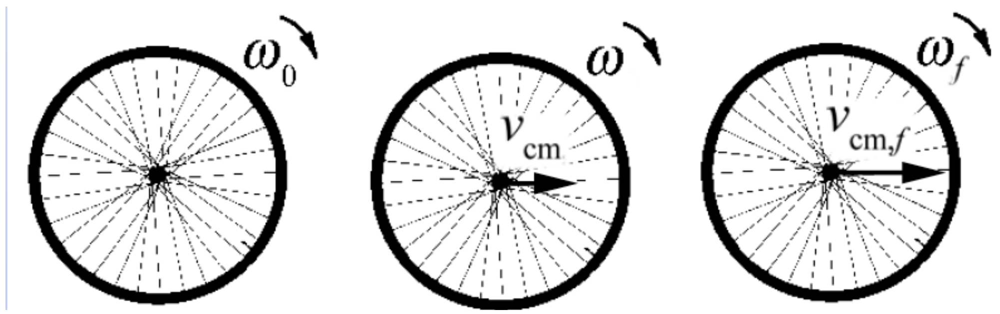
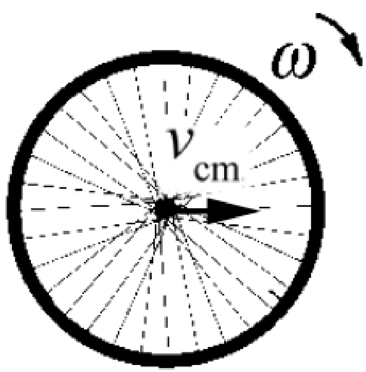
Two Dimensional Kinematics/Translation
Concept Question

Bicycle Wheel Solutions

Consider a bicycle wheel of radius R and mass m with moment of inertia I_{cm} about an axis passing perpendicular to the plane of the wheel and through the center-of-mass. The bicycle wheel is initially spinning with angular velocity ω_0 about the center-of-mass. The wheel is lowered to the ground without bouncing. As soon as the wheel touches the level ground, the wheel starts to accelerate forward until it begins to roll without slipping with a final angular velocity ω_f and center-of-mass velocity $v_{cm,f}$. Find the center-of mass velocity $v_{cm,f}$ when the wheel rolls without slipping.



Bicycle Wheel: Forces
How many forces (in a frame fixed to the ground) are acting on the bicycle when it is translating and rotating but is not yet rolling without slipping?



1. One.
2. Two.
3. Three.
4. Four.
5. Five.

Answer 2. Two.
There are two forces acting on the wheel, the contact force between the wheel and the ground and the gravitational force between the wheel and the earth. If you said three, that's ok because the contact force has two components the normal force and kinetic

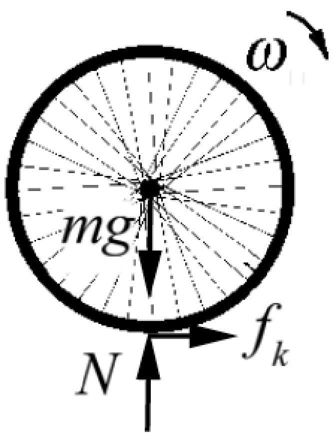
friction.

Bicycle Wheel: Direction of
Kinetic Friction

What is the direction of the kinetic friction force on the bicycle wheel in the figure?

1. Points to the right.
2. Points to the left.
3. Points up.
4. Points down.

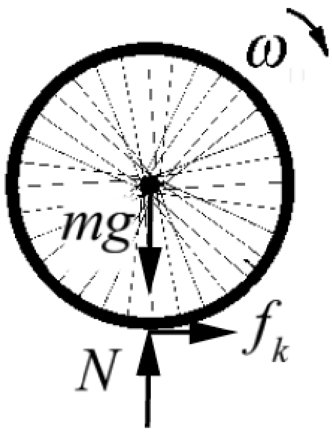
Answer 1. Points to the right.



