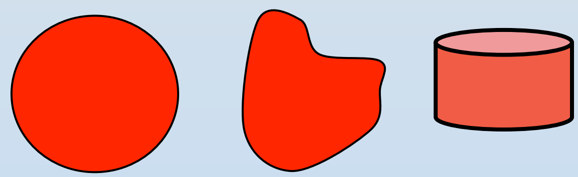
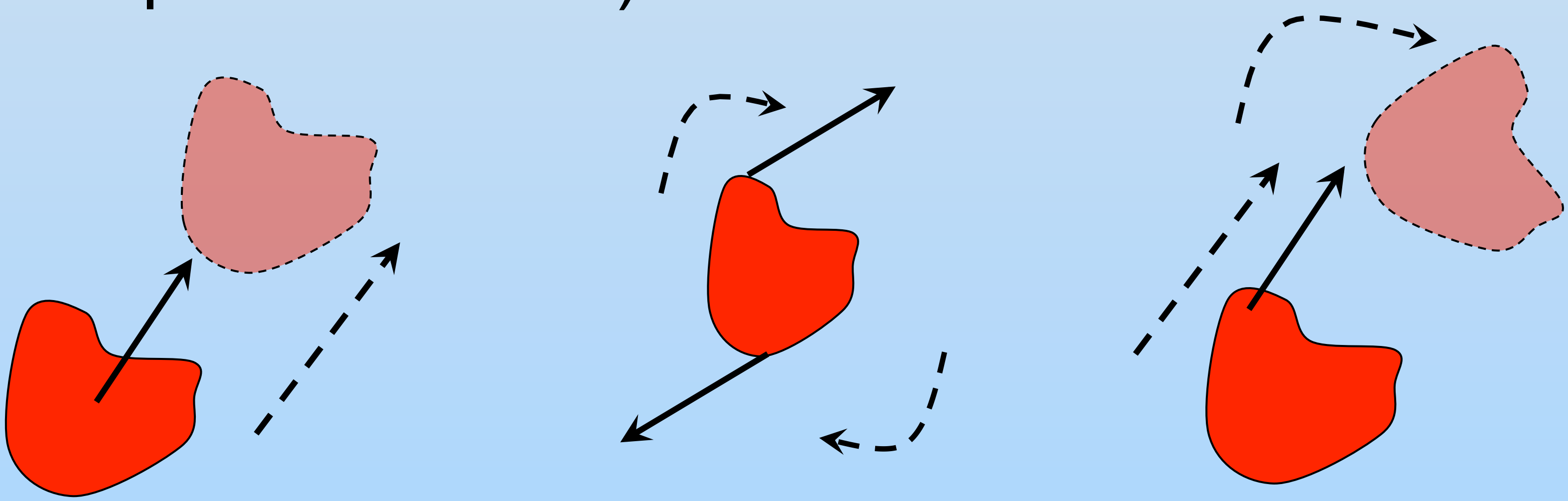
Torque and Static Equilibrium

Rigid Bodies

- Rigid body: An extended object in which the distance between any two points in the object is constant in time. Examples: sphere, disk …



- Effect of external forces (the solid arrows represent forces):

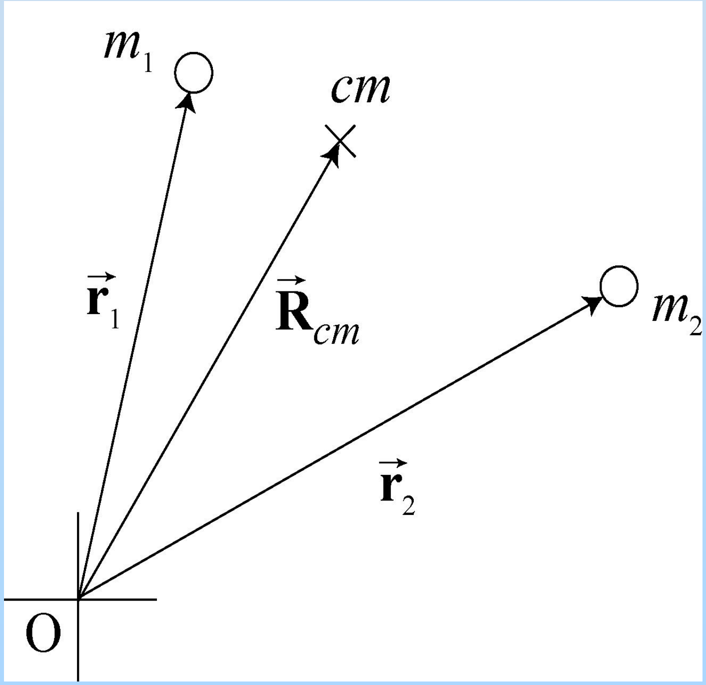


Translate   Rotate   Both translate and rotate

Center of Mass (C.M.)

