Problem 1 Solution

First of all 420 $\sim$ 440 feet correspond to circa $h = 130 \pm 3$ m. The sea area is more or less .7of the total surface of the earth that is :

$A_{sea} \sim .7 \cdot 4\pi R^2 \sim 3.6\ 10^{14} m^2$

where we used for the earth radius $R = 6.4\ 10^6 m$.

So the volume of water is $V \sim A \cdot h = 4.6\ 10^{16} m^3$ As the density of ice is 0.92 of the density of water the corresponding volume of ice will be $5.0\ 10^{16} m^3$

The volume of ice actually present around the north pole is instead circa $2.5\ 10^{15} m^3$and can be neglected in what follows

Assume an ice cap of mean thickness $2\ 10^3 m$ (Greenland average) we get that the area of the ice cap around the north pole will be:

$A_{ice} = .7 \cdot \frac{1}{2} A_{earth} \frac{1.3 \cdot 10^2}{2 \cdot 10^3} \cdot \frac{1}{0.92} \sim \frac{2.5}{100}\ A_{earth}$

So the colatitude θ of the southern edge of the north pole will be such that: $\frac{1}{2}(1 - \cos\theta) \sim 2.5\ 10^{-2}$ that is: $\theta \sim 18^0$.

Problem 2

a) The speed of both stones along an axis pointing downwards will increase linearly with time according to:

$v_1 = g(t - t_1) \quad v_2 = g(t - t_2)$

where $t_1$and $t_2$are the release times.

Then $v_1 - v_2 = g(t_2 - t_1)$is constant.

b) The distance travelled by the two stones from the starting point along the downward axis is:

$s_1 = \frac{1}{2} \cdot g(t - t_1)^2 \quad s_2 = \frac{1}{2} \cdot g(t - t_2)^2$

so that their distance will increase linearly in time according to: $s_1 - s_2 = \frac{1}{2} \cdot g(t_1^2 - t_2^2 + 2t(t_1 - t_2))$

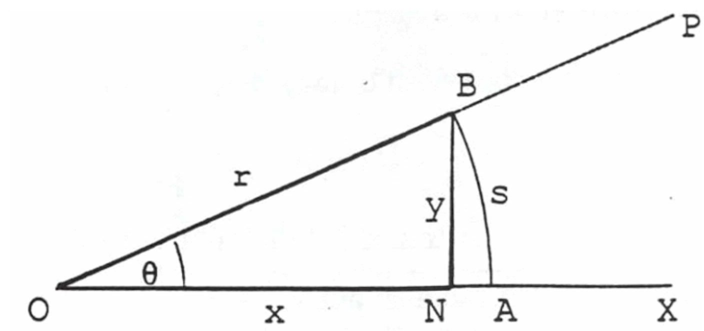
c) As the time the stones will employ to fall is the same, the time interval at which they hit the ground will be $t_1 - t_2$. More formally supposing the ground to be at a distance $s_0$from the release point the hitting times for the two stones will be:

$t_{H1} = t_1 + \sqrt{\frac{2s_0}{g}} \quad t_{H2} = t_2 + \sqrt{\frac{2s_0}{g}}$

Problem 3: ***Radians and Estimation (see section 1.11 in course notes)***

a) **Hold a dime at arm's length. What angle in radians is subtended by the diameter of the dime?**

When I hold a dime at arm's length, the distance from my eye to my fingertips is $x \cong 7.1\times10^{1}\ cm$ (see figure below.) The diameter of the dime is $y \cong 1.8\,cm$.



The radian measure of θ is given by

$$\theta = s/r \cong y/x = 1.8\,cm/7.1\times10^{1}\ cm = 2.5\times10^{-2}\ rad\ .$$

b) **Using your result from part a), estimate the length of the infinite corridor at MIT. In order to do this, choose a reference height at one end of the corridor and estimate its height. Then go to the other end of the corridor and measure what fraction of the diameter of the dime corresponds to your reference height. You can now calculate the length of the corridor by using similar triangles. The length is published by MIT. Can you find the published value?**

I used a doorway at the end of the infinite corridor (building 8) as a reference height which I measured to be $h = 9.0\ ft = 2.7\,m$. I measured the size of this height from the other end of the corridor compared to the dime and found it was approximately $1/2$ of the diameter of the dime or $y \cong 1cm$. Therefore the angular diameter of the door is approximately

$$\theta \cong y/x = 1cm/7.1\times10^{1}\ cm = 1.4\times10^{-2}\ rad \cong 1\times10^{-2}\ rad\ .$$

The length of the infinite corridor is then

$$d \cong h/\theta \cong \left(2.7\,m/1\times10^{-2}\ rad\right) \cong 2.7\times10^{2}\ m\ .$$

In the MIT Bulletin 1999-2000, pp.10 , the published result for the length of the infinite corridor is $8.25\times10^{2}\ ft = 2.51\times10^{2}\ m$. So our estimate is $\left(2.7\times10^{2}\ m\right)/\left(2.7\times10^{2}\ m\right) \cong 1.1$ of the actual value.

c) **Now use your dime and wait until the moon is out (the moon is new on Sept. 14 so you need to make a measurement as quickly as possible on the quarter moon) to try an estimate the angular diameter of the moon. Once you have this estimate, what additional information would you need in order to estimate the mass of the moon? Make some estimates regarding these additional quantities and then estimate the mass of the moon. Look up the actual value and compare it with your estimate. How did you do?**