The kinetic friction force on the bicycle wheel points to the right. This is the force that is responsible for the acceleration of the wheel. (All the forces are shown in the figure.)

Bicycle Wheel: Torque about the Center-of-Mass

Is the torque about the center-of-mass zero?



1. Yes.
2. No.
3. Not sure.

Answer 2. No.
The kinetic friction produces a torque about the center-of mass that points out of the page.

Bicycle Wheel: Angular Momentum about the Center-of-Mass
Is the angular momentum constant about the center-of-mass?

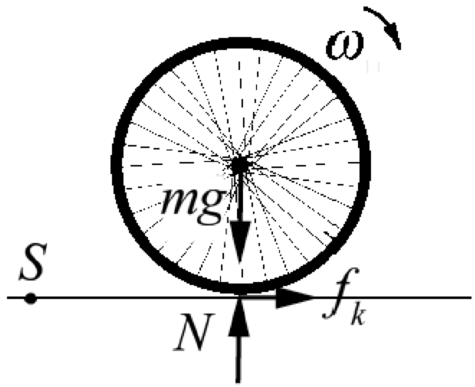
1. Yes.
2. No.
3. Not sure.

Answer 2. No.

Since the torque is non-zero, the angular momentum about the center-of-mass is changing. The torque due to the kinetic friction decreases the angular velocity.

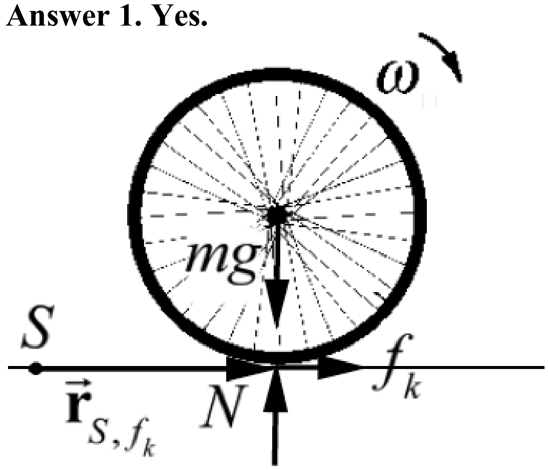
Bicycle Wheel: Torque about S due to Friction Force

Consider the point S shown in the figure. S lies on the line of contact between the wheel and the ground. Is the torque about S due to the kinetic friction force zero?



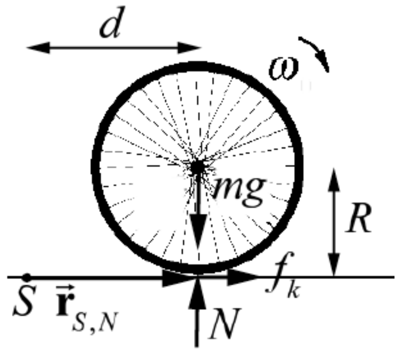
1. Yes.
2. No.
3. Not sure.

Answer 1. Yes.



Since the vector from S to the where the kinetic friction is acting is parallel to the kinetic friction the torque about S is zero.

Bicycle Wheel: Torque about S due to Normal Force

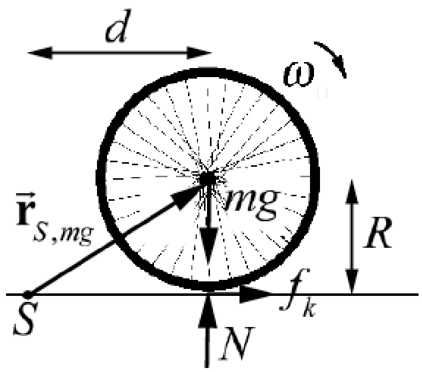


Consider the point S shown in the figure. S lies on the line of contact between the wheel and the ground. Is the magnitude and direction of the torque about S due to the normal force

1. $\left|\vec{\boldsymbol{\tau}}_{S,N}\right| = Nd$, out of the page?
2. $\left|\vec{\boldsymbol{\tau}}_{S,N}\right| = Nd$, into the page?
3. $\left|\vec{\boldsymbol{\tau}}_{S,N}\right| = NR$, out of the page?
4. $\left|\vec{\boldsymbol{\tau}}_{S,N}\right| = NR$, into the page?
5. None of the above.

