}} = F_{\text{grav},y}\Delta y = -mg\Delta y$$

Potential Energy: $$\Delta U = -W_{\text{grav}} = mg\Delta y = mg\left(y_f - y_0\right)$$

Choice of Zero Point: Whatever "ground" is convenient

Worked Example: Change in Potential Energy for Inverse Square Gravitational Force

Consider an object of mass m_1 moving towards the sun (mass m_2). Initially the object is at a distance r_0 from the center of the sun. The object moves to a final distance r_f from the center of the sun. For the object-sun system, what is the change in potential during this motion?

Worked Example Solution: Inverse Square Gravity

Force:

$$\vec{\mathbf{F}}_{m_1,m_2} = -\frac{Gm_1m_2}{r^2}\hat{\mathbf{r}}$$

Work done:

$$W = \int_{r_0}^{r_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}} = \int_{r_0}^{r_f} \left(-\frac{Gm_1m_2}{r^2}\right) dr = \left.\frac{Gm_1m_2}{r}\right|_{r_0}^{r_f} = Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{r_0}\right)$$

Potential Energy Change:

$$\Delta U_{\text{grav}} = -W_{\text{grav}} = -Gm_1m_2\left(\frac{1}{r_f} - \frac{1}{r_0}\right)$$

Zero Point:

$$U_{\text{grav}}(r_0 = \infty) = 0$$

Potential Energy Function

$$U_{\text{grav}}(r) = -\frac{Gm_1m_2}{r}$$

Checkpoint Problem: Change in Potential Energy Spring Force

Connect one end of a spring of length l_0 with spring constant k to an object resting on a smooth table and fix the other end of the spring to a wall. Stretch the spring until it has length l and release the object. Consider the object-spring as the system. When the spring returns to its equilibrium length what is the change in potential energy of the system?

Change in PE: Spring Force

Force: $$\vec{\mathbf{F}} = F_x\,\hat{\mathbf{i}} = -kx\,\hat{\mathbf{i}}$$

Work done: $$W_{\text{spring}} = \int_{x=x_0}^{x=x_f} (-kx)\,dx = -\frac{1}{2}k\left(x_f^{\,2} - x_0^{\,2}\right)$$

Potential Energy Change: $$\Delta U_{\text{spring}} = -W_{\text{spring}} = \frac{1}{2}k\left(x_f^{\,2} - x_0^{\,2}\right)$$

Zero Point: $$U_{\text{spring}}(x=0) = 0$$

Potential Energy Function $$U_{\text{spring}}(x) = \frac{1}{2}kx^2$$

Summary: Change in Mechanical Energy

Total force:

$$\vec{\mathbf{F}}^{\text{total}} = \vec{\mathbf{F}}_{\text{c}}^{\text{total}} + \vec{\mathbf{F}}_{\text{nc}}^{\text{total}}$$

Total work:

$$W^{\text{total}} = \int_{\text{initial}}^{\text{final}} \vec{\mathbf{F}}^{\text{total}} \cdot d\vec{\mathbf{r}} = \int_{\text{initial}}^{\text{final}} \left(\vec{\mathbf{F}}_{\text{c}}^{\text{total}} + \vec{\mathbf{F}}_{\text{nc}}^{\text{total}}\right) \cdot d\vec{\mathbf{r}}$$

Change in potential energy:

$$\Delta U^{\text{total}} = -\int_{\text{initial}}^{\text{final}} \vec{\mathbf{F}}_{\text{c}}^{\text{total}} \cdot d\vec{\mathbf{r}}$$

Total work done is change in kinetic energy:

$$W^{\text{total}} = -\Delta U^{\text{total}} + W_{\text{nc}} = \Delta K$$

Mechanical Energy Change:

$$\Delta E^{\text{mechanical}} \equiv \Delta K + \Delta U^{\text{total}}$$

Conclusion:

$$W_{\text{nc}} = \Delta K + \Delta U^{\text{total}}$$

Demo slide: potential to kinetic energy B97

http://scripts.mit.edu/~tsg/www/index.php?page=demo.php?letnum=B 97&show=0

This demonstration consists of dropping a ball and a pendulum released from the same height. Both balls are identical. The vertical velocity of the ball is shown to be equal to the horizontal velocity of the pendulum when they both pass through the same height.