Center of mass (*c.m.*) is a special point in a rigid body:

- The body can be balanced by pivoting it on the *c.m*.
- The gravitational force between the Earth and the body is exerted at the *c.m*.



For a rigid body with two point particles:

$$\boxed{\vec{\mathbf{R}}_{cm} = \frac{m_1\vec{\mathbf{r}}_1 + m_2\vec{\mathbf{r}}_2}{m_1 + m_2}}$$

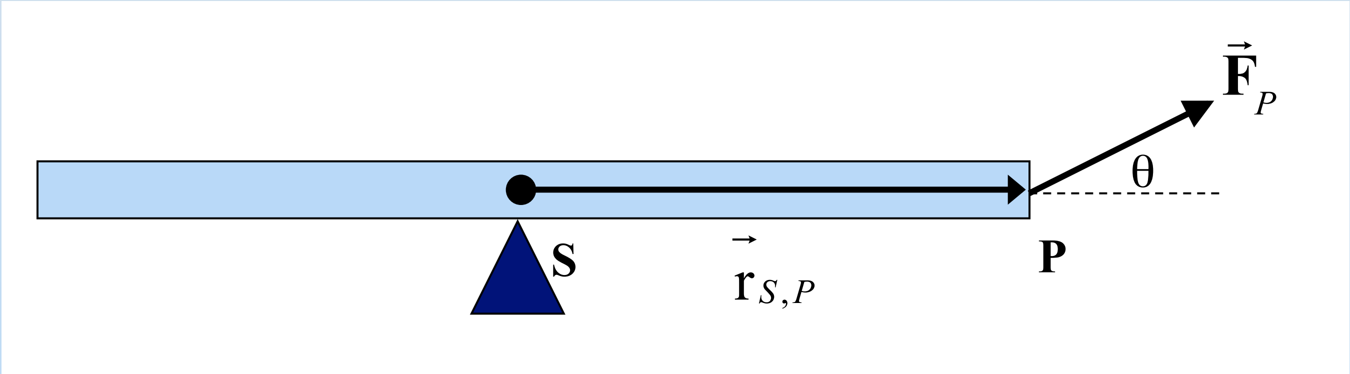
For a rigid body with n- point particles:

$$\boxed{\vec{\mathbf{R}}_{\mathrm{cm}} = \frac{\sum_{i=1}^{i=N} m_i\vec{\mathbf{r}}_i}{\sum_{i=1}^{i=N} m_i}}$$

Review: Torque

$\vec{\mathbf{F}}_P$: Force exerted at a point P on a rigid body.

$\vec{\mathbf{r}}_{S,P}$: Vector from a point *S* to the point *P.*



Torque about point S due to the force exerted at point P:

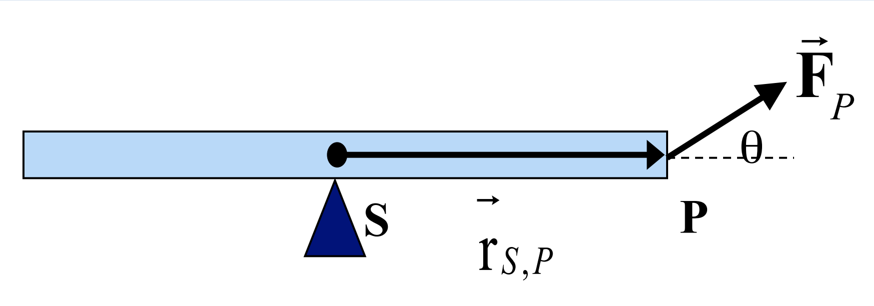
$$\boxed{\vec{\tau}_S = \vec{\mathbf{r}}_{S,P} \times \vec{\mathbf{F}}_P}$$

