MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01T **Fall Term 2004**

Problem Set 11: Angular Momentum, Rotation and Translation

Available on-line November 12; Due: November 23 at 4:00 p.m.

Please write your name, subject, lecture section, table, and the name of the recitation instructor on the top right corner of the first page of your homework solutions. Please place your solutions in your lecture section table box.

Nov 12

Hour One: Problem Solving Session 16: Angular Momentum

Problem Set 10: Due Tues Nov 16 at 4:00 pm.

Nov 15

Hour One: Experiment 9: Angular Momentum
Reading: Experiment 9

Hour Two: Planetary Motion Reading Class Notes: Planetary Orbits: The Kepler Problem, Energy Diagrams
Readings: YF 7.1, 7.5, 12.3-12.5

Nov 17

Hour One: Problem Solving Session 17: Rotation and Translation Galactic Black Hole
Reading: YF 10.3

Hour Two: Test Review

Nov 18 QUIZ THREE: Energy, Momentum, and Rotational Motion 7:30-9:30 pm

Nov 19 No Class

Problem Set 11: Due Tues Nov 23 at 4:00 pm.

Nov 22

Hour One: Kinetic Theory
Reading: YF 18.1-18.6

Hour Two: Problem Solving Session 18: Ideal Gas Law
Reading: YF 18.1-18.6

Nov 24
Hour One: Archimedes Principle
Reading: YF 14.1-14.3

Hour Two: Archimedes Principle PRS Contest

Nov 26 No Class
Problem Set 12: Due Fri Dec 3 at 4:00 pm.

Problem 1 : ***Bohr hydrogen atom***

The Bohr hydrogen atom models the atom as an electron orbiting the proton under the influence of an electric force producing uniform circular motion with radius a_0. The mass of the electron is $m_e = 9.1\times 10^{-31}\, kg$; the electric charge is $e = 1.6\times 10^{-19}\, C$; the Planck constant is $h = 6.63\times 10^{-34}\, J-s$, and the magnitude of the electric force is given by Coulomb's Law $\left|\vec{F}\right| = ke^2/r^2$ where $k = 9.0\times 10^9\, N-m^2/C^2$. The angular momentum is quantized according to $L = nh/2\pi$ where $n = 1, 2, \ldots$

a) Write down the equation that arises from the application of Newton's 2nd Law to the electron.

b) What is the angular momentum of the electron about the center of the atom?

c) Using your results from parts a) and b) derive an equation for the radius a_0 of the atom as a function of n, e, h, m_e, and k.

d) What is the energy E_n for the atom? Express your answer in terms of n, e, h, m_e, and k.

e) A hydrogen atom emits a photon which arise from an energy transition from the $n = 3$ to $n = 1$ energy level. Calculate the frequency of the light emitted

$$f = -\Delta E/h = -\left(E_1 - E_3\right)/h\,.$$

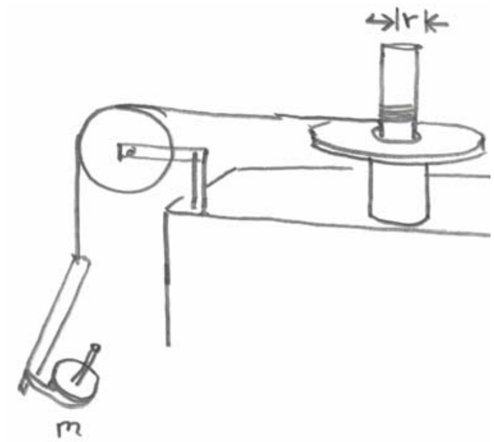
f) What is the wavelength of the emitted light from part e)?

Problem 2: ***Experiment 11 Angular Momentum: Analysis***

Part One: Rotor Moment of Inertia

Enter the results from your experiment report into the table below.

α_{down} $\left[rad \cdot s^{-2}\right]$	α_{up} $\left[rad \cdot s^{-2}\right]$	a $\left[m \cdot s^{-2}\right]$	T $[N]$	τ_{up} $[N \cdot m]$



Note: The rotational dynamics for the two stages are given by

$$RT - \tau_f = I\alpha_{up}$$

$$-\tau_f = I\alpha_{down}$$

Note that $\alpha_{down} < 0$.

