Collision Theory

Collisions

Any interaction between (usually two) objects which occurs for short time intervals Δt when forces of interaction dominate over external forces.

- Of classical objects like collisions of motor vehicles.
- Of subatomic particles – collisions allow study force law.
- Sports, medical injuries, projectiles, etc.

Collision Theory: Energy

Types of Collisions

Elastic:

$$K_0^{\text{sys}} = K_f^{\text{sys}}$$

$$\frac{1}{2}m_1 v_{1,0}{}^2 + \frac{1}{2}m_2 v_{2,0}{}^2 + \cdots = \frac{1}{2}m_1 v_{1,f}{}^2 + \frac{1}{2}m_2 v_{2,f}{}^2 + \cdots$$

Inelastic:

$$K_0^{\text{sys}} > K_f^{\text{sys}}$$

Completely Inelastic: Only one body emerges.

Superelastic:

$$K_0^{\text{sys}} < K_f^{\text{sys}}$$

Demo: Ball Bearing and Glass B60

- http://tsgphysics.mit.edu/front/index.php?page=demo.php?letnum=B%2060&show=0

Drop a variety of balls and let students guess order of elasticity.

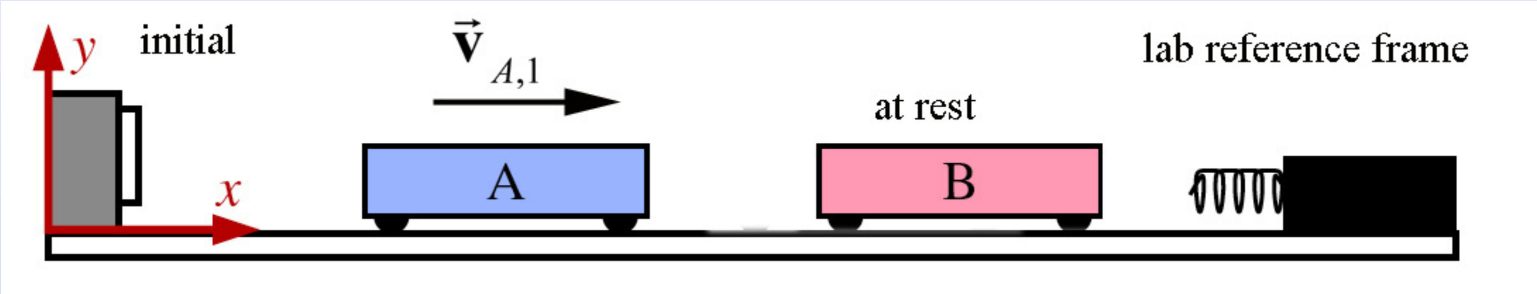
Demo: B62 Colliding Balls

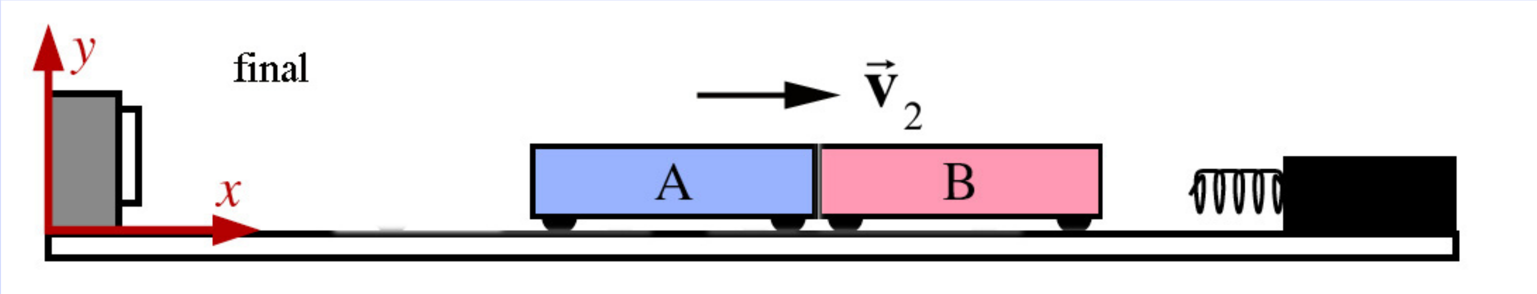
http://tsgphysics.mit.edu/front/index.php?page=demo.php?letnum=B%2062&show=0

A series of eight balls are mounted on two wires so that they slide. When one ball collides with the remaining seven, one is knocked off the other end. Similar results are obtained when several balls are slid.