Review: Torque: Magnitude and Direction

Magnitude of torque about a point S:

$$\boxed{\tau_S = rF_\perp = rF\sin\theta}$$

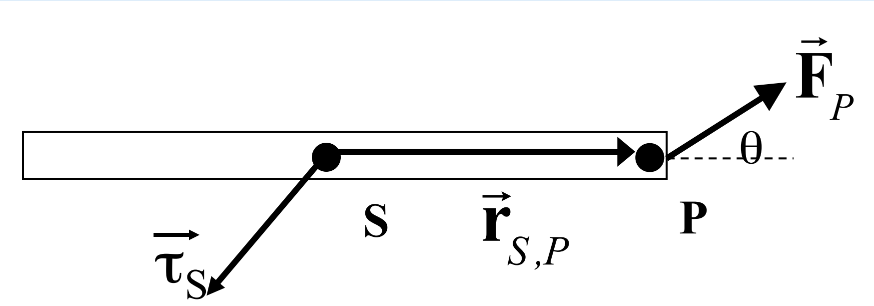
where F is the magnitude of the force $\vec{\mathbf{F}}_P$.



Direction of torque:

Perpendicular to the plane formed by $\vec{\mathbf{F}}_P$ and $\vec{\mathbf{r}}_{S,P}$.

Determined by the ***Right-Hand-rule.***



Conditions for Static Equilibrium

(1) Translational equilibrium: the sum of the forces acting on the rigid body is zero.

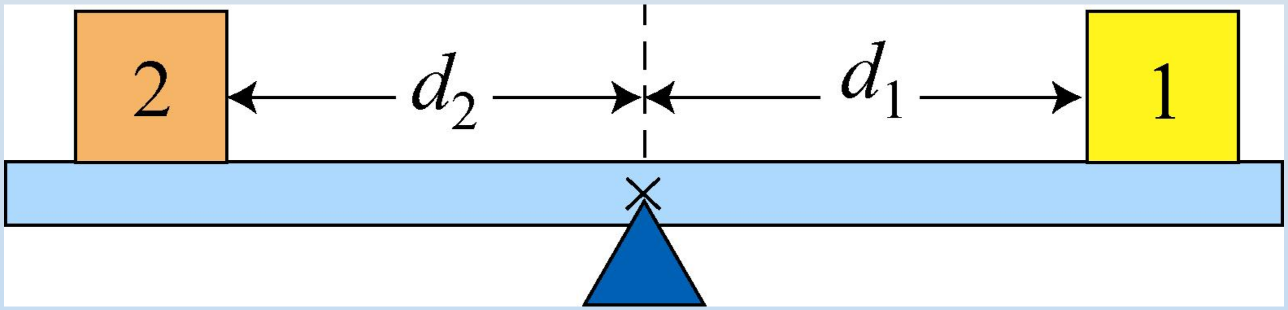
$$\vec{\mathbf{F}}_{\text{total}} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \ldots = \vec{\mathbf{0}}$$

(2) Rotational Equilibrium: the vector sum of the torques about any point S in a rigid body is zero.

$$\vec{\boldsymbol{\tau}}_{\mathbf{S}}^{\,\mathbf{total}} = \vec{\boldsymbol{\tau}}_{S,1} + \vec{\boldsymbol{\tau}}_{S,2} + \ldots = \vec{\mathbf{0}}$$

Worked Example: Lever Law

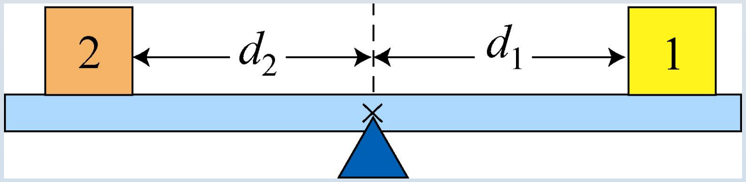
Pivoted Lever at Center of Mass in Equilibrium