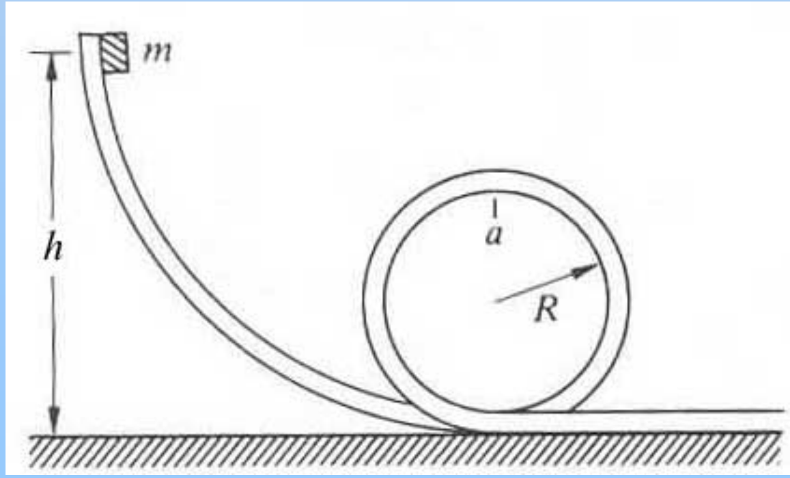
Strategy: Using Multiple Ideas

Mechanical Energy Conservation

Newton’s Second Law for radial direction

Checkpoint Problem: Loop-the-Loop

An object of mass *m* is released from rest at a height *h* above the surface of a table. The object slides along the inside of the loop-the-loop track consisting of a ramp and a circular loop of radius *R* shown in the figure. Assume that the track is frictionless. When the object is at the top of the track (point a) it pushes against the track with a force equal to three times it's weight. What height was the object dropped from?



Demo slide: Loop-the-Loop B95

http://scripts.mit.edu/~tsg/www/index.php?page=demo.php?letnum=B 95&show=0

A ball rolls down an inclined track and around a vertical circle. This demonstration offers opportunity for the discussion of dynamic equilibrium and the minimum speed for safe passage of the top point of the circle.

Potential Energy and Force

In one dimension, the potential difference is

$$U(x) - U(x_0) = -\int_A^B F_x \, dx$$

Force is the derivative of the potential energy

$$\boxed{F_x = -\frac{dU}{dx}}$$

Examples: (1) Spring Potential Energy: $U_{\text{spring}}(x) = \frac{1}{2}kx^2$

$$F_{x,spring} = -\frac{dU}{dx} = -\frac{d}{dx}\left(\frac{1}{2}kx^2\right) = -kx$$

(2) Gravitational Potential Energy: $U_{\text{grav}}(r) = -\frac{Gm_1m_2}{r}$

$$F_{r,gravity} = -\frac{dU}{dr} = -\frac{d}{dr}\left(-\frac{Gm_1m_2}{r}\right) = -\frac{Gm_1m_2}{r^2}$$

Energy Diagram

Choose zero point for potential energy:

$$U(x=0)=0$$

Potential energy function:

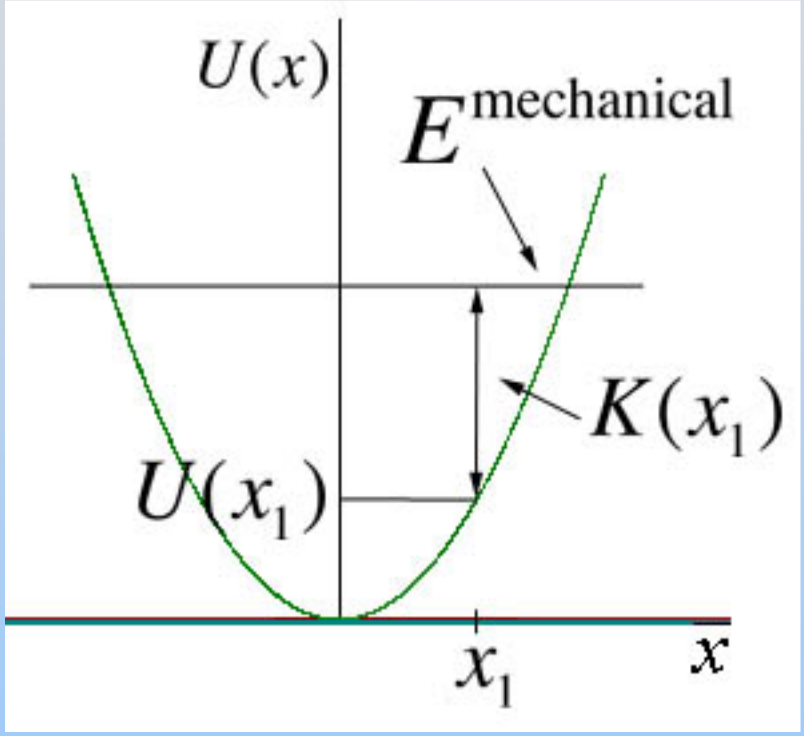
$$U(x)=\frac{1}{2}kx^2, \quad U(x=0)=0$$

