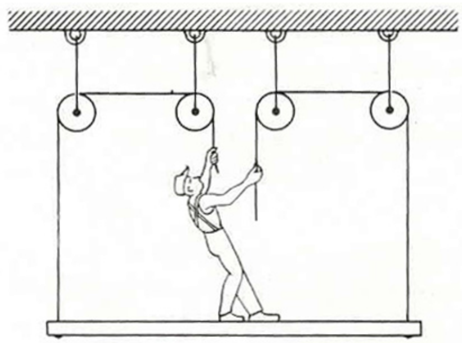
Application of Newton's Second Law
Challenge Problems

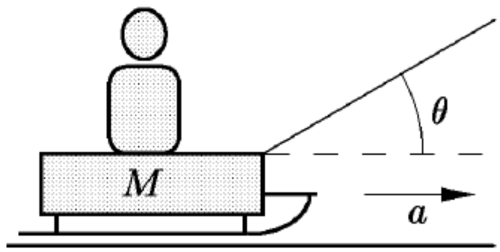
Problem 1: Painter on a Platform

A painter of mass m_1 stands on a platform of mass m_2 and pulls himself up by two ropes that run over massless pulleys, as shown. He pulls on each rope with a force of magnitude F and accelerates upward with a uniform acceleration a. Find the acceleration a.



Problem 2: Towing a Sled

A mother tows her daughter on a sled on level ice. The friction between the sled and the ice is negligible, and the tow rope makes an angle of θ to the horizontal. The combined mass of the sled and the child is M. The sled has an acceleration in the horizontal direction of magnitude a. As we will learn to justify in a few weeks, the child and sled can be treated in this problem as if they comprised a single particle.



a) Calculate the tension, T, in the rope and the magnitude of the normal force, N, exerted by the ice on the sled. Briefly describe how you model the problem and your strategy for solving this problem. Show all relevant free body diagrams.

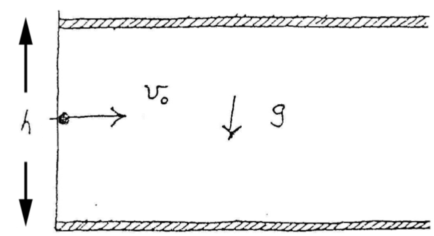
b) Calculate the new tension, T', in the rope and the magnitude of the new normal force, N', exerted by the ice on the sled. Briefly describe any changes you need to make to your previous model and strategy. Show all relevant free body diagrams.

Problem 3:

A particle of mass m enters a region horizontally at time $t = 0$ with speed v_0 midway between two plates that are separated by a distance h as shown in the figure. The particle is acting on by both gravity and by a time varying force that points upward and has magnitude

$$\left|\vec{\mathbf{F}}\right| = bt$$

where b is a positive constant that is sufficiently large such that the particle hits the top plate without ever touching the bottom plate.



a) Sketch the motion of the particle.

b) How long does it take for the particle to reach its lowest point?

c) What is the minimum possible value of b such that the particle does not hit the lower plate?

Problem 4: Emergency Landing

An airliner made an emergency landing at the Los Angeles airport with its nose wheel locked in a position perpendicular to its normal rolling position. The horizontal component of the plane's velocity at touchdown is an unknown quantity $v_{\text{horiz},0}$. The forces acting to stop the airliner arose from friction due to the nose wheel and the braking effort of the engine in reverse thrust mode. The sum of horizontal forces can be modeled as

$$F_{\text{horiz}}(t) = -F_0 + B\,t \tag{4.1}$$

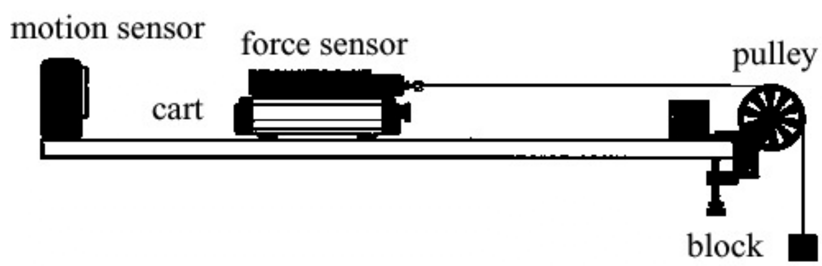
from touchdown at $t = 0$ until the plane comes to rest at $t = t_s$. Assume the mass of the plane is m. The point of this problem is to figure how far the plane traveled from touchdown until it came to a stop. Express your answer in terms of the known quantities F_0, B, t_s, m but not in terms of $v_{\text{horiz},0}$.

(a) Explain how you will model this problem and your strategy for solving it. Also include graphs of the horizontal position, horizontal component of the velocity, and horizontal component of the acceleration as functions of time.

(b) How far did the plane travel from touchdown until it came to a stop?