Answer 1.
The torque about S due to the normal force is out of the page and has magnitude $\left|\vec{\boldsymbol{\tau}}_{S,N}\right| = N r_{S,\perp} = Nd$.

Bicycle Wheel: Torque about S due to Gravitational Force



Consider the point S shown in the figure. S lies on the line of contact between the wheel and the ground. Is the magnitude and direction of the torque about S due to the gravitational force

1. $\left|\vec{\boldsymbol{\tau}}_{S,F_{grav}}\right| = mgd$, out of the page?
2. $\left|\vec{\boldsymbol{\tau}}_{S,F_{grav}}\right| = mgd$, into the page?
3. $\left|\vec{\boldsymbol{\tau}}_{S,F_{grav}}\right| = mgR$, out of the page?
4. $\left|\vec{\boldsymbol{\tau}}_{S,F_{grav}}\right| = mgR$, into the page?
5. None of the above.

Answer 2.
The torque about S due to the gravitational force is into the page and has magnitude $\left|\vec{\boldsymbol{\tau}}_{S,F_{grav}}\right| = mgr_{S,\perp} = mgd$.

Bicycle Wheel: Torque about S

Do the torques about S due to the normal force and the gravitational force sum to zero?

1. Yes.
2. No.
3. Not sure.

Answer 1. Yes.

The directions of the torques are opposite. They
have the same moment arm. By Newton's Second Law, the vertical component of the acceleration of the wheel is zero, so $N = mg$. Hence the two torques sum to zero.

Bicycle Wheel: Angular Momentum about S

Is the angular momentum constant about the point S?

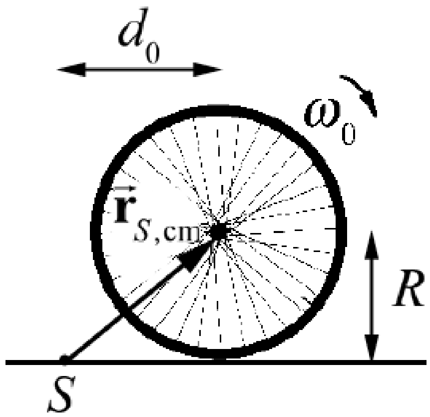
1. Yes.
2. No.
3. Not sure.

Answer 1. Yes.
Since the torque about S due to all three forces sums to zero, the angular momentum about S is zero.

$$\vec{\boldsymbol{\tau}}_S^{ext} = \frac{d\vec{\mathbf{L}}_S^{sys}}{dt} = \vec{\mathbf{0}}$$

Bicycle Wheel: Initial Angular Momentum

What is the magnitude and direction of the angular momentum of the wheel about the point S just before the wheel touches the ground?