The first two equations above imply that

$$RT + I\alpha_{down} = I\alpha_{up} .$$

The force equation for the first stage is given by

$$mg - T = ma_{up} .$$

The linear acceleration and angular acceleration are constrained by

$$a_{up} = R\alpha_{up} .$$

Combining these last two equations and solving for the tension yields

$$T = mg - m\alpha R_{up}$$

Substituting the tension into the combined torque equation yields

$$(mg - m\alpha_{up}R)R + I\alpha_{down} = I\alpha_{up}$$

We can solve this for the moment of inertia

$$I = \frac{mgR - m\alpha_{up}R^2}{\alpha_{up} - \alpha_{down}} = \frac{mgR - m\alpha_{up}R^2}{\alpha_{up} + |\alpha_{down}|}$$

(Here $m = 0.055$ kg is the mass of the weight, $r = 0.0127$ m, α_{up} and α_{down} are obtained from your measurements).

What is your numerical value for I_R ?

Part Two: Inelastic Collision:

Write your measurement results into the table below.

$\omega_1\left[\mathrm{rad\cdot s^{-1}}\right]$	$\omega_2\left[\mathrm{rad\cdot s^{-1}}\right]$	$\delta t\ [\mathrm{s}]$

What is your numerical value for $I_W = \frac{1}{2}m_w\left(r_0^{\ 2} + r_i^{\ 2}\right)$?

1. Use the moments of inertia I_R and I_W along with ω_i and ω_f to calculate the angular momentum before and after the collision and compare them.

2. Use the values you found for the friction torque τ_f and δt to estimate the angular impulse of τ_f during the collision. Compare it to the angular momentum difference that you just calculated.

3. Calculate the rotational kinetic energies $K_1 = \frac{1}{2}I_R w_1^2$, before, and $K_2 = \frac{1}{2}(I_R + I_W)w_2^2$, after the collision.

Part Three: Slow Inelastic Collision:

Write your measurement results into the table below.

$\omega_1\left[\mathrm{rad}\cdot\mathrm{s}^{-1}\right]$	$\omega_2\left[\mathrm{rad}\cdot\mathrm{s}^{-1}\right]$	$\delta t\left[\mathrm{s}\right]$	$\alpha_c\left[\mathrm{rad}\cdot\mathrm{s}^{-2}\right]$

1. Use the moments of inertia I_R and I_W along with ω_1 and ω_2 to calculate the angular momentum before and after the collision and compare them.

2. Use the value you found for the friction torque τ_f and δt to estimate the angular impulse of τ_f during the collision. Compare it to the difference in angular momentum before and after the collision.

3. Use the value you found for the angular acceleration during the collision, α_c, to estimate the total torque τ_f on the rotor during the collision.

4. The torque τ_c is made of two parts: the friction torque τ_f from the bearings and the torque τ_{RW} due exerted by the washer you dropped on the rotor. By the 3rd law, the rotor exerts an equal and opposite torque on the washer. Since you know τ_f, subtract it from τ_c to find an estimate for τ_{RW}.

Part Four: Nonconservative Work in the Slow Collision

You can find τ_{RW} a different way, because it produces the angular acceleration of the dropped washer, whose average value is $\alpha_W = \omega_2/\delta t$. Use this relation to estimate τ_{RW} and compare it to the previous estimate you made:

The torque τ_{RW} comes from the sliding friction between the washer on the rotor and the washer you dropped. Thus there must be some non-conservative work. You may calculate it if you know the angular "distance" the washer slides over the rotor before it reaches the same angular velocity as the rotor, and you do have enough information to find that. The angle the rotor rotates through during the collision is (analogous to linear motion with constant acceleration)

$$\Delta\theta_R = \omega_1 \delta t - \frac{1}{2}|\alpha_c|\delta t^2$$

while the dropped washer rotates through an angle

$$\Delta\theta_W = \frac{1}{2}|\alpha_W|\delta t^2.$$

Thus the non-conservative work done by the sliding friction between the two washers will be

$$W_{NC,W} = \tau_{RW}(\Delta\theta_R - \Delta\theta_W).$$

You can also calculate the non-conservative work done by the bearing friction during the collision

$$W_{NC,B} = \tau_f \Delta\theta_R.$$

Calculate these two amounts of non-conservative work and compare their sum to the change in rotational kinetic energy $\frac{1}{2}I_R\omega_1^2 - \frac{1}{2}(I_R + I_W)\omega_2^2$ during the collision.