MASSACHUSETTS INSTITUTE OF TECHNOLOGY
Department of Physics

Physics 8.01 TEAL **Fall Term 2004**

Final Exam: Equation Summary

First Law of Thermodynamics:
$\Delta U \equiv U_f - U_i = -W_{i\to f} + Q_{i\to f}$

Thermistor Calibration:
$R(T) = R_0 e^{-\alpha T}$
$T = ln(R_0/R)/\alpha$

Mechanical Equivalent of Heat:
$(dE_{mech}/dt) = -k(dQ/dt)$
$(dQ/dt) = cm(dT/dt)$
Power: $P = \tau\omega \quad P = \Delta VI$
Specific Heat:
$c_{H_2O} = 1 cal/g - {}^0 C$
$1 cal = 4.186 J$

Ideal Gas Law:
$PV = n_m RT = NkT$
$P_{pressure} = (1/3)\rho(v^2)_{ave}$

Equipartition of Energy:
$U = \frac{(\#\ of\ degrees\ of\ freedom)}{2} n_m RT = \frac{3}{2} n_m RT$

Constants:
$k = 1.38 \times 10^{-23} J \cdot K^{-1}$
$N_A = 6.022 \times 10^{23}\ molecules \cdot mole^{-1}$
$R = N_A k = 8.31 J \cdot mole^{-1} \cdot K^{-1}$

Pressure:
$P = \frac{dF_\perp}{dA}$
$1 bar = 10^5 Pa$
$1 atm = 1.013 \times 10^5 Pa$

Momentum:
$\vec{\mathbf{p}} = m\vec{\mathbf{v}}$, $\vec{\mathbf{F}}_{ave}\Delta t = \Delta\vec{\mathbf{p}}$, $\vec{\mathbf{F}}_{ext}^{total} = \frac{d\vec{\mathbf{p}}^{total}}{dt}$

Impulse: $\vec{\mathbf{I}} \equiv \int_{t=0}^{t=t_f} \vec{\mathbf{F}}(t)dt = \Delta\vec{\mathbf{p}}$

Torque: $\vec{\boldsymbol{\tau}}_S = \vec{\mathbf{r}}_{S,P} \times \vec{\mathbf{F}}_P$
$|\vec{\boldsymbol{\tau}}_S| = |\vec{\mathbf{r}}_{S,P}||\vec{\mathbf{F}}_P| sin\theta = r_\perp F = rF_\perp$

Static Equilibrium:
$\vec{\mathbf{F}}_{total} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + ... = \vec{\mathbf{0}}$;
$\vec{\boldsymbol{\tau}}_S^{total} = \vec{\boldsymbol{\tau}}_{S,1} + \vec{\boldsymbol{\tau}}_{S,2} + ... = \vec{\mathbf{0}}$.

Rotational dynamics: $\vec{\boldsymbol{\tau}}_S^{total} = \frac{d\vec{\mathbf{L}}_S}{dt}$

Angular Velocity: $\vec{\boldsymbol{\omega}} = (d\theta/dt)\hat{\mathbf{k}}$

Angular Acceleration: $\vec{\boldsymbol{\alpha}} = (d^2\theta/dt^2)\hat{\mathbf{k}}$

Fixed Axis Rotation: $\vec{\boldsymbol{\tau}}_S = I_S\vec{\boldsymbol{\alpha}}$
$\tau_S^{total} = I_S\alpha = I_S\frac{d\omega}{dt}$

Moment of Inertia: $I_S = \int_{body} dm(r_\perp)^2$

Angular Momentum: $\vec{\mathbf{L}}_S = \vec{\mathbf{r}}_{S,m} \times m\vec{\mathbf{v}}$,
$|\vec{\mathbf{L}}_S| = |\vec{\mathbf{r}}_{S,m}||m\vec{\mathbf{v}}| sin\theta = r_\perp p = rp_\perp$

