One Dimensional Kinematics
Challenge Problems

Problem 1:

As a space shuttle burns up its fuel after take-off, it gets lighter and lighter and its acceleration larger and larger. Between the moment it takes off and the time at which it has consumed nearly all of its fuel, is the magnitude of the average velocity larger than, equal to, or smaller than half its final speed? Explain why.

Problem 2

An airliner made an emergency landing at the Los Angeles airport with its nose wheel locked in a position perpendicular to its normal rolling position. The forces acting to stop the airliner arose from friction due to the nose wheel and the braking effort of the engine in reverse thrust mode. The sum of horizontal forces can be modeled as

$$F_{\text{horiz}}(t) = -F_0 + Bt \tag{2.1}$$

from touchdown at $t = 0$ until the plane comes to rest at $t = t_s$ where $F_0 > 0, B > 0$. Assume the mass of the plane is m. The point of this problem is to figure how far the plane traveled from touchdown until it came to a stop. In the questions below express your answers in terms of the known quantities F_0, B, t_s, m but not in terms of $v_{\text{horiz},0}$.

(a) Find the horizontal component of the velocity when the airplane first touches the runway at $t = 0$.

(b) How far did the airplane travel on the ground before stopping?

Problem 3

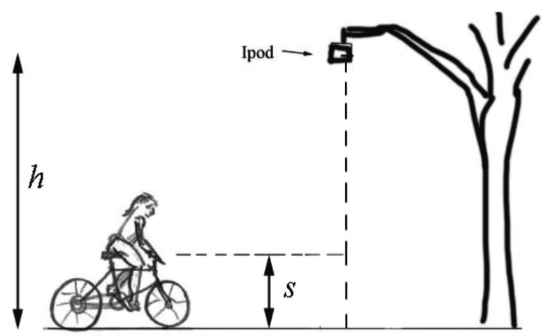
A bicycle rider is traveling at a constant speed along a straight road and then gradually applies the brakes during a time interval $0 < t < t_f$ until the bicycle comes to a stop. The combined mass of the cyclist and bicycle is m. Assume that the magnitude of the braking acceleration increases linearly in time according to

$$|\vec{\mathbf{a}}| = bt / m, \qquad\qquad 0 < t < t_f$$

where $b > 0$ is a constant. At the instant the person applies the brakes, a horizontal distance from the rider, the wind blows and snaps an iPod off the branch of the tree. With a speed v_{po} directed downwards. The ipod was initially a height h above the ground. The cyclist catches the iPod at the instant the cyclist has come to a stop. You may assume that the cyclist catches it at a height s above the ground. How far did the bicycle travel while it was braking? The gravitational constant is g.

a) At what time t_1 did the cyclist catch the ipod? Express your answer in terms of the quantities, b, m, h, s, v_{po}, d, and g as needed.

b) What was the initial speed of the cyclist? You may leave your answer in terms of t_1 from part a) and any other quantities as needed.



Problem 4: Model Rocket

Roxana launches a home-built model rocket straight up into the air. At $t = 0$ the rocket is at rest at $y = 0$ with $v_{y,0} = 0$. The acceleration of the rocket is given by

$$a_y = \begin{cases} -g + \alpha g - \beta t^2; & 0 < t < t_b \\ -g; & t > t_b \end{cases} \tag{4.1}$$

where $t_b = (\alpha g / \beta)^{1/2}$ is the time that the fuel burns out, g is the acceleration of gravity, and $\alpha > 3/2$ is a positive dimensionless number greater than 3/2. After the fuel burns out, the rocket is still moving upwards.

The goal of this problem is to figure out how high the rocket went up.

You may find the following integration formula useful

$$\int_{t_1}^{t_2} t^n dt = \frac{1}{n+1}\left(t_2^{\,n+1} - t_1^{\,n+1}\right);\ n \neq -1 \tag{4.2}$$

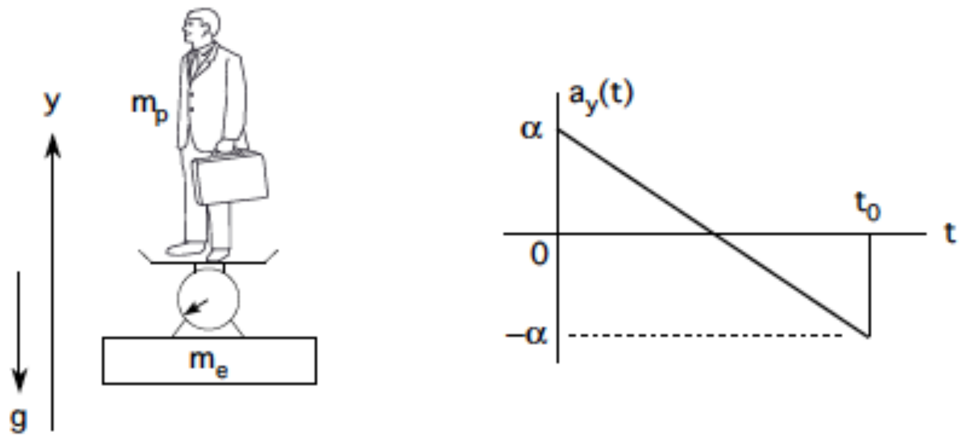
a) Make a qualitative graphical representation of the velocity and position of the rocket before answering the questions below.

b) Explain how you will model the motion of the rocket i.e. how will you determine the equations for the position and velocity of the rocket.

c) Explain your strategy that you will use to determine how high the rocket went up.

d) How high did the rocket go up? (This part will tax your integration and algebra skills so please be careful.)

Problem 5: Elevator



A person of mass m_p stands on a scale in an elevator of mass m_p. The scale reads the magnitude of the force F exerted on it from above in a downward direction. Starting at rest at $t = 0$ the elevator moves upward, coming to rest again at time $t = t_0$. The downward acceleration of gravity is g. The acceleration of the elevator during this period is shown graphically above and is given analytically by

$$a_y(t) = \alpha - \frac{2\alpha}{t_0}t. \tag{5.1}$$

a) Find the maximum speed of the elevator.

b) Find the total distance traveled by the elevator.

Problem 6: Periodic Motion

The motion of an object moving in one dimension is given by the function

$$x(t) = A\cos(2\pi t / T) \tag{6.1}$$

a) In your own words, describe the meaning of the constants T and A that appears in the above equation.

b) Find the velocity and acceleration of the object as functions of time.

c) Graph the position, velocity, and acceleration as functions of time. Be sure to indicate clearly on your graph the constants T and A.