Show that (Lever Law):

$$d_1 m_1 g = d_2 m_2 g$$

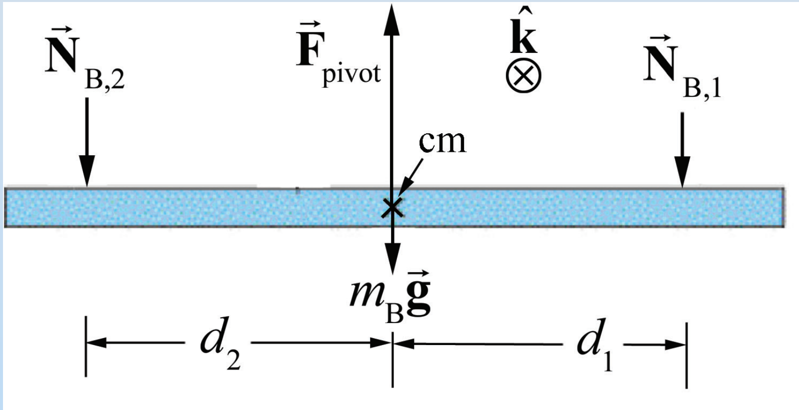
Worked Example: Pivoted Lever



Apply Newton's 2nd law to each body:

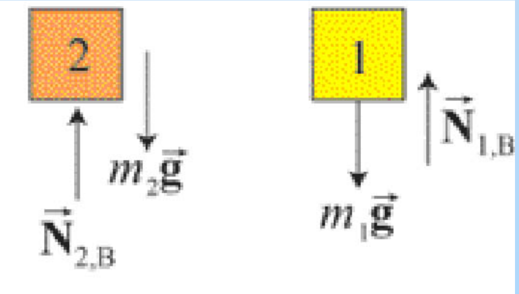
$$F_{\text{pivot}} - m_{\text{B}}g - N_{\text{B},1} - N_{\text{B},2} = 0$$

$$N_{1,B} - m_1 g = 0 \qquad N_{2,\text{B}} - m_2 g = 0$$



Third Law: $N_{1,B} = N_{B,1} \qquad N_{2,B} = N_{B,2}$

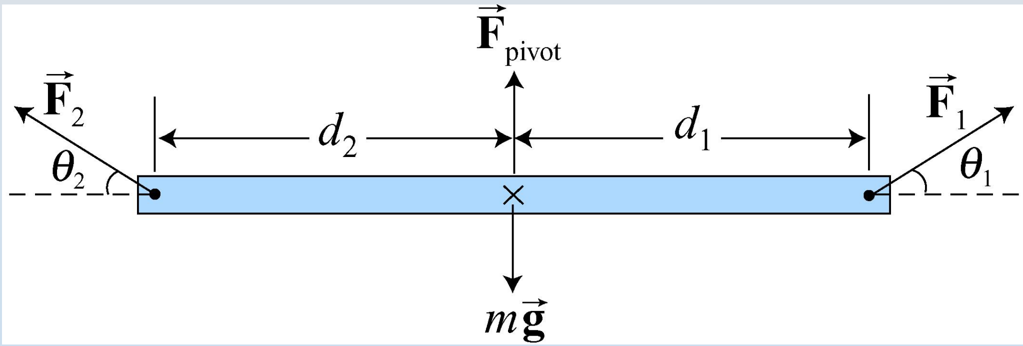
Pivot force: $F_{\text{pivot}} = (m_{\text{B}} + m_1 + m_2)g$



Torque about pivot: $d_1 N_1 \hat{\mathbf{k}} - d_2 N_2 \hat{\mathbf{k}} = \vec{\mathbf{0}}$

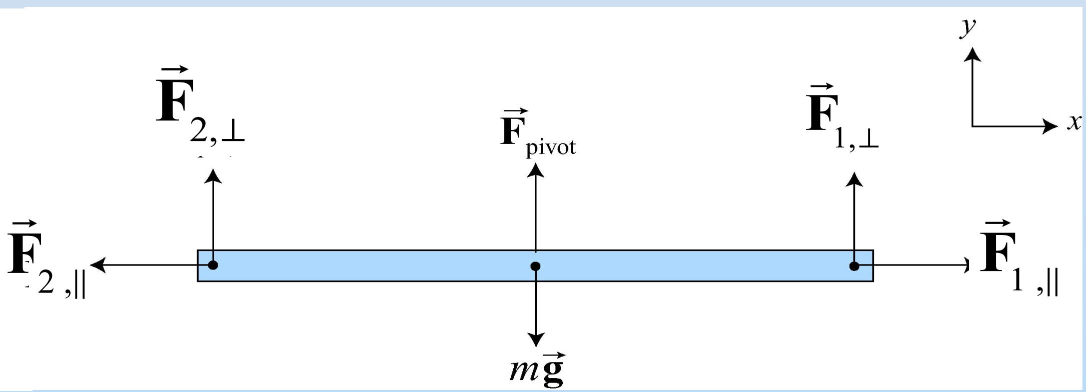
Lever Law: $d_1 m_1 g = d_2 m_2 g$

Generalized Lever Law



$$\vec{\mathbf{F}}_1 = \vec{\mathbf{F}}_{1,\parallel} + \vec{\mathbf{F}}_{1,\perp}$$