Mechanical energy is represented by a horizontal line since it is a constant

$$E^{\text{mechanical}}=K(x)+U(x)=\frac{1}{2}mv_x^2+\frac{1}{2}kx^2$$

Kinetic energy is difference between mechanical energy and potential energy (independent of choice of zero point)

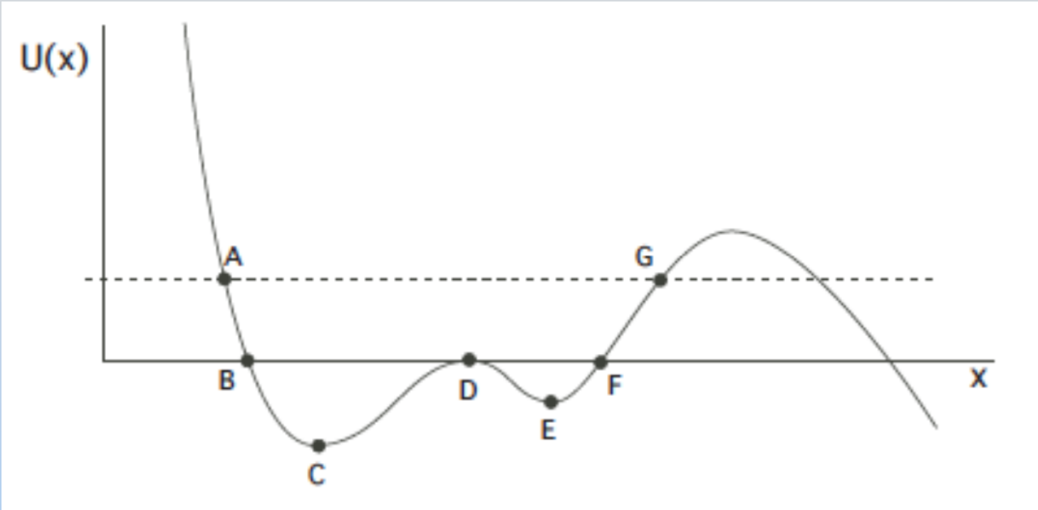
$$K=E^{\text{mechanical}}-U$$



Graph of Potential energy function

$U(x)$ vs. x

Checkpoint Problem: Energy Diagrams



The figure above shows a graph of potential energy verses position for a particle executing one dimensional motion along the axis. The total mechanical energy of the
the
system is indicated by the dashed line. At the particle is somewhere between points A and G. For later times, answer the following questions.

At which point will the magnitude of the force be a maximum?

At which point will the kinetic energy be a maximum?

At how many of the labeled points will the velocity be zero?

At how many of the labeled points will the force be zero?

Checkpoint Problem: Force and Potential Energy

A particle of mass , moving in the x-direction, is acting on by a potential

,

$$U(x) = -U_1\left(\left(\frac{x}{x_1}\right)^3 - \left(\frac{x}{x_1}\right)^2\right)$$

where U_1 and x_1 are positive constants and $U(0)=0$.

Sketch $U(x)/U_1$ as a function of x/x_1.

Find the points where the force on the particle is zero. Classify them as stable or unstable. Calculate the value of $U(x)/U_1$ at these equilibrium points.

For energies that lies in the range $0<E<(4/27)\,U_1$ find an equation whose solution yields points along the x-axis about which the particle will undergo periodic motion.

Suppose $E=(4/27)\,U_1$ and that the particle starts at $x = 0$ with speed v_0. Find v_0.