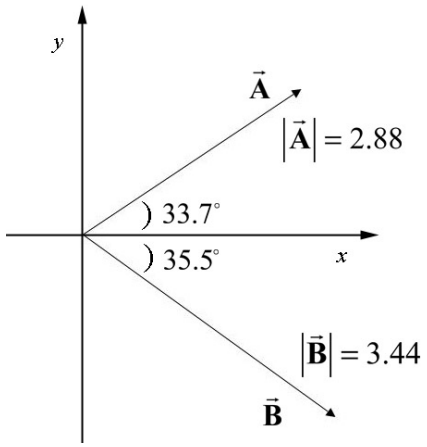
Problem Solving Vectors
Challenge Problems

Problem 1: Vector Addition

1.1 Consider the two vectors shown in the figure below. The magnitude of $|\vec{\mathbf{A}}| = 2.88$ and the vector $\vec{\mathbf{A}}$ makes an angle 33.7° with the positive x-axis. The magnitude of $|\vec{\mathbf{B}}| = 3.44$ and the vector $\vec{\mathbf{B}}$ makes an angle 35.5° with the positive x-axis pointing down to the right as shown in the figure below. Find the x and y components of the vectors $\vec{\mathbf{A}}$ and $\vec{\mathbf{B}}$.



1.2 Runner

At 2 am one morning you decide to take a jogging run through the MIT buildings. You take 54 seconds to run 250 m along the infinite corridor at MIT from Mass Ave to the end of Building 8, you turn right at the end of the corridor and take 42 seconds to run 178 m to the end of Building 2, and then you turn right, run down the hall for 9 seconds covering 30 m until you to stop.

a) Construct a vector diagram that represents your motion. Indicate your choice of unit vectors.

b) What are the directions and magnitudes of your average velocity for each leg of your trip?

c) What is the direction and magnitude your average velocity for the entire trip?

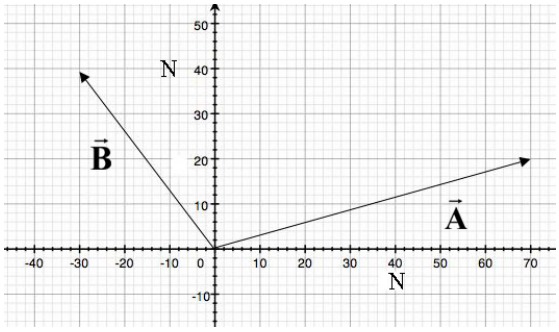
d) Is your average speed for the entire trip greater than, equal to, or less than your average speed for each individual leg? Explain your answer.

1.3 Sinking Sailboat

A Coast Guard ship is located $35\ \text{km}$ away from a checkpoint in a direction 42° north of west. A distressed sailboat located in still water $20\ \text{km}$ from the same checkpoint in a direction 36° south of east is about to sink. Draw a diagram indicating the position of both ships. In what direction and how far must the Coast Guard ship travel to reach the sailboat?

1.4 Balancing Forces on a Post

Two horizontal ropes are attached to a post that is stuck in the ground. The ropes pull the post producing the vector forces $\vec{\mathbf{A}} = 70\ \text{N}\,\hat{\mathbf{i}} + 20\ \text{N}\,\hat{\mathbf{j}}$ and $\vec{\mathbf{B}} = -30\ \text{N}\,\hat{\mathbf{i}} + 40\ \text{N}\,\hat{\mathbf{j}}$ as shown in the figure. Find the direction and magnitude of the horizontal component of a third force on the post that will make the vector sum of forces on the post equal to zero.



Problem 2:

2.1 In the methane molecule, CH_4, each hydrogen atom is at the corner of a tetrahedron with the carbon atom at the center. In a coordinate system centered on the carbon atom, if the direction of one of the C—H bonds is described by the vector $\vec{\mathbf{A}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} + \hat{\mathbf{k}}$ and the direction of an adjacent C—H is described by the vector $\vec{\mathbf{B}} = \hat{\mathbf{i}} - \hat{\mathbf{j}} - \hat{\mathbf{k}}$, what is the angle between these two bonds.

2.2 Show that the diagonals of an equilateral parallelogram are perpendicular.

2.3 Let $\hat{\mathbf{a}}$ and $\hat{\mathbf{b}}$ be unit vectors in the *xy* plane making angles θ and ϕ with the *x* axis, respectively. Show that $\hat{\mathbf{a}} = \cos\theta\hat{\mathbf{i}} + \sin\theta\hat{\mathbf{j}}$, $\hat{\mathbf{b}} = \cos\phi\hat{\mathbf{i}} + \sin\phi\hat{\mathbf{j}}$, and using vector algebra prove that $\cos(\phi - \theta) = \cos\theta\cos\phi + \sin\theta\sin\phi$.

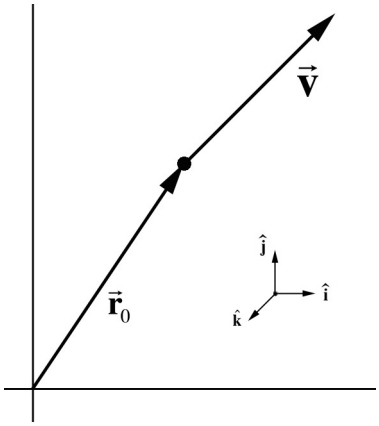
Problem 3: Problem Solving Cross Product

Problem 3.1 Find a unit vector perpendicular to $\vec{\mathbf{A}} = \hat{\mathbf{i}} + \hat{\mathbf{j}} - \hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = -2\hat{\mathbf{i}} - \hat{\mathbf{j}} + 3\hat{\mathbf{k}}$.

Problem 3.2 Let $\vec{\mathbf{A}}$ be an arbitrary vector and let $\hat{\mathbf{n}}$ be a unit vector in some fixed direction. Show that $\vec{\mathbf{A}} = \left(\vec{\mathbf{A}} \cdot \hat{\mathbf{n}}\right)\hat{\mathbf{n}} + \left(\hat{\mathbf{n}} \times \vec{\mathbf{A}}\right) \times \hat{\mathbf{n}}$.

Problem 3.3 Angular Momentum of a Point-like Particle

A particle of mass $m = 2.0\,\text{kg}$ moves as shown in the sketch with a uniform velocity $\vec{\mathbf{v}} = 3.0\,\text{m}\cdot\text{s}^{-1}\,\hat{\mathbf{i}} + 3.0\,\text{m}\cdot\text{s}^{-1}\,\hat{\mathbf{j}}$. At time t, the particle passes through the point $\vec{\mathbf{r}}_{0,m} = 2.0\,\text{m}\,\hat{\mathbf{i}} + 3.0\,\text{m}\,\hat{\mathbf{j}}$. Find the direction and the magnitude of the angular momentum about the origin at time t.



8.01SC Physics I: Classical Mechanics