$$\vec{\mathbf{F}}_2 = \vec{\mathbf{F}}_{2,\parallel} + \vec{\mathbf{F}}_{2,\perp}$$



$$F_{1,\perp} = F_1 \sin\theta_1$$

$$F_{2,\perp} = F_2 \sin\theta_2$$

$$\vec{\boldsymbol{\tau}}_{cm}^{\,\text{total}} = \vec{\boldsymbol{\tau}}_{cm,1} + \vec{\boldsymbol{\tau}}_{cm,2} = \vec{\mathbf{0}} \quad \rightarrow (d_1 F_{1,\perp} - d_2 F_{2,\perp})\hat{\mathbf{k}} = 0$$

$$\boxed{d_1 F_{1,\perp} = d_2 F_{2,\perp}}$$

Problem Solving Strategy

Force:

1. Identify System and draw all forces and where they act on Free Body Force Diagram

2. Write down equations for static equilibrium of the forces: sum of forces is zero

Torque:

1. Choose point to analyze the torque about.

2. Choose sign convention for torque

3. Calculate torque about that point for each force. (Note sign of torque.)

4. Write down equation corresponding to condition for static equilibrium: sum of torques is zero

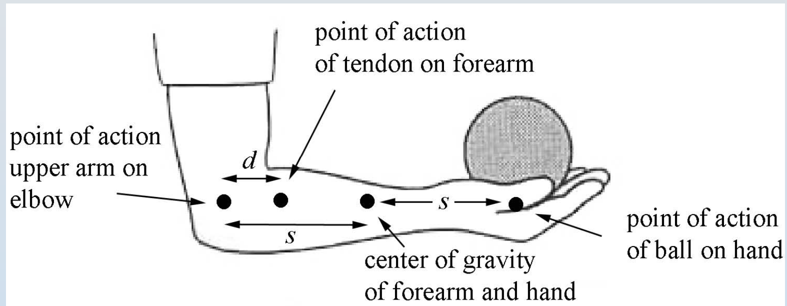
Checkpoint Problem: Lever Law

Suppose a beam of length s = 1.0 m and mass m = 2.0 kg is balanced on a pivot point that is placed directly beneath the center of the beam. Suppose a mass m_1 = 0.3 kg is placed a distance d_1 = 0.4 m to the right of the pivot point. A second mass m_2 = 0.6 kg is placed a distance d_2 to the left of the pivot point to keep the beam static.

(1) What is the force that the pivot exerts on the beam?

(2) What is the distance d_2 that maintains static equilibrium?

Problem: Forearm



You are able to support a ball of mass m_b when you hold out your arm in an outstretched horizontal position thanks to the action of the tendon on the forearm. The forearm and hand have a mass m. The center of mass of the forearm and hand is a distance s from the point where the upper arm meets the elbow, the tendon attaches to the forearm a distance d from the point where the upper arm meets the elbow, and the angle the tendon makes with the horizontal is α, and the ball is a distance 2s from the point where the upper arm meets the elbow. The upper arm exerts an force on the forearm. The direction and magnitude of this force depends on the other parameters (that are fixed) m, d, α , s, and m_b

a) Draw a free body diagram for all the forces that are acting on the forearm and hand. Indicate on your free body diagram your choice of unit vectors.

b) Choose a point about which to calculate the torques acting on the forearm and hand. Explain why you decided on that point.

c) What is the tension T in the tendon?

Checkpoint Problem: Person on Ladder

A person of mass m is standing on a rung one-third of the way up a ladder of length d . The mass of the ladder, which is uniformly distributed, is m_1. The ladder is initially inclined at an angle θ with respect to the horizontal. Assume that there is no friction between the ladder and the wall but that there is friction between the base of the ladder and the floor with a coefficient of static friction μ_s. Find an expression for the minimum value of the coefficient of static friction necessary so that the ladder does not slip. Let g be the gravitational acceleration.