I measured the moon and found the diameter of the moon corresponded to approximately $1/2$ of the diameter of the dime or $y \cong 1cm$. Note that this means that if you stand at one end of the infinite corridor and look at the sun or moon it will entirely fill the doorway. Try it out. There is one day a year when the sun sets exactly in front of the infinite corridor. Therefore the angular diameter of the moon is

$$\theta \cong y/x = 1cm/7.1\times10^{1}\,cm = 1.4\times10^{-2}\,rad \cong 1\times10^{-2}\,rad\ .$$

I estimate the distance from the earth to the moon to be approximately $R_{e,m} \cong 3\times10^{8}\,m$. The diameter of the moon is then

$$D = R_{e,m}\theta \cong \left(3\times10^{8}\,m\right)\left(1.4\times10^{-2}\,rad\right) = 4\times10^{6}\,m\,.$$

The volume of the moon is

$$V_m = (4/3)\pi(D/2)^3 = (\pi/6)D^3 \cong (\pi/6)\left(4\times10^{6}\,m\right)^3 \cong 3\times10^{19}\,m^3\,.$$

I estimate the density of the moon to be $\rho_m \cong 5\times10^{3}\,kg-m^{-3}$, similar to the density of rocks found on Earth. The mass of the moon is then

$$m_m = \rho_m V_m = \rho_m(\pi/6)D^3 \cong \left(5\times10^{3}\,kg-m^{-3}\right)(\pi/6)\left(4\times10^{6}\,m\right)^3 \cong 1.7\times10^{23}\,kg \cong 1\times10^{23}\,kg$$

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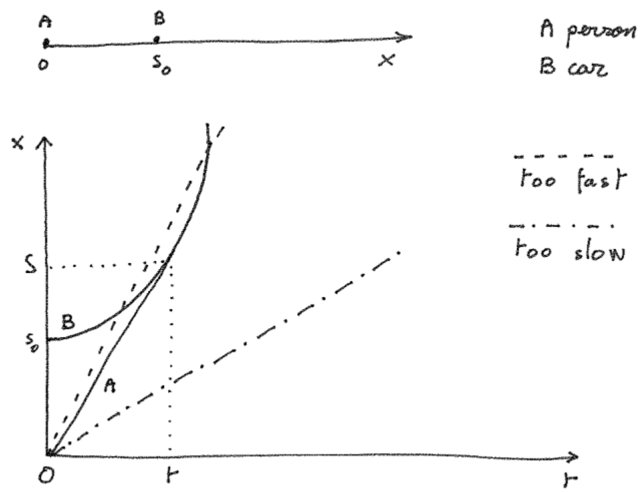
The accepted standard value for the mass of the moon is $7.35\times10^{22}\,kg$. So our estimate was

$1\times10^{23}\,kg/7\times10^{22}\,kg \cong 1.4$ times the accepted value. Not bad!!!

Problem 4

a) The following picture shows the ccordinate axis we have chosen along the direction of motion $\hat{x}$.

The person starts at the origin while the car starts at $x = s_0 = 20m$. Moreover the graph represents the position along the x axis in function of time for the car and the person. It also shows that if the speed of the person is too high there will be two intersection points between the two functions. If the speed of the person is just sufficient to hop on the car there will be only 1 intersection point and the two functions will be tangent at (t, S) that is when the person hops on the car.



b) When the person hops on the car the following equation is satisfied (v is the person speed a is the car acceleration):

$s_0 + \frac{1}{2}at^2 = v$t

As the person is just able to hop on the car there must be only one solution to this equation that is

$\Delta = v - 2as_0^2 = 0$ must be satisfied.

This gives:$v = \sqrt{2as_0} = 6\ ms^{-1}$. The only solution to the quadratic equation will be: $t = \sqrt{\frac{2s_0}{a}} = 6.67\ s$.

The person will have run for a distance $S = vt = 2s_0 = 40\ m$.

Problem 5

a) The derivative of the constant pieces is zero while it diverges where the function jumps so that the correct answer is 2

b) The integral of the function from zero must start from zero quadratically as the sin is linear near zero, the answer is then 2) and in fact
$\int_0^x dx \sin(x) = 1 - \cos(x)$

c) The derivative of the function is constant and circa 2 on the rising slopes and constant ~ -2 on the decreasing slopes so the answer is 1)

Problem 6

The speed of the ball in the upward vertical direction just leaving the photogate is: $v_v = \frac{D}{\Delta T}\sin\theta$, while the speed horizontal component is:$v_h = \frac{D}{\Delta T}\cos\theta$.

The ball will reach the maximum height after a time $t_1 = \frac{v_v}{g}$. At that time its height above the ground will be: $H = h + \frac{v_v^2}{2g}$.

So the ball will reach the ground after a time: $t_2 = t_1 + \sqrt{\frac{2H}{g}} = t_1 + \sqrt{\frac{2h}{g} + t_1^2}$covering in this time an horizontal distance equal to:

$S = v_h t_2 = \frac{D}{\Delta T}\cos\theta(\frac{D}{g\Delta T}\sin\theta + \sqrt{\frac{2h}{g} + \frac{D^2}{g^2\Delta T^2}\sin^2\theta})$