Checkpoint Problem: totally inelastic collision

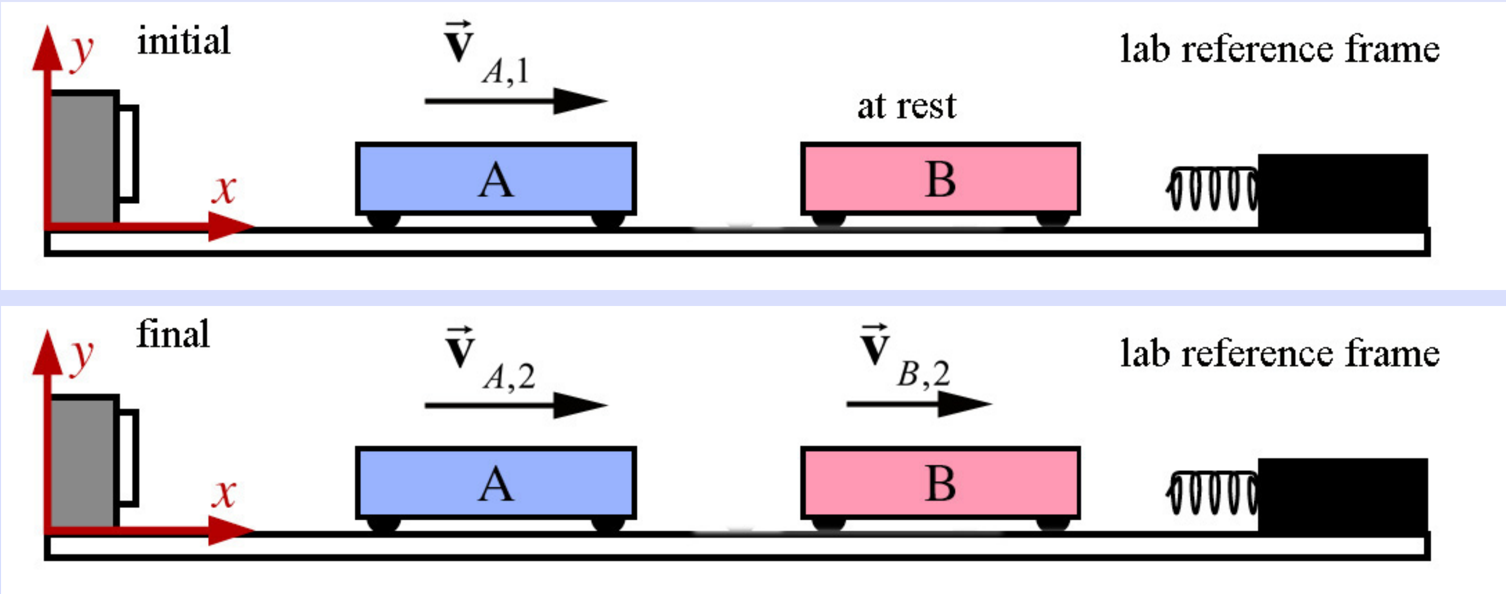
A car of mass m_A moving with speed v_{A1} collides with another car that has mass m_B and is initially at rest. After the collision the cars stick together. What is the change in mechanical energy due to the collision?





Checkpoint Problem: elastic collision

Consider the elastic collision of two carts along a track; the incident cart A has mass m_A and moves with initial speed $v_{A,1}$. The target cart B has mass has mass $m_B = 2\ m_A$ and is initially at rest. Immediately after the collision, the incident cart has final speed $v_{A,2}$ and the target cart has final speed $v_{B,2.}$ Calculate the final velocities of the carts as a function of the initial speed .