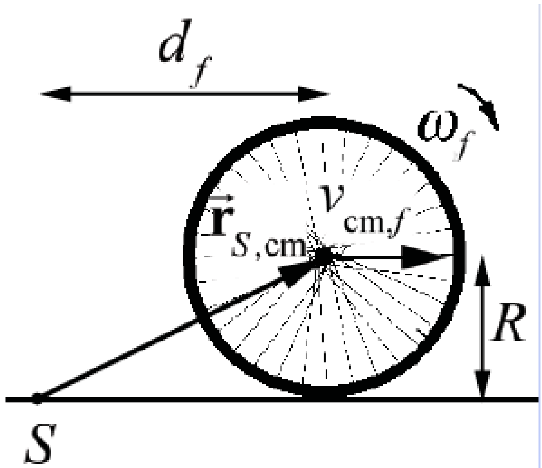


1. $\left|\vec{\mathbf{L}}_{S,0}^{sys}\right| = \left|\vec{\mathbf{L}}_{cm,0}^{sys}\right| = mR\omega_0$, out of the page.
2. $\left|\vec{\mathbf{L}}_{S,0}^{sys}\right| = \left|\vec{\mathbf{L}}_{cm,0}^{sys}\right| = mR\omega_0$, into the page.
3. $\left|\vec{\mathbf{L}}_{S,0}^{sys}\right| = \left|\vec{\mathbf{L}}_{cm,0}^{sys}\right| = I_{cm}\omega_0$, out of the page.
4. $\left|\vec{\mathbf{L}}_{S,0}^{sys}\right| = \left|\vec{\mathbf{L}}_{cm,0}^{sys}\right| = I_{cm}\omega_0$, into the page.
5. $\left|\vec{\mathbf{L}}_{S,0}^{sys}\right| = \left|\vec{\mathbf{L}}_{cm,0}^{sys}\right| = md\omega_0$, into the page.
6. $\left|\vec{\mathbf{L}}_{S,0}^{sys}\right| = \left|\vec{\mathbf{L}}_{cm,0}^{sys}\right| = md\omega_0$, out of the page.
7. None of the above.

Answer 4.
Since the velocity of the center-of- mass of the wheel is initially zero when the wheel just touches the ground, the only contribution to the angular momentum about S is do the rotation about the center-of-mass. Since the wheel is spinning clockwise, the angular momentum points into the page and has magnitude $\left|\vec{\mathbf{L}}_{S,0}^{sys}\right| = \left|\vec{\mathbf{L}}_{cm,0}^{sys}\right| = I_{cm}\omega_0$.

Bicycle Wheel: Final Angular Momentum

What is the magnitude and direction of the angular momentum of the wheel about the point S when the wheel is rolling without slipping?



1. $\left|\vec{\mathbf{L}}^{sys}_{S,f}\right| = \left|\vec{\mathbf{L}}^{sys}_{cm,f}\right| = -mRv_{cm,f} + I_{cm}\omega_f$, out of the page.
2. $\left|\vec{\mathbf{L}}^{sys}_{S,f}\right| = \left|\vec{\mathbf{L}}^{sys}_{cm,f}\right| = -mRv_{cm,f} + I_{cm}\omega_f$, into the page.
3. $\left|\vec{\mathbf{L}}^{sys}_{S,f}\right| = \left|\vec{\mathbf{L}}^{sys}_{cm,f}\right| = +md_f v_{cm,f} + I_{cm}\omega_f$, out of the page.
4. $\left|\vec{\mathbf{L}}^{sys}_{S,f}\right| = \left|\vec{\mathbf{L}}^{sys}_{cm,f}\right| = +md_f v_{cm,f} + I_{cm}\omega_f$, into the page.
5. $\left|\vec{\mathbf{L}}^{sys}_{S,f}\right| = \left|\vec{\mathbf{L}}^{sys}_{cm,f}\right| = +mRv_{cm,f} + I_{cm}\omega_f$, into the page.
6. $\left|\vec{\mathbf{L}}^{sys}_{S,f}\right| = \left|\vec{\mathbf{L}}^{sys}_{cm,f}\right| = +mRv_{cm,f} + I_{cm}\omega_f$, out of the page.
7. $\left|\vec{\mathbf{L}}^{sys}_{S,f}\right| = \left|\vec{\mathbf{L}}^{sys}_{cm,f}\right| = -md_f v_{cm,f} + I_{cm}\omega_f$, into the page.
8. $\left|\vec{\mathbf{L}}^{sys}_{S,f}\right| = \left|\vec{\mathbf{L}}^{sys}_{cm,f}\right| = -md_f v_{cm,f} + I_{cm}\omega_f$,out of the page.
9. None of the above.