Rotation and Translation:
$\vec{\mathbf{L}}_S^{total} = \vec{\mathbf{L}}_S^{orbital} + \vec{\mathbf{L}}_{cm}^{spin}$,
$\vec{\mathbf{L}}_{cm}^{spin} = I_{cm}\vec{\boldsymbol{\omega}}_{cm}^{spin}$
$\vec{\mathbf{L}}_S^{orbital} = \vec{\mathbf{r}}_{S,cm} \times \vec{\mathbf{p}}^{total}$,
$\vec{\boldsymbol{\tau}}_S^{orbit} = \frac{d\vec{\mathbf{L}}_S^{orbit}}{dt}$ $\vec{\boldsymbol{\tau}}_{cm}^{spin} = \frac{d\vec{\mathbf{L}}_{cm}^{spin}}{dt}$

Rotational Energy:

$$K_{cm} = \frac{1}{2} I_{cm} \omega_{cm}^{2}$$

Rotational Power:

$$P_{rot} \equiv \frac{dW_{rot}}{dt} = \vec{\boldsymbol{\tau}}_S \cdot \vec{\boldsymbol{\omega}} = \tau_S \omega = \tau_S \frac{d\theta}{dt}$$

Angular Impulse:

$$\vec{\mathbf{J}}_S = \int_{t_0}^{t_f} \vec{\boldsymbol{\tau}}_S dt = \Delta \vec{\mathbf{L}}_S = \vec{\mathbf{L}}_{S,f} - \vec{\mathbf{L}}_{S,0}$$

One Dimensional Kinematics:

$\vec{\mathbf{v}} = d\vec{\mathbf{r}} / dt$, $\vec{\mathbf{a}} = d\vec{\mathbf{v}} / dt$

$$v_x(t) - v_{x,0} = \int_{t'=0}^{t'=t} a_x(t') dt'$$

$$x(t) - x_0 = \int_{t'=0}^{t'=t} v_x(t') dt'$$

Constant Acceleration:

$$x(t) = x_0 + v_{x,0}(t - t_0) + \frac{1}{2} a_x (t - t_0)^2$$

$$v_x(t) = v_{x,0} + a_x (t - t_0)$$

$$y(t) = y_0 + v_{y,0}(t - t_0) + \frac{1}{2} a_y (t - t_0)^2$$

$$v_y(t) = v_{y,0} + a_y (t - t_0)$$

where $x_0, v_{x,0}, y_0, v_{y,0}$ are the initial position and velocities components at $t = t_0$

Newton's Second Law:

$\vec{\mathbf{F}} \equiv m\vec{\mathbf{a}}$ $\quad \vec{\mathbf{F}}^{total} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2$

Newton's Third Law: $\vec{\mathbf{F}}_{1,2} = -\vec{\mathbf{F}}_{2,1}$

Force Laws:

Universal Law of Gravity:

$\vec{\mathbf{F}}_{1,2} = -G \dfrac{m_1 m_2}{r_{1,2}^{2}} \hat{\mathbf{r}}_{1,2}$, attractive

Gravity near surface of earth:

$\vec{\mathbf{F}}_{grav} = m_{grav} \vec{\mathbf{g}}$, towards earth

Contact force:

$\vec{\mathbf{F}}_{contact} = \vec{\mathbf{N}} + \vec{\mathbf{f}}$, depends on applied forces

Static Friction:

$0 \le f_s \le f_{s,\max} = \mu_s N$

direction depends on applied forces

Kinetic Friction:

$f_k = \mu_k N$ opposes motion

Hooke's Law:

$F = k\left|\Delta x\right|$, restoring

Kinematics Circular Motion:

arc length: $s = R\theta$;

angular velocity: $\omega = d\theta / dt$

tangential velocity: $v = R\omega$;

angular acceleration: $\alpha = d\omega / dt = d^2\theta / dt^2$;

tangential acceleration $a_\theta = R\alpha$.