Problem Solving Strategies: Momentum Flow Diagram

- Identify the objects that comprise the system

- Identify your choice if reference frame with an appropriate choice of positive directions and unit vectors

- Identify your initial and final states of the system

- Construct a momentum flow diagram as follow:

  Draw two pictures; one for the initial state and the other for the final state. In each picture: choose symbols for the mass and velocity of each object in your system, for both the initial and final states. Draw an arrow representing the momentum. (Decide whether you are using components or magnitudes for your velocity symbols.)

Review: External Force and Momentum Change

The total momentum of a system of N particles is defined as the sum of the individual momenta of the particles

$$\vec{\mathbf{p}}_{\text{sys}} \equiv \sum_{i=1}^{i=N} \vec{\mathbf{p}}_i$$

Total force changes the momentum of the system

$$\vec{\mathbf{F}}^{\text{total}} = \sum_{i=1}^{i=N} \vec{\mathbf{F}}_i^{\text{total}} = \sum_{i=1}^{i=N} \frac{d\vec{\mathbf{p}}_i}{dt} \equiv \frac{d\vec{\mathbf{p}}_{\text{sys}}}{dt}$$

Total force equals total external force

$$\vec{\mathbf{F}}^{\text{total}} = \vec{\mathbf{F}}_{\text{ext}}^{\text{total}}$$

Newton's Second and Third Laws for a system of particles: The total external force is equal to the change in momentum of the system

$$\vec{\mathbf{F}}_{\text{ext}}^{\text{total}} = \frac{d\vec{\mathbf{p}}_{\text{sys}}}{dt}$$

Review: Translational Motion of the Center of Mass of Rigid Body

- Total momentum $\qquad \vec{\mathbf{p}}_{\text{sys}} = m_{\text{sys}} \vec{\mathbf{V}}_{cm}$

- External force and acceleration of center of mass

$$\vec{\mathbf{F}}_{\text{ext}}^{\text{total}} = \frac{d\vec{\mathbf{p}}_{\text{sys}}}{dt} = m_{\text{sys}} \frac{d\vec{\mathbf{V}}_{cm}}{dt} = m_{\text{sys}} \vec{\mathbf{A}}_{cm}$$

Modeling: Instantaneous Interactions

- Decide whether or not an interaction is instantaneous.

- External impulse changes the momentum of the system.

$$\vec{\mathbf{I}}[t, t+\Delta t_{col}] = \int_{t}^{t+\Delta t_{col}} \vec{\mathbf{F}}_{ext} dt = (\vec{\mathbf{F}}_{ext})_{ave} \Delta t_{col} = \Delta \vec{\mathbf{p}}_{sys}$$

- If the collision time is approximately zero,

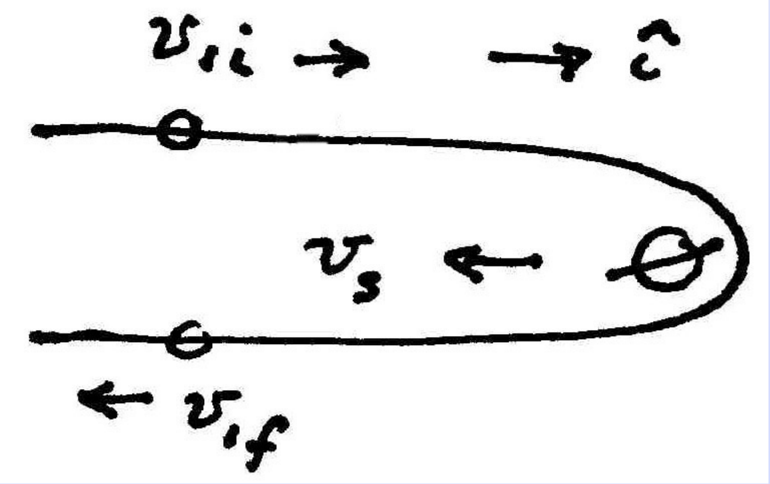
$$\Delta t_{col} \simeq 0$$

then the change in momentum is approximately zero.

