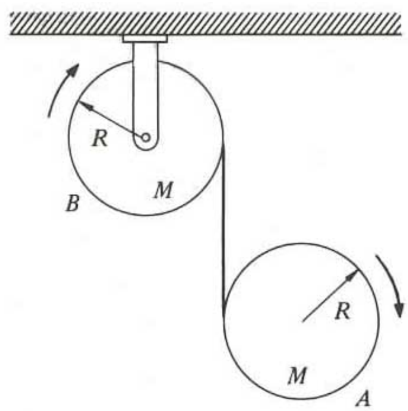
Rotation and Translation
Challenge Problems

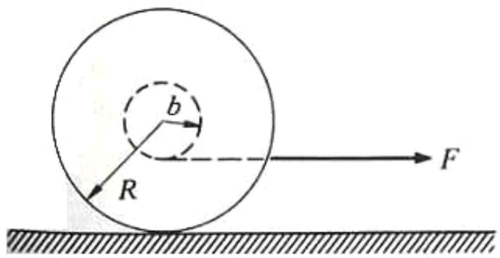
Problem 1:

A drum A of mass m and radius R is suspended from a drum B also of mass m and radius R, which is free to rotate about its axis. The suspension is in the form of a massless metal tape wound around the outside of each drum, and free to unwind. Gravity is directed downwards. Both drums are initially at rest. Find the initial acceleration of drum A, assuming that it moves straight down.



Problem 2: ***Yo-Yo rolling on a plane Rotation and Translation***

A Yo-Yo of mass m has an axle of radius b and a spool of radius R. It's moment of inertia about an axis passing through the center of the Yo-Yo can be approximated by $I_0 = (1/2)mR^2$. The Yo-Yo is placed upright on a table and the string is pulled with a horizontal force $\vec{\mathbf{F}}$ to the right as shown in the figure.



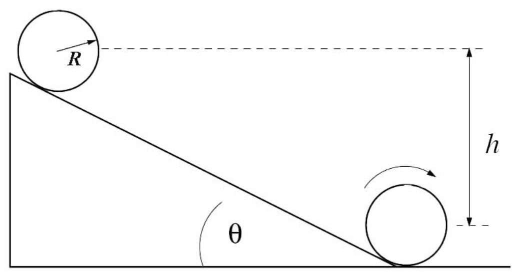
The coefficient of static friction between the Yo-Yo and the table is μ_s.

a) Which way will the Yo-Yo rotate if the string is pulled very gently? If the string is jerked hard, which way will the Yo-Yo rotate?

b) What is the maximum magnitude of the pulling force, $|\vec{\mathbf{F}}|$, for which the Yo-Yo will roll without slipping?

Problem 3

A hollow cylinder of outer radius R and mass M with moment of inertia about the center of mass $I_{\text{cm}} = M R^2$ starts from rest and moves down an incline tilted at an angle θ from the horizontal. The center of mass of the cylinder has dropped a vertical distance h when it reaches the bottom of the incline. Let g denote the gravitational constant. The coefficient of static friction between the cylinder and the surface is μ_s. The cylinder rolls without slipping down the incline. The goal of this problem is to find an expression for the smallest possible value of μ_s such that the cylinder rolls without slipping down the incline plane and the velocity of the center of mass of the cylinder when it reaches the bottom of the incline.



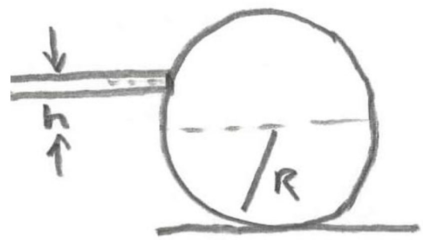
a) Write down a plan for solving this problem. Make sure you clearly state which concepts you plan to use to calculate any relevant physical quantities. Also clearly state any assumptions you make. Be sure you include any diagrams or sketches that you plan to use.

b) What is the minimum value for the coefficient of static friction μ_s such that the cylinder rolls without slipping down the incline plane? Express your answer in terms of M, R, g, θ and h as needed.

c) What is the magnitude of the velocity of the center of mass of the cylinder when it reaches the bottom of the incline? Express your answer in terms of M, R, g, θ and h as needed.

Problem 4: ***Billiards Challenge***

A spherical billiard ball of uniform density has mass m and radius R and moment of inertia about the center of mass $I_{\text{cm}} = (2/5)mR^2$. The ball, initially at rest on a table, is given a sharp horizontal impulse by a cue stick that is held an unknown distance h above the centerline (see diagram below). The coefficient of sliding fiction between the ball and the table is μ_k. You may ignore the friction during the impulse. The ball leaves the cue with a given speed v_0 and an unknown angular velocity ω_0. Because of its initial rotation, the ball eventually acquires a maximum speed of $(9/7)v_0$. The point of the problem is to find the ratio h/R.



a) Write up your plan for solving this problem. You may find some of the following concepts useful: angular impulse is equal to the change in angular momentum; linear impulse is equal to the change in momentum; Newton's Second Law; torque about the center-of-mass is proportional to the angular acceleration; if the torque about a point is zero, then the angular momentum about that point is constant; etc.

