Let's now go to problem 10.9.

This is a classic I have here-- an angle alpha, and here an angle beta, a rope, a mass m2, a mass m1, and here I have a pulley. The pivot of the pulley is frictionless, but the pulley itself has a radius R, and it has a certain mass, so it has a moment of inertia in rotating about its center C. There's no slip on the pulley, so I can again use that alpha, which is omega dot for the pulley, equals a divided by R at any moment in time.

I have three options, three possibilities: either a on the right side is down, or a on the right side is up, or a is 0. It is by no means obvious to predict what will happen. You will have to know m1, you have to know numerical values for m2, for alpha, for beta, for the kinetic friction coefficient, and for the static fiction coefficient to decide whether it will accelerate like this, or like this, or whether it will not accelerate at all.

Let us try to make diagrams. This force is m2 g, then there is T2, and now I'm going to make the assumption that it is going in this direction, accelerated in that direction. If that's the case, then the frictional force must be upwards, because it's sliding down. I can decompose the force of gravity in this direction, which is m2 g sine beta, and I can decompose in this direction, which is m2 g cosine theta. The normal force N2 equals exactly a magnitude the same equal to m2 g cosine theta.

I can do exactly the same here, except that if the object is accelerated downwards, that the frictional force here is in this direction. Here I get m1 g, I get here tension T1, I get here a normal force N1, which is m1 times g times cosine alpha-- in a similar way, I did that here-- I now get a frictional force, which is a maximum value here. It's in this direction because the object is going to be accelerated in this direction-- that has been my assumption-- and then there is the component of gravity, which is m1 g sine theta. I actually should have but this one in blue, because it is a component of the real gravity. Only the red forces are the ones that are the forces acting upon it, but this one can be replaced by these two, since with each of these we're only dealing with this one, this one, and this one.

Now we are ready to set up our differential equations-- quite a job, actually. For object number one, I have T1, and I call that the positive direction, minus this one, which is m1 g sine alpha minus mu times m1 g cosine alpha, and that must be m1 times a-- that's the acceleration along the slope. For object number two-- you can check this now-- m2 g sine beta, and this one is not obviously the positive direction. If this is the positive direction, this is also the positive direction, and all the other forces are negative-- T2 is up, so that's negative, minus mu times m2 g cosine beta equals m2 times a. This is my equation number one, and this is my equation number two.

I now have here the pulley: point C, radius r, the pulley experiences here a tension T1, and it experiences here a tension T2. I can write down for the pulley that T2 minus T1 times R-- it is the torque, and T2 is clockwise, so it's positive, and T1 is counterclockwise, so that's negative. That is I c times alpha-- I about this axis of rotation times alpha, but I'm going to get rid of my alpha-- so it is I relative to point C times a divided by R, and this is my third equation. I realize that every time where I have mu, I should really have written mu k, but I'm a little lazy, so I write down simply mu.

Notice that I have three equation with three unknowns: I have T1, I have T2, and I have a, and so I can solve. The solution is not all that difficult, except that now-- and this is crucial-- my a must be positive. If my a is not positive, then my whole solution is wrong, because the directions of the frictional force I have adjusted in such a way that it's only correct if the [UNINTELLIGIBLE] acceleration is that m2 is accelerated downwards. You solve for a, and a better be larger than 0-- if a is not larger than 0, then your solution is wrong.

I massaged it a little bit for you: when a is larger than 0, you can express that in terms of a mu if you want to-- it's nothing special, it's nothing new, and it's the same thing-- and mu, then, must be smaller than m 2 sine beta minus m1 sine alpha divided by m2 cosine beta plus m1 cosine alpha. That is only, then, the case if a is downstairs. This must be met.

Let's assume that the acceleration is in this direction. I now have to redo the whole problem, and I can leave all the forces as they are, with two exceptions: I must flip over the frictional force of 1. This frictional force now is in this direction, and this frictional force is now in this direction. I have to rewrite the differential equations accordingly, and I have to solve again for a. It better be now so that a is larger than 0-- otherwise, my assumption would have been wrong, and my frictional forces will be in the wrong direction.