$$\Delta \vec{\mathbf{p}}_{system} \cong \vec{\mathbf{0}}$$

Checkpoint Problem: Slingshot

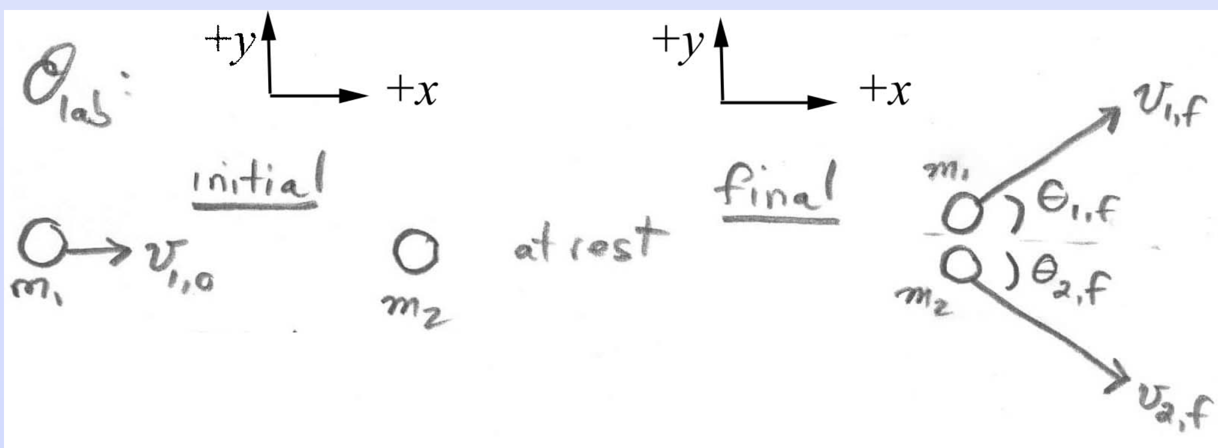
A spacecraft of mass m_1 = 2150 kg with a speed v_{1i} = 10.4 km/s approaches Saturn which is moving in the opposite direction with a speed v_s = 9.6 km/s . After interacting gravitationally with Saturn, the spacecraft swings around Saturn and heads off in the opposite direction it approached . The mass of Saturn is $m_s = 5.69 \times 10^{26}$ kg. Find the final speed of the spacecraft after it is far enough away from Saturn to be nearly free of Saturn's gravitational pull.



Particle Collisions: Problem Solving Strategies

Two Dimensional Collisions: Momentum Flow Diagram

Consider a collision between two particles. In the laboratory reference frame, the 'incident' particle with mass m_1, is moving with an initial given velocity $v_{1,0}$. The second 'target' particle is of mass m_2 and at rest. After the collision, the first particle moves off at an angle $\theta_{1,f}$ with respect to the initial direction of motion of the incident particle with a final velocity $v_{1,f}$. Particle two moves off at an angle $\theta_{2,f}$ with a final velocity $v_{2,f}$. Draw a momentum diagram representing this collision. Answer:



Strategies: Conservation of Momentum

- If system is isolated, write down the condition that momentum is constant in each direction

$$(p_{sys,0})_x = (p_{sys,f})_x$$

$$m_1\left(v_x\right)_{1,0} + m_2\left(v_x\right)_{2,0} + \cdots = m_1\left(v_x\right)_{1,f} + m_2\left(v_x\right)_{2,f} + \cdots$$

$$(p_{sys,0})_y = (p_{sys,f})_y$$

$$m_1\left(v_y\right)_{1,0} + m_2\left(v_y\right)_{2,0} + \cdots = m_1\left(v_y\right)_{1,f} + m_2\left(v_y\right)_{2,f} + \cdots$$

Momentum Analysis

Since momentum is a vector quantity, identify the initial and final vector components of the total momentum

Initial State

$$\vec{\mathbf{p}}_0^{total} = m_1 \vec{\mathbf{v}}_{1,0} + m_2 \vec{\mathbf{v}}_{2,0} + \cdots$$

x-comp:

$$p_{x,0}^{total} = m_1 \left(v_x\right)_{1,0} + m_2 \left(v_x\right)_{2,0} + \cdots$$

y-comp:

$$p_{y,0}^{total} = m_1 \left(v_y\right)_{1,0} + m_2 \left(v_y\right)_{2,0} + \cdots$$