Problem 5: Track with Pulley

Consider a track with a pulley located at one end. The force sensor and cart have total mass m_1. (The force sensor was not used in the first experiment.) They are connected by an inextensible rope of length l (passing over the pulley) to a block of mass m_2. You may vary the mass of the block by adding additional weights. You may ignore the small mass of the rope and pulley. The coefficient of static friction between the car and the track is μ_s, and the coefficient of kinetic friction is μ_k. Assume that the coefficient of kinetic friction is less than the coefficient of static friction, $\mu_k < \mu_s$. Assume that the gravitational constant is g. (The friction is actually between the cart wheels and the axles, but this model, using coefficients of friction, is valid for low speeds. Friction in these carts is quite small.)

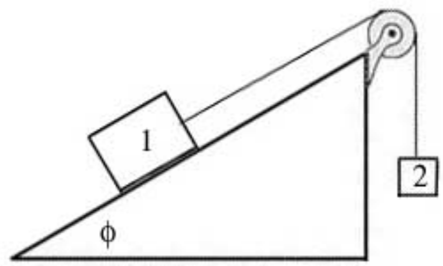


a) What additional mass m_w do you need to add to the block such that the force sensor and cart just start slipping? Express your answer in terms of the relevant given information.

b) Suppose you now add an extra weight m_3 so that the total mass of the block and weights is $m_2 + m_w + m_3$. What is the acceleration of the block?

Problem 6: Blocks and Pulley

A block 1 of mass m_1, constrained to move along a plane inclined at angle ϕ to the horizontal, is connected via a massless inextensible string that passes over a massless pulley, to a second block 2 of mass m_2. Assume the coefficient of static friction between the block and the inclined plane is μ_s and the coefficient of kinetic friction is μ_k. Assume the gravitational constant is g.



a) What is the relation between the masses of block 1 and block 2 such that the system just starts to slip?

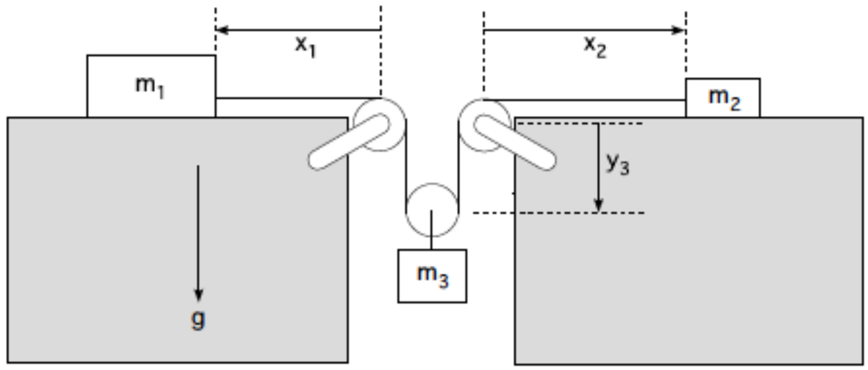
For the following questions suppose block 2 has a mass greater than the value you found in part a).

b) Calculate the acceleration of the blocks.

c) Calculate the tension in the string.

d) Block 2 starts out at a height h above the bottom of the inclined plane and is released at rest. How long does it take to fall a distance s? Assume that block 1 starts off a distance greater than s from the pulley.

Problem 7: Blocks and Pulleys



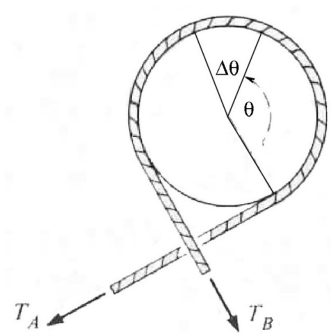
In the system shown above $m_1 > m_2$. The pulleys are massless and frictionless, and the rope joining the blocks has no mass. The coefficient of static friction between the blocks and the tables is greater than the coefficient of kinetic friction: $\mu_s > \mu_k$. The downward acceleration of gravity is g.

a) Imagine that when the system is released from rest body 3 accelerates downward at a constant rate of magnitude a, but only one of the other blocks moves. Which block does not move, and what is the magnitude and direction of the friction force holding it back.

b) Now consider the case where, when released from rest, all three blocks begin to move. Find the accelerations of all three blocks and the tension in the rope.

Problem 8:

A device called a capstan is used aboard ships in order to control a rope that is under great tension. The rope is wrapped around a fixed drum of radius R, usually for several turns (the drawing below shows about three fourths turn as seen from overhead).



The load on the rope pulls it with a force T_A, and the sailor holds the other end of the rope with a much smaller force T_B. The coefficient of static friction between the rope and the drum is μ_s. The sailor is holding the rope so that it is just about to slip. Can you show that $T_B = T_A e^{-\mu_s \theta}$, where μ_s is the coefficient of static friction and θ is the total angle subtended by the rope on the drum?