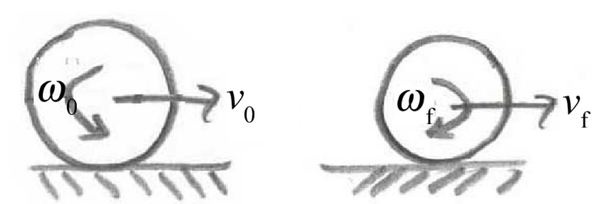
b) Find the ratio h/R.

Problem 5: Bowling Ball

A bowling ball of mass m and radius R is initially thrown down an alley with an initial speed v_0 and backspin with angular speed ω_0, such that $v_0 > R\omega_0$. The moment of inertia of the ball about its center of mass is $I_{\mathrm{cm}} = (2/5)mR^2$. Your goal is to determine the speed v_{f} of the bowling ball when it just starts to roll without slipping.

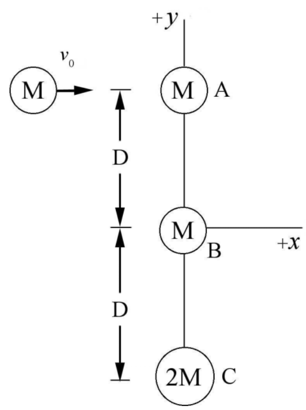
a) Write up your plan for solving this problem. You may find some of the following concepts useful: angular impulse is equal to the change in angular momentum; linear impulse is equal to the change in momentum; Newton's Second Law; torque about the center-of-mass is proportional to the angular acceleration; if the torque about a point is zero, then the angular momentum about that point is constant; etc.

b) What is the speed v_{f} of the bowling ball when it just starts to roll without slipping?



Problem 6: Rotational Collision

Three point-like objects located at the points A, B and C of respective masses $M_A = M$, $M_B = M$ and $M_C = 2M$, as shown in the figure below. The three objects are initially oriented along the y-axis and connected by rods of negligible mass each of length D, forming a rigid body. A fourth object of mass M moving with velocity $v_0\,\hat{\mathbf{i}}$ collides and sticks to the object at rest at point A. Neglect gravity. Give all your answers in terms of M, v_0 and D as needed. The z-axis points out of the page.



a) Describe qualitatively in words how the system moves after the collision: direction, translation and rotation.

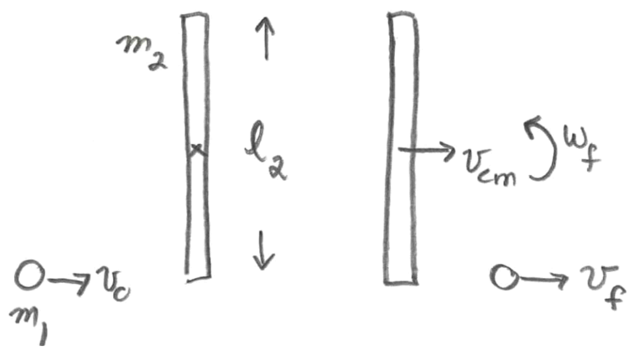
b) What is the direction and magnitude of the linear velocity of the center of mass after the collision?

c) What is the magnitude of the angular velocity of the system after the collision?

d) What is the direction and magnitudes of the velocity $\vec{\mathbf{v}}_C$ and acceleration $\vec{\mathbf{a}}_C$ of the object located at the point C immediately after the collision?

Problem 7: Hockey Puck Collision

A hockey puck of mass m_1 slides along ice with a velocity v_0 and strikes one end of a stick lying on the ice of length l_2 and mass m_2. The center of mass of the stick moves with an unknown magnitude v_{cm}. The stick also rotates about the center of mass with unknown angular velocity ω_f. The puck continues to move in the same straight line as before it hit the stick with velocity v_f. Assume the ice is frictionless and there is no loss of mechanical energy during the collision.



a) Write down the equation for conservation of momentum.

b) Write down the equation for conservation of energy.

c) Is there any external torques acting on the system consisting of the puck and the stick? Write down the equation for conservation of angular momentum about a convenient point.

d) Find the velocity of the center of mass of the stick.

e) Find the velocity of the puck after the collision.

f) Find the angular velocity of the stick after the collision.