Final State

$$\vec{\mathbf{p}}_f^{total} = m_1 \vec{\mathbf{v}}_{1,f} + m_2 \vec{\mathbf{v}}_{2,f} + \cdots$$

x-comp:

$$p_{x,f}^{total} = m_1 \left(v_x\right)_{1,f} + m_2 \left(v_x\right)_{2,f} + \cdots$$

y-comp:

$$p_{y,f}^{total} = m_1 \left(v_y\right)_{1,f} + m_2 \left(v_y\right)_{2,f} + \cdots$$

Conservation Laws Analysis

No external forces are acting on the system:

$$p_{x,0}^{\mathrm{sys}} = p_{x,f}^{\mathrm{sys}}$$

$$m_1 v_{1,0} = m_1 v_{1,f} \cos\theta_{1,f} + m_2 v_{2,f} \cos\theta_{2,f}$$

$$p_{\mathrm{y},0}^{\mathrm{sys}} = p_{y,f}^{\mathrm{sys}}$$

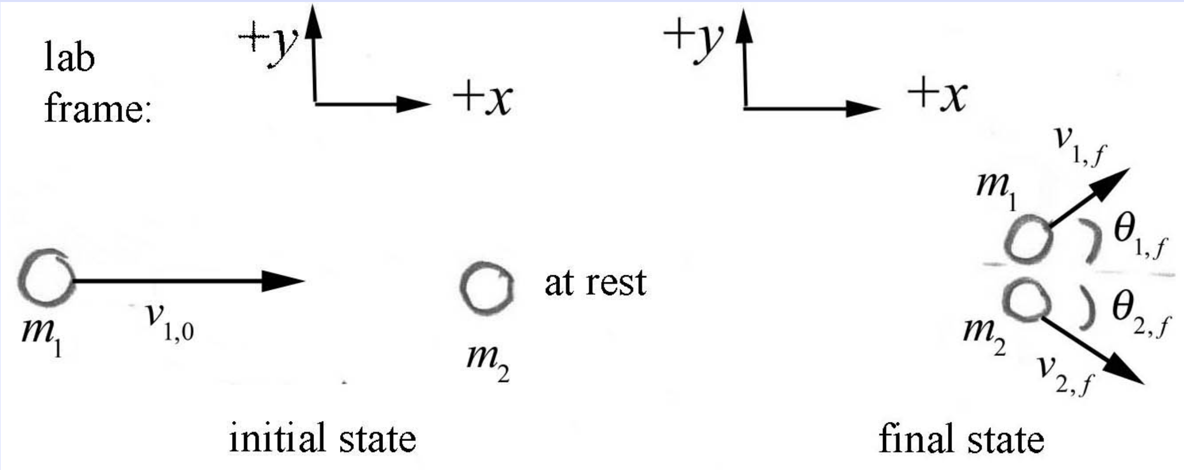
Collision is elastic: $0 = m_1 v_{1,f} \sin\theta_{1,f} - m_2 v_{2,f} \sin\theta_{2,f}$

$$\frac{1}{2} m_1 v_{1,0}{}^2 = \frac{1}{2} m_1 v_{1,f}{}^2 + \frac{1}{2} m_2 v_{2,f}{}^2$$

Checkpoint Problem: Elastic Collision in 2-d

In the laboratory reference frame, an "incident" particle with mass m_1, is moving with given initial speed v_{10}. The second "target" particle is of mass m_2 and at rest. After an elastic collision, the first particle moves off at an angle $\theta_{1,f}$ with respect to the initial direction of motion of the incident particle with final speed $v_{1,f}$. Particle two moves off at an angle $\theta_{2,f}$ with final speed $v_{2,f}$. (i) Find the equations that represent conservation of momentum and energy. Assume no external forces. (ii)

Find $v_{1,f}$, $v_{2,f}$, and $\theta_{2,f}$.



Strategy:

- Three unknowns: v_{1f} , v_{2f}, and $\theta_{2,f}$
- First squaring then adding the momentum equations and equations and solve for v_{2f} in terms of $v_{1f.}$
- Substitute expression for v_{2f} kinetic energy equation and solve quadratic equation for v_{1f}
- Use result for v_{1f} to solve expression for v_{2f}
- Divide momentum equations to obtain expression for $\theta_{2,f}$