MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL **Fall Term 2003**

Exam 2: Equation Summary

One Dimensional Kinematics:

$\vec{\mathbf{v}} = d\vec{\mathbf{r}} / dt$, $\vec{\mathbf{a}} = d\vec{\mathbf{v}} / dt$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t')dt' \qquad x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t')dt'$$

Constant Acceleration:

$$x(t) = x_0 + v_{x,0}(t - t_0) + \frac{1}{2}a_x(t - t_0)^2 \qquad v_x(t) = v_{x,0} + a_x(t - t_0)$$

$$y(t) = y_0 + v_{y,0}(t - t_0) + \frac{1}{2}a_y(t - t_0)^2 \qquad v_y(t) = v_{y,0} + a_y(t - t_0)$$

where $x_0, v_{x,0}, y_0, v_{y,0}$ are the initial position and velocities components at $t = t_0$

Newton's Second Law: Force, Mass, Acceleration

$$\vec{\mathbf{F}} \equiv m\vec{\mathbf{a}} \quad \vec{\mathbf{F}}^{total} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 \quad F_x^{\ total} = ma_x \quad F_y^{\ total} = ma_y \quad F_z^{\ total} = ma_z$$

Newton's Third Law:

$\vec{\mathbf{F}}_{1,2} = -\vec{\mathbf{F}}_{2,1}$

Force Laws:

Universal Law of Gravity: $\vec{\mathbf{F}}_{1,2} = -G\dfrac{m_1 m_2}{{r_{1,2}}^2}\hat{\mathbf{r}}_{1,2}$, attractive

Gravity near surface of earth: $\vec{\mathbf{F}}_{grav} = m_{grav}\vec{\mathbf{g}}$, towards earth

Contact force: $\vec{\mathbf{F}}_{contact} = \vec{\mathbf{N}} + \vec{\mathbf{f}}$, depends on applied forces

Static Friction: $0 \le f_s \le f_{s,\max} = \mu_s N$ direction depends on applied forces

Kinetic Friction: $f_k = \mu_k N$ opposes motion

Hooke's Law: $F = k\left|\Delta x\right|$, restoring

Kinematics Circular Motion:

arc length: $s = R\theta$; **angular velocity:** $\omega = d\theta/dt$ **tangential velocity:** $v = R\omega$; **angular acceleration:** $\alpha = d\omega/dt = d^2\theta/dt^2$; **tangential acceleration** $a_\theta = R\alpha$.

Period: $T = \dfrac{2\pi R}{v} = \dfrac{2\pi R}{R\omega} = \dfrac{2\pi}{\omega}$; **frequency:** $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$,

Radial Acceleration: $|a_r| = R\,\omega^2$; $|a_r| = \dfrac{v^2}{R}$; $|a_r| = 4\pi^2 R\, f^2$; $|a_r| = \dfrac{4\pi^2 R}{T^2}$

Center of Mass: $\vec{\mathbf{R}}_{cm} = \dfrac{\sum_{i=1}^{i=N} m_i \vec{\mathbf{r}}_i}{\sum_{i=1}^{i=N} m_i} \to \dfrac{\int_{body} dm\vec{\mathbf{r}}}{\int_{body} dm}$;

Velocity of Center of Mass: $\vec{\mathbf{V}}_{cm} = \dfrac{\sum_{i=1}^{i=N} m_i \vec{\mathbf{v}}_i}{\sum_{i=1}^{i=N} m_i} \to \dfrac{\int_{body} dm\vec{\mathbf{v}}}{\int_{body} dm}$

Torque: $\vec{\boldsymbol{\tau}}_S = \vec{\mathbf{r}}_{S,P} \times \vec{\mathbf{F}}_P$ $|\vec{\boldsymbol{\tau}}_S| = |\vec{\mathbf{r}}_{S,P}||\vec{\mathbf{F}}_P| \sin\theta = r_\perp F = rF_\perp$

Static Equilibrium: $\vec{\mathbf{F}}_{total} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \ldots = \vec{\mathbf{0}}$; $\vec{\boldsymbol{\tau}}_S^{total} = \vec{\boldsymbol{\tau}}_{S,1} + \vec{\boldsymbol{\tau}}_{S,2} + \ldots = \vec{\mathbf{0}}$.

Kinetic Energy: $K = \dfrac{1}{2}mv^2$; $\Delta K = \dfrac{1}{2}mv_f{}^2 - \dfrac{1}{2}mv_0{}^2$

Work: $W = \int_{r_0}^{r_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$; **Work-Change in Kinetic Energy:** $W^{total} = \Delta K$

Power: $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = dK/dt$

Potential Energy: $\Delta U = -W_{conservative} = -\int_A^B \vec{\mathbf{F}}_c \cdot d\vec{\mathbf{r}}$

Potential Energy Functions with Zero Points:

Constant Gravity: $U(y) = mgy$ with $U(y_0 = 0) = 0$.

Inverse Square Gravity: $U_{gravity}(\mathrm{r}) = -\dfrac{Gm_1m_2}{r}$ with $U_{gravity}(\mathrm{r}_0 = \infty) \equiv 0$.

Hooke's Law: $U_{spring}(x) = \dfrac{1}{2}kx^2$ with $U_{spring}(x = 0) \equiv 0$.

Work- Change in Mechanical Energy: $\vec{\mathbf{F}}^{total} = \vec{\mathbf{F}}_c{}^{total} + \vec{\mathbf{F}}_{nc}{}^{total}$;

$W^{total} = W_c{}^{total} + W_{nc}{}^{total} = \Delta K$; $W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech}$, $E_{mech} = K + U^{total}$

$W_{nc} = \Delta E_{mech} = E_f - E_0 = \left(K_f + U_f{}^{total}\right) - \left(K_0 + U_0{}^{total}\right)$