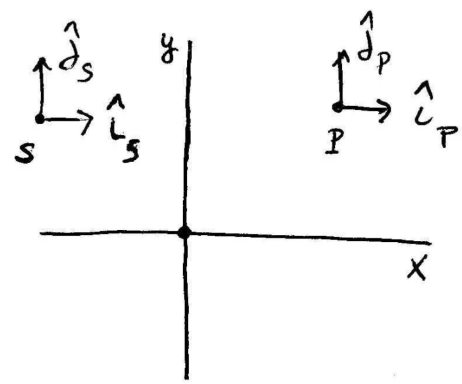
Vectors
Concept Questions

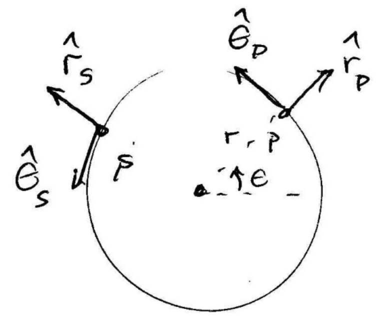
Question 1. Consider the pair of units vectors $(\hat{\mathbf{i}}_P, \hat{\mathbf{j}}_P)$ located at the point P, and the pair of units vectors $(\hat{\mathbf{i}}_S, \hat{\mathbf{j}}_S)$ located at the point S. Which of the following statements is true?



1) $\hat{\mathbf{i}}_P \neq \hat{\mathbf{i}}_S$
2) $\hat{\mathbf{j}}_P \neq \hat{\mathbf{j}}_S$
3) $\hat{\mathbf{i}}_P = \hat{\mathbf{i}}_S$
4) $\hat{\mathbf{j}}_P = \hat{\mathbf{j}}_S$

Answer 3 and 4. Vectors are equal if they have the same magnitude and point in the same direction. It doesn't matter where they are drawn.

Question 2. Consider the pair of units vectors $(\hat{\mathbf{r}}_P, \hat{\boldsymbol{\theta}}_P)$ located at the point P, and the pair of units vectors $(\hat{\mathbf{r}}_S, \hat{\boldsymbol{\theta}}_S)$ located at the point S. Which of the following statements is true?



1) $\hat{\mathbf{r}}_P \neq \hat{\mathbf{r}}_S$
2) $\hat{\boldsymbol{\theta}}_P \neq \hat{\boldsymbol{\theta}}_S$
3) $\hat{\mathbf{r}}_P = \hat{\mathbf{r}}_S$
4) $\hat{\boldsymbol{\theta}}_P = \hat{\boldsymbol{\theta}}_S$

Answer 1 and 2. Vectors cannot be equal if they do not point in the same direction

Question 3. Consider two vectors $\vec{\mathbf{A}} = 2\hat{\mathbf{i}} + 3\,\hat{\mathbf{k}}$ and $\vec{\mathbf{B}} = -6\hat{\mathbf{i}} + 4\,\hat{\mathbf{k}}$. The two vectors are

1. parallel.
2. perpendicular.
3. neither parallel or perpendicular.

Answer 2. We can calculate the scalar product between two vectors in a Cartesian coordinates system as follows:

$$\vec{\mathbf{A}} \cdot \vec{\mathbf{B}} = A_x B_x + A_y B_y + A_z B_z = (2)(-6) + (3)(4) = 0$$

When the scalar product of two vectors is zero then are perpendicular.

Question 4 Consider a vector $\vec{\mathbf{A}}$ with $\left|\vec{\mathbf{A}}\right| > 1$. The unit vector pointing in the same direction as the vector $\vec{\mathbf{A}}$ is given by

1) $\dfrac{\left|\vec{\mathbf{A}}\right|}{\vec{\mathbf{A}}}$

2) $\dfrac{\vec{\mathbf{A}}}{\left|\vec{\mathbf{A}}\right|}$

3) $\left|\vec{\mathbf{A}}\right|\vec{\mathbf{A}}$

4) $\dfrac{1}{\left|\vec{\mathbf{A}}\right|\vec{\mathbf{A}}}$

Answer 3. If you divide a vector by its lenth then the resulting vector has length one hence is a unit vector pointing in the same direction as the original vector.

Question 5 Consider two vectors $\vec{\mathbf{A}} = A_x \hat{\mathbf{i}}$, $\vec{\mathbf{B}} = B_x \hat{\mathbf{i}} + B_z \hat{\mathbf{k}}$ with $A_x < 0$, $B_x < 0$, and $B_z > 0$. The cross product $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ points in the

1) + x-direction
2) -x-direction
3) +y-direction
4) -y-direction
5) +z-direction
6) -z-direction
7) None of the above directions

Answer 3. $\vec{\mathbf{A}} \times \vec{\mathbf{B}} = A_x \hat{\mathbf{i}} \times (B_x \hat{\mathbf{i}} + B_z \hat{\mathbf{k}}) = A_x B_z (-\hat{\mathbf{j}})$. Because $A_x < 0$ and $B_z > 0$, $C_y > 0$ therefore the direction of $\vec{\mathbf{A}} \times \vec{\mathbf{B}}$ is in the positive y-direction.