Period: $T = \dfrac{2\pi R}{v} = \dfrac{2\pi}{\omega}$;

frequency: $f = \dfrac{1}{T} = \dfrac{\omega}{2\pi}$,

Radial Acceleration:

$\left|a_r\right| = R\,\omega^2$; $\left|a_r\right| = \dfrac{v^2}{R}$;

$\left|a_r\right| = 4\pi^2 R\, f^2$; $\left|a_r\right| = \dfrac{4\pi^2 R}{T^2}$

Center of Mass:

$$\vec{\mathbf{R}}_{cm} = \sum_{i=1}^{i=N} m_i \vec{\mathbf{r}}_i / m^{total} \to \int_{body} dm\vec{\mathbf{r}} / m^{total};$$

Velocity of Center of Mass:

$$\vec{\mathbf{V}}_{cm} = \sum_{i=1}^{i=N} m_i \vec{\mathbf{v}}_i / m^{total} \to \int_{body} dm\vec{\mathbf{v}} / m^{total}$$

Torque: $\vec{\boldsymbol{\tau}}_S = \vec{\mathbf{r}}_{S,P} \times \vec{\mathbf{F}}_P$

$$\left|\vec{\boldsymbol{\tau}}_S\right| = \left|\vec{\mathbf{r}}_{S,P}\right|\left|\vec{\mathbf{F}}_P\right| \sin\theta = r_\perp F = rF_\perp$$

Static Equilibrium:

$\vec{\mathbf{F}}_{total} = \vec{\mathbf{F}}_1 + \vec{\mathbf{F}}_2 + \ldots = \vec{\mathbf{0}}$;

$\vec{\boldsymbol{\tau}}_S^{total} = \vec{\boldsymbol{\tau}}_{S,1} + \vec{\boldsymbol{\tau}}_{S,2} + \ldots = \vec{\mathbf{0}}$.

Kinetic Energy:

$$K = \frac{1}{2}mv^2;\ \Delta K = \frac{1}{2}mv_f^{\,2} - \frac{1}{2}mv_0^{\,2}$$

Work: $W = \int_{r_0}^{r_f} \vec{\mathbf{F}} \cdot d\vec{\mathbf{r}}$;

Work- Kinetic Energy: $W^{total} = \Delta K$

Power: $P = \vec{\mathbf{F}} \cdot \vec{\mathbf{v}} = dK/dt$

Potential

Energy: $\Delta U = -W_{conservative} = -\int_A^B \vec{\mathbf{F}}_c \cdot d\vec{\mathbf{r}}$

Potential Energy Functions with Zero Points:

Constant Gravity:

$U(y) = mgy$ with $U(y_0 = 0) = 0$.

Inverse Square Gravity:

$U_{gravity}(\mathrm{r}) = -\dfrac{Gm_1m_2}{r}$ with

$U_{gravity}(\mathrm{r}_0 = \infty) = 0$.

Hooke's Law:

$U_{spring}(x) = \dfrac{1}{2}kx^2$ with $U_{spring}(x = 0) = 0$.

Work- Mechanical Energy:

$W_{nc} = \Delta K + \Delta U^{total} = \Delta E_{mech}$

Planetary Motion:

Energy:

$$E = (1/2)\mu v^2 - (Gm_1m_2/r)$$

$$E = (1/2)(dr/dt)^2 + U_{effective}$$

$$U_{effective} = \left(L^2/2\mu r^2\right) - \left(Gm_1m_2/r\right)$$

Angular Momentum:

$L = \mu r v_{tangential}$

Orbit Equation:

$$r = \left(r_0 / \left(1 - \varepsilon\cos\theta\right)\right)$$

radius of lowest energy circular orbit

$$r_0 = \left(L^2 / \mu G m_1 m_2\right)$$

eccentricity

$$\varepsilon = \left(1 + \left(2EL^2 / \left(\mu\left(Gm_1m_2\right)^2\right)\right)\right)^{\frac{1}{2}}$$

Equal Area Law:

$$\left(dA/dt\right) = \left(L/2\mu\right)$$

Period Law:

$$T^2 = \frac{4\pi^2 a^3}{G\left(m_1 + m_2\right)}$$