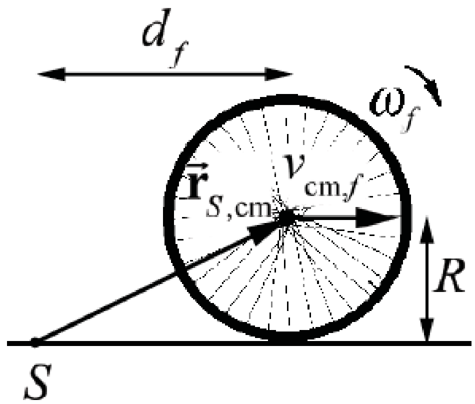
Answer 5. We need to consider both the motion of the center-of-mass and the rotation about the center-of- mass.

$$\vec{\mathbf{L}}^{sys}_{S,f} = \vec{\mathbf{R}}_{cm,f} \times \vec{\mathbf{p}}^{sys}_f + I_{cm}\vec{\omega}_f$$

The direction is into the page and the magnitude is

$$\left|\vec{\mathbf{L}}^{sys}_{S,f}\right| = r_{S,\perp,f} m v_{cm,f} + I_{cm}\omega_f$$
$$\left|\vec{\mathbf{L}}^{sys}_{S,f}\right| = r_{S,\perp,f} m v_{cm,f} + I_{cm}\omega_f$$
$$r_{S,\perp,f} = R$$
$$\left|\vec{\mathbf{L}}^{sys}_{S,f}\right| = Rmv_{cm,f} + I_{cm}\omega_f$$

Bicycle Wheel: Rolling Without Slipping



When the wheel is rolling without slipping what is the relation between the final center-of-mass velocity and the final angular velocity?

1. $v_{cm,f} = -R\omega_f$.
2. $\omega_f = Rv_{cm,f}$.
3. $v_{cm,f} = R\omega_f$.
4. $\omega_f = -Rv_{cm,f}$.

Answer 3.

When the wheel is rolling without slipping, in a time interval Δt, a point on the rim of the wheel travels a distance $\Delta s = r\Delta\theta$ In the same time interval Δt, the center-of-mass of the is displaced the same distance $\Delta x = v_{cm,f}\Delta t$ Equating these two distances,

$$r\Delta\theta = v_{cm,f}\Delta t$$

Dividing through by Δt, and taking limits, the rolling without slipping condition becomes

$$v_{cm,f} = \lim_{\Delta t \to 0} r\frac{\Delta\theta}{\Delta t} = r\omega_f .$$

Bicycle Wheel: Final Centerof- Mass Velocity

What is the magnitude of the final center-of mass velocity when the wheel is rolling without slipping?

1. $v_{cm,f} = \dfrac{I_{cm}\omega_0}{I_{cm} - mR^2}$
2. $v_{cm,f} = \dfrac{I_{cm}\omega_0}{I_{cm} + mR^2}$
3. $v_{cm,f} = \dfrac{I_{cm}R\omega_0}{I_{cm} + mR^2}$
4. $v_{cm,f} = \dfrac{I_{cm}R\omega_0}{I_{cm} - mR^2}$
5. $v_{cm,f} = \dfrac{I_{cm}R\omega_0}{I_{cm} + mR}$
6. $v_{cm,f} = \dfrac{I_{cm}R\omega_0}{I_{cm} - mR}$
7. $v_{cm,f} = \dfrac{R\omega_0}{I_{cm} + mR^2}$
8. $v_{cm,f} = \dfrac{R\omega_0}{I_{cm} - mR^2}$
9. None of the above.

Answer 3. Use the fact that angular momentum is constant about the point S,

$$\vec{\mathbf{L}}_{cm,0} = \vec{\mathbf{L}}_{S,f}$$

Now use the rolling without slipping condition

$$v_{cm,f} = \lim_{\Delta t \to 0} r\frac{\Delta\theta}{\Delta t} = r\omega_f$$

$$I_{cm}\omega_0 = Rmv_{cm,f} + I_{cm}\frac{v_{cm,f}}{R}$$

$$v_{cm,f} = \frac{I_{cm}R\omega_0}{I_{cm} + mR^2}$$

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