When I do that, I find a different criterion for mu-- I've expressed it in terms of mu. Now I find m1 sine alpha minus m2 sine beta, and it shouldn't surprise you that the upstairs changes sine, and the downstairs is exactly the same-- m2 cosine beta plus m1 cosine alpha. In all other cases-- this is not the case, and that this is not the case-- a equals 0, and the frictional force will be, in general, smaller then the maximum frictional force, and could even be 0.

I have chose a particular example. I have chosen alpha equal 30 degrees, I've chosen beta equals 5 degrees, m1 equals 3 kilograms, m2 equals 10 kilograms, and I have chosen for mu-- which is really the kinetic friction coefficient-- 0.04. The question now is it being accelerated like this only on two sides, or like this, only on two sides, or does it equals 0? What I did was-- I said, let's calculate m 1 sine alpha, let's calculate m 2 sine beta, let's calculate m 1 cosine alpha, and let's calculate m2 cosine beta. I substitute these numbers in, and if I didn't make a mistake, I find plus 1.5, 0.87, 2.60, and 9.96.

Let's first evaluate the situation as to whether perhaps a is in this direction. In other words, is mu smaller then 1.5 minus 0.7 divided by 2.6 plus 9.96? The answer is yes, because this answer is 0.05, and the actual friction coefficient is 0.04, and since 0.04 is indeed smaller than 0.05, the acceleration is nonnegotiable-- it will be in this direction.

Is it possible somehow that if we adjust mu that a would be in this direction? The answer is no, because you look at our criterion of what mu should be for a to go down, and then you would need 0.07 minus 1.5 divided by 2.6 plus 9.96. This is smaller than 0, this is negative, and so this has no physical meaning. This is not meaningful.

If you take mu static equals 0.1, the only conclusion I must draw is a must be 0, because I don't meet either of my conditions. I don't meet the mu condition for acceleration going downhill, nor do I meet the mu condition for a going uphill. I want you to work on this a little bit, and I'll help you.

If the pulley is not rotating-- T1 equals T2 equals T that you can use, and I want you to calculate now what the frictional force 1 is, what the frictional force 2 is, and what T is. What you may find to your great surprise is that this combination is not unique-- nature has several possibilities on doing this. In other words, there are several values of T that are allowed that meet the condition that a is 0. The only thing that I can tell you which may interest you is that you will always find that the frictional force 1 plus the frictional force 2 is always the same, and I believe if I don't have that mistaken, that it is always 6.3, but you better check that. Nature has various ways of solving this problem-- it's not one unique answer, and not so intuitive.

Now I have an interesting problem for you, and I want you to try this at home. This is a yo-yo, and the yo-yo has an inner radius R1, and it has outer radius R2. I attach to his yo-yo a string, which is wrapped around the inner core. I'm going to pull, and I'm going to pull at an angle alpha. If alpha is small enough, the yo-yo will come in this direction, and if alpha is large enough, the yo-yo will move away from me in his direction.

This is by no means so obvious, but I will show it to you. I want you to prove it at what angle, depending upon R1 and R2, this is going to happen. I have here a box, I have here a yo-yo, and I'm going to pull at a very small angle alpha. What do you see? The yo-yo moves in my direction.

I now am going to increase the angle alpha substantially, and I'm going to pull it again-- what do you see? It moves away from me. Isn't that interesting? Depending upon the angle alpha, the yo-yo comes to you, or goes away from you. There has to be friction here.

You can do it in your dormitory, you can do it on the floor, and you'll be amazed, but what is more important than anything else-- what is that angle of alpha at which the transitional curves from moving towards me to moving away from me?

I'm going to watch my tape now-- I have one minute and five seconds left, and I'll take out all the slips of the tongue and all the slips of the pen. It's over for now, but not yet out.

In problem 10.9, I don't know why, but all of a sudden in the drawing-- I introduce an angle theta, which of course shouldn't be that at all. Twice will you see in the drawing that I say cosine theta, which should in both cases be cosine beta. Once I say sine theta in the drawing, which would be sine alpha. In the algebra, I did it correctly, so there's nothing wrong with the solution-- it's only the drawing which is not quite kosher. That's all I have to add-- honestly, I think you will easily be able to correct for yourself.